

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ( $g_0 = 1$ ,  $\omega_c = 1$ ,  $N = 1$  to 10)

$N$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$	$g_{11}$
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

be tabulated [1]. Table 8.3 gives such element values for maximally flat low-pass filter prototypes for  $N = 1$  to 10. (Notice that the values for  $N = 2$  agree with the above analytical solution.) This data is used with either of the ladder circuits of Figure 8.25 in the following way. The element values are numbered from  $g_0$  at the generator impedance to  $g_{N+1}$  at the load impedance, for a filter having  $N$  reactive elements. The elements alternate between series and shunt connections, and  $g_k$  has the following definition:

$$g_0 = \begin{cases} \text{generator resistance (network of Figure 8.25a)} \\ \text{generator conductance (network of Figure 8.25b)} \end{cases}$$

$$g_k \quad (k=1 \text{ to } N) = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{load resistance if } g_N \text{ is a shunt capacitor} \\ \text{load conductance if } g_N \text{ is a series inductor} \end{cases}$$

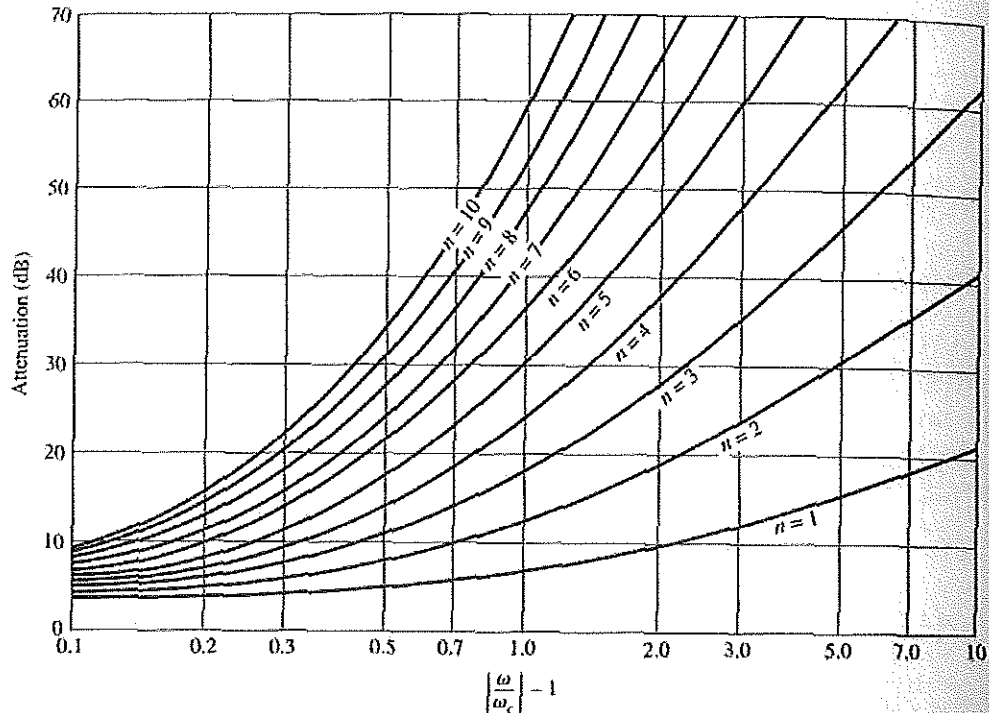
Then the circuits of Figure 8.25 can be considered as the dual of each other, and both will give the same response.

Finally, as a matter of practical design procedure, it will be necessary to determine the size, or order, of the filter. This is usually dictated by a specification on the insertion loss at some frequency in the stopband of the filter. Figure 8.26 shows the attenuation characteristics for various  $N$ , versus normalized frequency. If a filter with  $N > 10$  is required, a good result can usually be obtained by cascading two designs of lower order.



### EXAMPLE 8.3 Low-Pass Filter Design

A maximally flat low-pass filter is to be designed with a cutoff frequency of 8 GHz and a minimum attenuation of 20 dB at 11 GHz. How many filter elements are required?



**FIGURE 8.26** Attenuation versus normalized frequency for maximally flat filter prototypes. Adapted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

#### Solution

We have  $\omega/2\pi = 11$  GHz and  $\omega_c/2\pi = 8$  GHz, so

$$\left| \frac{\omega}{\omega_c} \right| - 1 = \frac{11}{8} - 1 = 0.375.$$

Then from Figure 8.26 we see that an attenuation of 20 dB at this frequency requires that  $N \geq 8$ . Further design details will be discussed in Section 8.4.  $\circ$

#### Equal-Ripple Low-Pass Filter Prototype

For an equal-ripple low-pass filter with a cutoff frequency  $\omega_c = 1$ , the power loss ratio from (8.54) is

$$P_{LR} = 1 + k^2 T_N^2(\omega), \quad 8.61$$

where  $1 + k^2$  is the ripple level in the passband. Since the Chebyshev polynomials have

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ( $g_0 = 1$ ,  $\omega_c = 1$ ,  $N = 1$  to 10, 0.5 dB and 3.0 dB ripple)

0.5 dB Ripple											
$N$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$	$g_{11}$
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

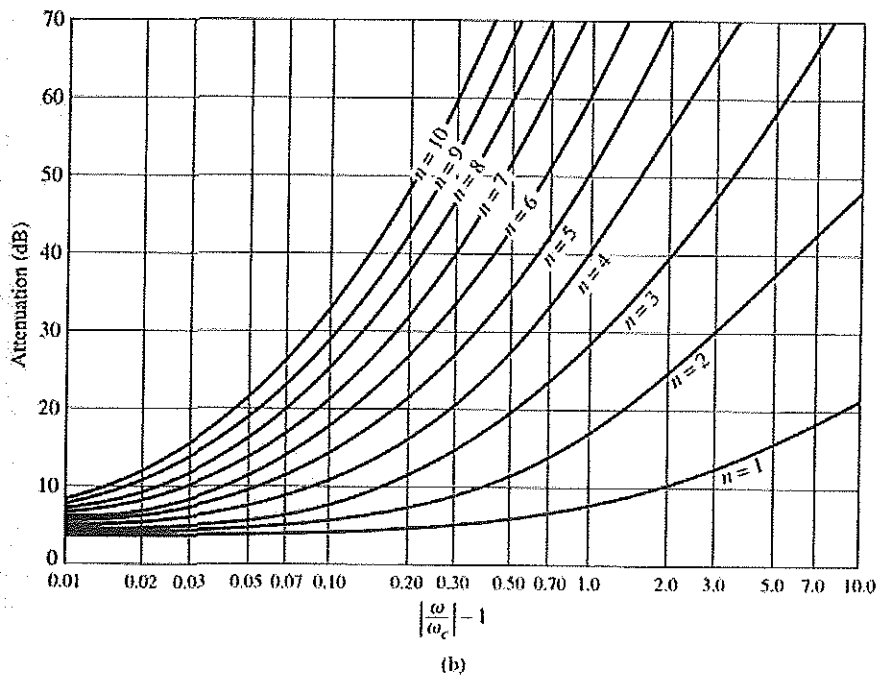
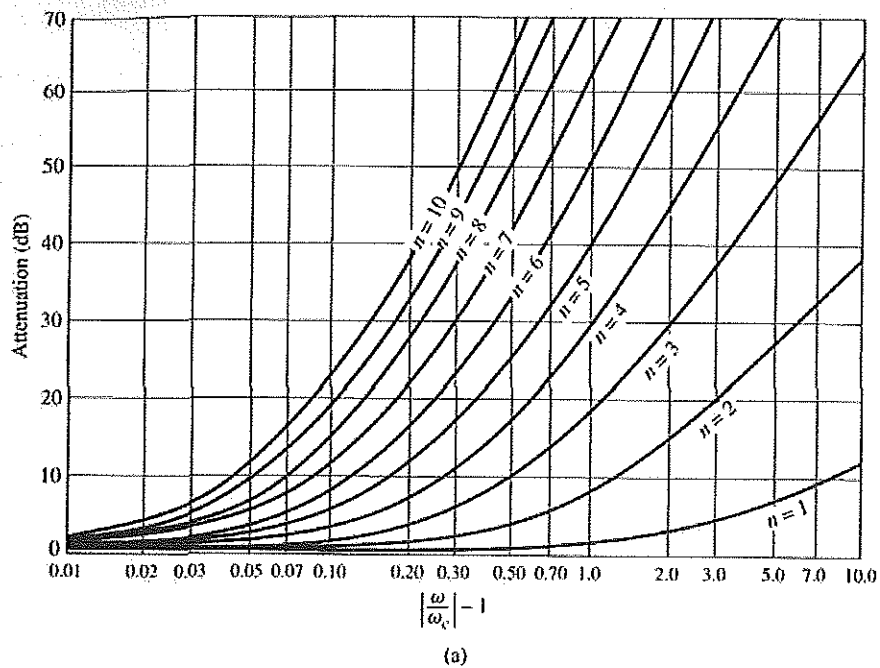
3.0 dB Ripple											
$N$	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$	$g_{11}$
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Source: Reprinted from G. L. Matthaci, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

values have been derived for such filters [1], however, again for the ladder circuits of Figure 8.25, and are given in Table 8.5 for a normalized source impedance and cutoff frequency ( $\omega'_c = 1$ ). The resulting group delay in the passband will be  $\tau_d = 1/\omega'_c = 1$ .

## 8.4 FILTER TRANSFORMATIONS

The low-pass filter prototypes of the previous section were normalized designs having a source impedance of  $R_s = 1 \Omega$  and a cutoff frequency of  $\omega_c = 1$ . Here we show how these designs can be scaled in terms of impedance and frequency, and converted to give high-pass, bandpass, or bandstop characteristics. Several examples will be presented to illustrate the design procedure.



**FIGURE 8.27** Attenuation versus normalized frequency for equal-ripple filter prototypes. (a) 0.5 dB ripple level. (b) 3.0 dB ripple level.

Adapted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.