Symmetric encryption

Symmetric Encryption Scheme

M: message space

C: ciphertext space

K: keyspace

$E: P \times K \rightarrow C$ encryption transformation

$D: C \times K \rightarrow P$ decryption transformation

Two properties

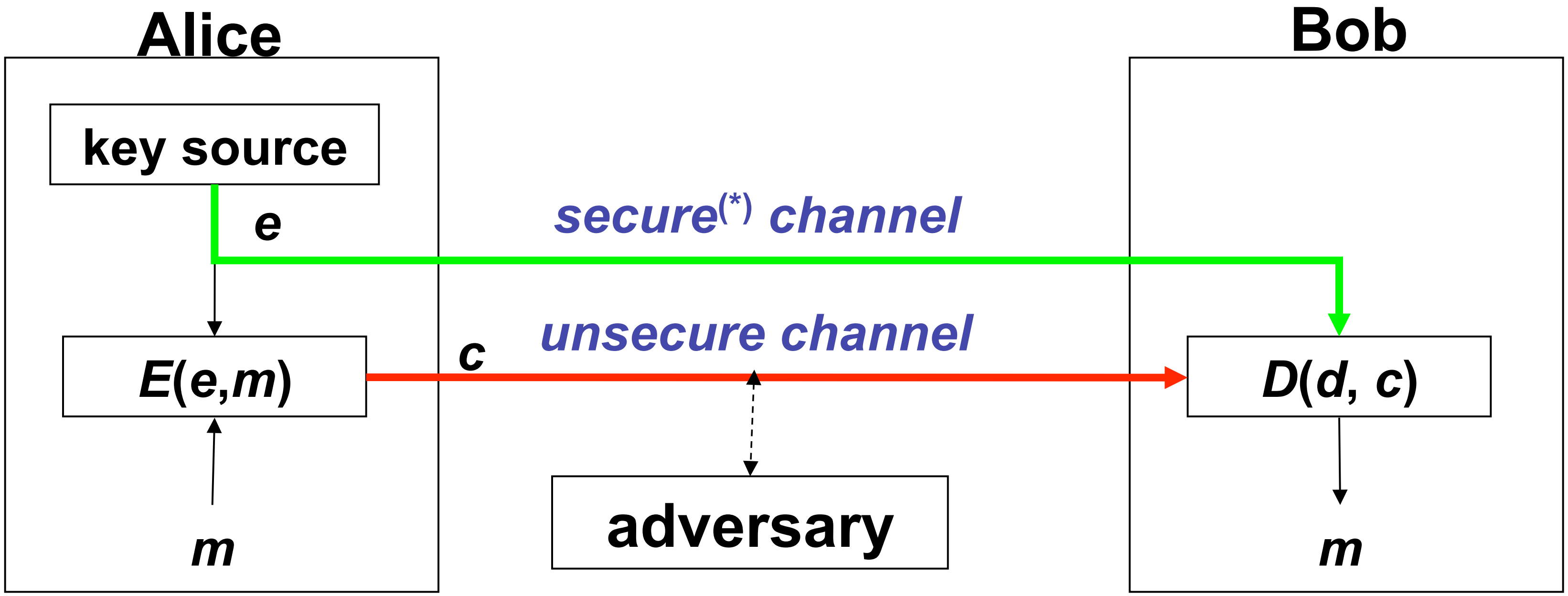
- $\forall m \in M, \forall e \in K; \exists d \in K : m = D(d, E(e, m))$
- It is *computationally* "*easy*" to compute ***d*** knowing **e**, and viceversa

In most practical symmetric encryption scheme ***e = d***

Security of a symmetric cipher

- ***An informal definition***
  - Let **(*E*, *D*)** a symmetric encryption scheme
  - For each pair **(m, c),** such that ***c* = *E*(e, *m*)** and ***m* = D(e, c)** the symmetric cipher **(E, D)** is secure iff
    - Given ***c*,** it is **difficult** to determine ***m*** without knowing **e**, and viceversa
    - Given ***c*** and **m**, it is **difficult** to determine ***e***, unless it is used just once

2-party comm with symmetric encryption



- Alice and Bob know ***E*** and ***D***
- Alice and Bob trust each other
- key **e** is a shared secret between Alice and Bob

(*) the channel is not ***physically*** accessible to the adversary and ensures both confidentiality and integrity

Discussion

- How can Bob be sure that ***m* = *D*(*k*,*c*)** is good?
  - Bob knows ***m*** in advance
  - Bob knows a part of ***m*** in advance (e.g., email)
  - Bob knows that ***m*** has certain structural redundancies (e.g., ASCII)

Discussion

EXAMPLE (DES-CBC)

- Bob receives

  c = `f3 9e 8a 73 fc 76 2d 0f`
  `59 43 bd 85 c3 c9 89 d2`
  `bf 96 b6 4f 34 b8 51 dd`

- Bob deciphers ***c*** with

  k = `0x3dd04b6d14a437a9`

- Bob obtains
  - ***m*** = "`Ci vediamo alle 20!`"

Discussion

What is the effect of a "small" change in the ciphertext?

- Single bit change
  - $c[0]_7 = \sim c[0]_7$ (`73 9e 8a 73 fc ...`)
  - ***m'***="`e8¢biö=}o alle 20:00!`"
- Single byte change
  - c[c.lenght() – 1] = 0x00 (`... 34 b8 51 00`)
  - ***m'***="`Ci vediamo alle "}2gÀlõ`"

Discussion

- Upon *seeing* ***m***, Bob *believes* that:
  - only Alice saw message ***m*** (privacy)
  - message ***m*** comes from Alice (?provenience?)
  - message ***m*** has not been modified (?integrity?)

On trust

What does "Alice and Bob trust each other" mean?

- Alice (Bob) believes that Bob (Alice) does not reveal ***m***
- Alice (Bob) believes that Bob (Alice) keeps key ***e*** secret, i.e.,
  - Alice (Bob) believes that Bob (Alice) is competent to do key management
  - Alice (Bob) believes that Bob (Alice) does not reveal the key

Perfect ciphers

Cifrario perfetto

- **Intuition**. By using a perfect cipher, an adversary analysing a ciphertext *c* cannot gain any additional information on the corresponding message *m*

- Shannon (1949) formalized this intuition
  - Let *M* be a stochastic variable taking values from the message space **M**
  - Let *C* be a stochastic variable taking values from the ciphertext space **C**
  - **Definition**. A cipher is perfect if for all $m \in \mathbf{M}$ and for all $c \in \mathbf{C}$, $Pr(M = m \mid C = c) = Pr(M = m)$

Cifrario perfetto

- **Theorem**. In a perfect cipher, the number of keys is not smaller than the number of clear-texts
- **Proof (by contradiction)**. Let N_m be the number of clear-texts, N_c be the number of ciphertexts and N_k the number of keys

1. *$N_m \leq N_c$ or otherwise the cipher is not invertible*
2. Let us assume that $N_k < N_m$. Thus $N_k < N_c$
3. Let *m s.t.* $Pr(M = m) \neq 0$. From (2) it follows that c′ ∈ **C** exists s.t. c′ is not image of *m*. Therefore

   $Pr(M = m \mid C = c') = 0 \neq Pr(M = m) \neq 0$ which contradicts the assumption of perfect cipher

Unconditional security

- **Unconditional security** (**perfect secrecy**)
  - An adversary is assumed to have **unlimited computational resources**
  - The uncertainty in the plaintext after observing the ciphertext must be equal to the a priori uncertainty about the plaintext
  - Observation of the ciphertext provides no information whatsoever to an adversary
- A **necessary condition** for a symmetric-key encryption scheme to be unconditionally secure is that the key bits are chosen randomly and independently and the key is at least as long as the message

One-time Pad (Vernam, 1917)

- Let ***m*** be a ***t***-bit message
  Let ***k*** be a sequence of ***t*** randomly chosen bits
- **Encryption and decryption functions**
  Encryption: $c_i = m_i \oplus k_i,\ 0 \leq i \leq t$
  Decryption: $m_i = c_i \oplus k_i,\ 0 \leq i \leq t$
- **An alternative view of the encryption function**

$$E_{k_i}(m_i) = \begin{cases} m_i & k_i = 0 \\ (m_i + 1) \bmod 2 & k_i = 1 \end{cases}$$

- **Esempio**
  - *m* = 01010101, *k* = 01001110, *c* = 00011011 (si noti che *m* è periodico ma *c* no)

One-Time Pad è un cifrario perfetto

THEOREM. One-Time Pad is a perfect cipher if

1. For each message a new key is chosen in perfect random way
2. All messages have bit-size t
3. Every sequence of t bits may be a possible message

Proof. Omitted

THEOREM. One-Time Pad utilises the smallest number of keys

Proof. Omitted

One-Time Pad

- One-time padding is *unconditionally secure* against ciphertext-only attack
  - Any ***t***-bit plaintext message ***m**** can be recovered from a ***t***-bit ciphertext ***c*** by using a proper key ***k**** = ***m****⊕***c***
- **OTP is vulnerable to a known-plaintext attack**
  - key *k* can be easily obtained from *m* and *c:* $k_i = m_i \oplus c_i$
- **The key must be used only once**.
  - Let us suppose that a key ***k* is used twice,** $c = m \oplus k$ and $c' = m' \oplus k$. $\Rightarrow c \oplus c' = m \oplus m'$.
  - This provides important information pieces to a cryptanalyst who has both *c* and *c*´.
    - Ex.: a sequence of zeros in $c \oplus c'$ corresponds to equal sequences in *m* and *m*´

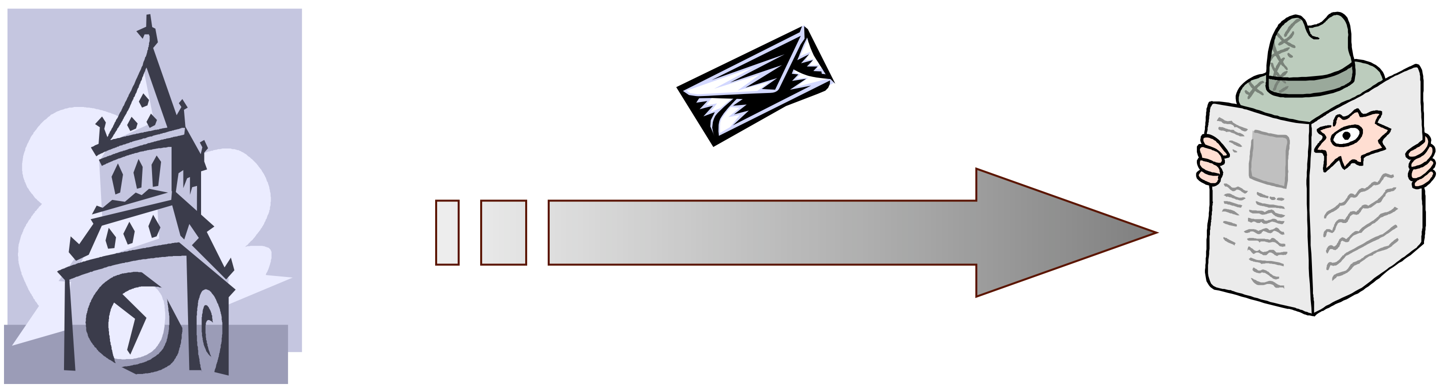
Security of one-time pad

- **OTP requires to generate a key of many random bits**
  - This problem is not trivial!
  - Key distribution and key management are complicated
  - Practical approach
    - For this reason, in practice, stream ciphers are used where the key stream is pseudo randomly generated from a smaller secret key. These ciphers are not unconditionally secure but, hopefully, practically secure
- OTP is vulnerable to integrity attacks

One-time pad

- **c[i] = m[i] + k[i] mod 26**
- **m = “SUPPORT JAMES BOND”**

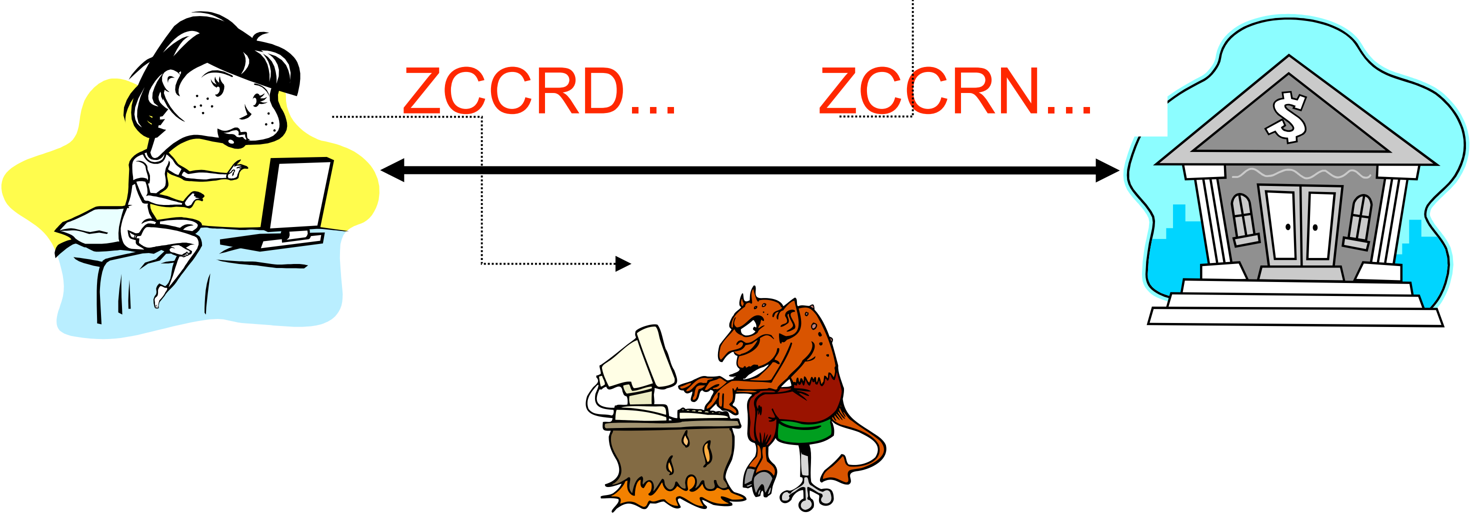
m =	S	U	P	P	O	R	T	J	A	M	E	S	B	O	N	D
k =	W	C	L	N	B	T	D	E	F	J	A	Z	G	U	I	R
c =	O	W	A	C	P	K	W	N	F	V	E	R	H	I	V	U



c =	O	W	A	C	P	K	W	N	F	V	E	R	H	I	V	U
k' =	M	W	L	J	V	T	S	E	F	J	A	Z	G	U	I	R
m =	C	A	P	T	U	R	E	J	A	M	E	S	B	O	N	D

OTP does not protect integrity

m =	D	A	R	E	C	E	N	T	O	E	U	R	O	A	B	O	B
k =	W	C	L	N	B	T	D	E	F	J	A	Z	G	U	I	R	X
c =	Z	C	C	R	D	X	Q	X	T	N	U	Q	U	U	J	F	Y



c' =	Z	C	C	R	N	B	O	P	J	N	U	Q	U	U	J	F	Y
k =	W	C	L	N	B	T	D	E	F	J	A	Z	G	U	I	R	X
m =	D	A	R	E	M	I	L	L	E	E	U	R	O	A	B	O	B

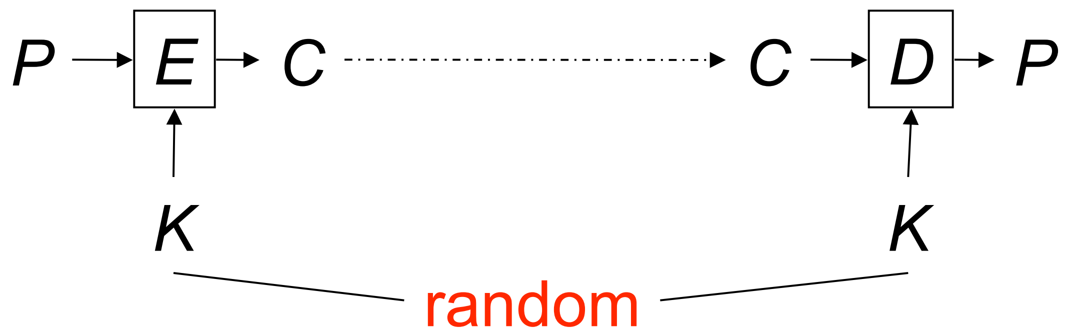
Symmetric encryption

BLOCK CIPHERS

Symmetric ciphers

- **Block ciphers** are encryption schemes which break up the plaintext in blocks of fixed lenght t bits and encrypt one block at time
- **Stream ciphers** are simple block ciphers in which t = 1 and the encryption function can change for each bit

Block cipher



$|P| = |C| = \boldsymbol{n}$ bits (block lenght)

$|K| = \boldsymbol{k}$ bits (key lenght)

$K \in \mathrm{K} \subseteq V_k$

$\mathrm{P} \in \Pi \subseteq V_n$

$\mathrm{C} \in \mathrm{X} \subseteq V_n$

V_i set of i-bits vectors

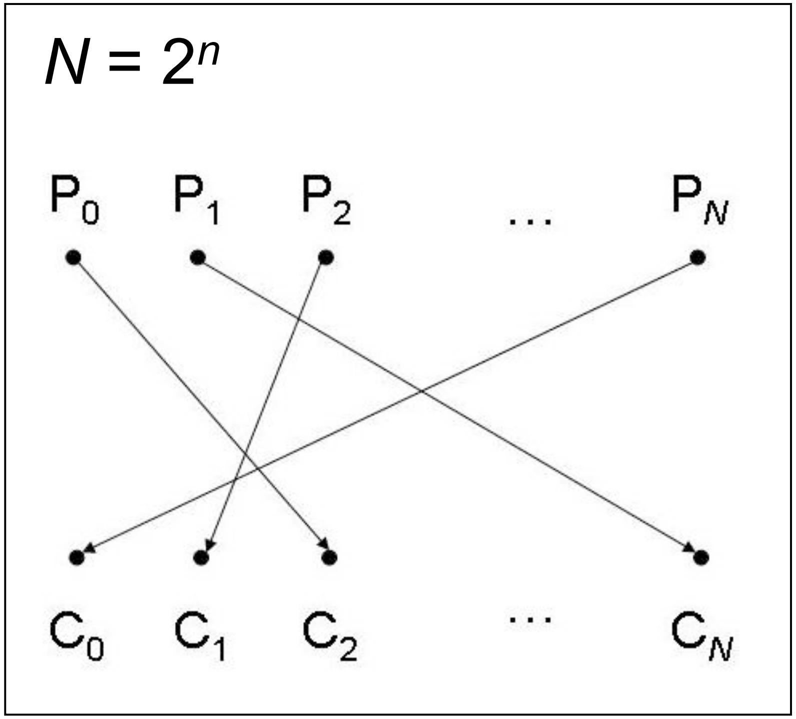
For any K,

- $E(K, P)$ must be an *invertible* mapping from V_n to V_n and
- $D(K, P)$ is the *inverse function*

- $E(K, P)$ will be often denoted by $E_K(P)$

True random cipher

For any key K, E_K defines a particular substitution (permutation)



- A true random cipher is a perfect cipher
- All the possible substitutions are $2^n!$
- Therefore the key lenght is $k = \lg(2^n!) \approx (n - 1.44)\, 2^n$
  - **key lenght is 2^n times the block lenght**
- A true random cipher is impractical

In practice, the encryption function corresponding to a randomly chosen key ***should appear*** a randomly chosen invertible function

Computational (practical) security

- A cipher is **computationally** (**practically**) **secure** if the perceived level of computation required to defeat it, ***using the best attack known***, exceeds, by a comfortable margin, the ***computation resources of the hypothesized adversary***
- **The adversary is assumed to have a limited computation power**

Standard assumptions

- **Objective of the adversary**
  - To recover the plaintext from the ciphertext (*partial break*) or even the key (*total break*)
- **Standard assumptions**.
  - An adversary

1. has access to all data transmitted over the ciphertext channel;
2. knows all details of the encryption function except the secret key (*Kerckhoff's assumption*)

Classification of attacks

- Attacks are classified according to what information an adversary has access to
  - **ciphertext-only attack**
  - **known-plaintext attack**
  - **chosen-plaintext attack**

stronger

- A cipher secure against chosen-plaintext attacks is also secure against ciphertext-only and known-plaintext attack
- It is customary to use ciphers resistant to a chosen-plaintext attack even when mounting that attack is not practically feasible

Attack complexity

- **Attack complexity** is the dominant of:
  - **data complexity** — expected number of input data units required
    - Ex.: exhaustive data analysis is $O(2^n)$
  - **storage complexity** — expected number of storage units required
  - **processing complexity** — expected number of operations required to processing input data and/or fill storage with data
    - Ex.: exhaustive key search is $O(2^k)$

Attack complexity

- A block cipher is **computationally secure** if
  - ***n*** is sufficiently large to preclude **exhaustive data analysis**, and
  - ***k*** is sufficiently large to preclude **exhaustive key search**, and
  - ***no known attack*** has data and processing complexity significantly less than, respectively, 2^n and 2^k

Exhaustive key search

- Number of processors necessary to break a key
- Every processor performs 10^6 encryption/second

Key size (bit)	1 Year	1 Month	1 Week	1 Day
56	2,300	28,000	120,000	830,000
64	590,000	7,100,000	3.1×10^{7}	2.1×10^{8}
128	$1,1\times10^{25}$	$1,3\times10^{26}$	$5,6\times10^{26}$	$3,9\times10^{27}$

Exhaustive key search

- Cost of a year-2005 hardware cracker

1 Year	1 Month	1 Week	1 Day
56 bit			
\$2000	\$24,000	\$100,000	\$730,000
64 bit			
\$510,000	\$6.2M	\$27M	\$190M
128 bit			
$\$9.4\times10^{24}$	$\$1.2\times10^{26}$	$\$4.9\times10^{26}$	3.3×10^{27}

Exhaustive key search

- Exhaustive key search is a known-plaintext attack
- Exhaustive key search may be a ciphertext-only attack if the plaintext has known redundancy
- Exhaustive key search has widespread applicability since cipher operations (including decryption) are generally designed to be computationally efficient
- Given $\lceil (k+4)/n \rceil$ pairs of plaintext-ciphertext, a key can be recovered by exhaustive key search in an expected time $O(2^{k-1})$
  - Exhaustive key search in Des requires 2^{55} decryptions and one plaintext-ciphertext pair

Exhaustive data analysis

- A dictionary attack requires to assemble plaintext-ciphertext pairs for a fixed key
- A dictionary attack is a known-plaintext attack
- A complete dictionary requires at most 2^n pairs
- Each pairs requires 2^n bits

Cryptoanalysis: an historical example

Monoalphabetic substitution

Cleartext alphabet	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Key	J	U	L	I	S	C	A	E	R	T	V	W	X	Y	Z	B	D	F	G	H	K	M	N	O	P	Q

- **The key is a permutation of the alphabet**
- **Encryption algorithm**: every cleartext character having position *p* in the alphabet is *substituted* by the character having the same position *p* in the key
- **Decryption algorithm**: every ciphertext character having position *p* in the key is *substituted* by the character having the same position *p* in the cleartext
- **Number of keys** = $26! - 1 \cong 4 \times 10^{26}$ (number of seconds since universe birth)

Cryptoanalysis: an historical example

P = "TWO HOUSEHOLDS, BOTH ALIKE IN DIGNITY,
IN FAIR VERONA, WHERE WE LAY OUR SCENE"

("Romeo and Juliet", Shakespeare)

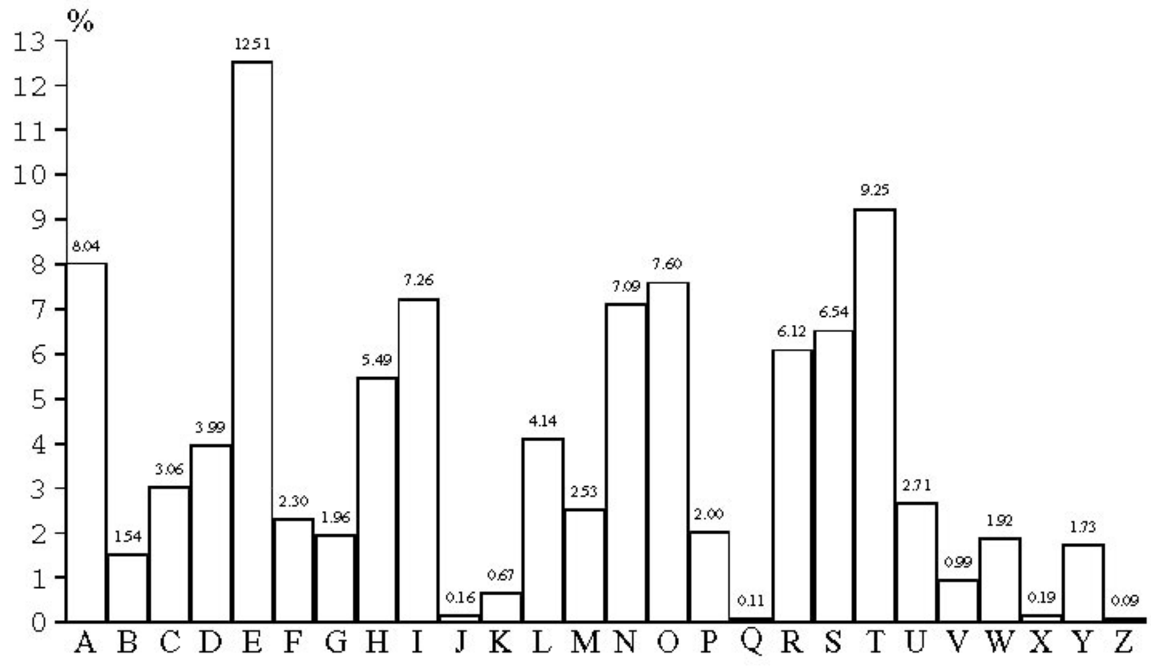
P′ = "TWOHO USEHO LDSBO THALI KEIND IGNIT
YINFA IRVER ONAWH EREWE LAYOU RSCEN E"

C = "HNZEZ KGSEZ WIGUZ HEJWR VSRYI RAYRH
PRYCJ RFMSF ZYJNE SFSNS WJPZK FGLSY S"

Cryptoanalysis: an historical example

- The monoalphabetic-substitution cipher maintains the redundancy that is present in the cleartext
- It can be "easily" cryptoanalized with a ciphertext-only attack based on language statistics

Frequency of single characters in English text



Linear/differential cryptoanalysis

- **Linear cryptonalysis**
  - è una tecnica di crittoanalisi per cifrari a blocchi ed a caratteri
  - Attribuita a Mitsuru Matsui (1992)
- **Differential cryptoanalysis**
  - è una tecnica di crittoanalisi principalmente concepita per cifrari a blocchi ma che può essere applicata anche ai cifrari a caratteri
  - Attribuita a to Eli Biham and Adi Shamir verso la fine degli anni `80

Security of DES

attack method	data complexity		storage complexity	processing complexity
	known	chosen		
exhaustive precomputation	—	1	2^{56}	1 *
exhaustive search	1	—	negligible	2^{55}
linear cryptanalysis	2^{43} (85%)	—	for texts	2^{43}
	2^{38} (10%)	—	for texts	2^{50}
differential cryptanalysis	—	2^{47}	for texts	2^{47}
	2^{55}	—	for texts	2^{55}

* Table lookup

% : probability of success

- Linear cryptanalysis is a known-plaintext attack
- Differential cryptanalysis is primarily a chosen-plaintext attack

Cryptoanalysis of DES

- **Linear cryptonalysis**
  - A known-plaintext attack has $O(2^{43})$ data complexity and $O(2^{43})$ computation complexity.
    - With a chosen-plaintext attack, data complexity can be reduced by a factor of 4.
- **Differential cryptoanalysis**
  - Known-plaintext attack has $O(2^{55})$ data complexity and $O(2^{55})$ computation complexity
  - Chosen-plaintext attack has $O(2^{47})$ data complexity and $O(2^{47})$ computation complexity
  - DES is "surprisingly" resilient to DC.
- **LC is the "best" analytical attack but is considered unpractical**

Encryption modes

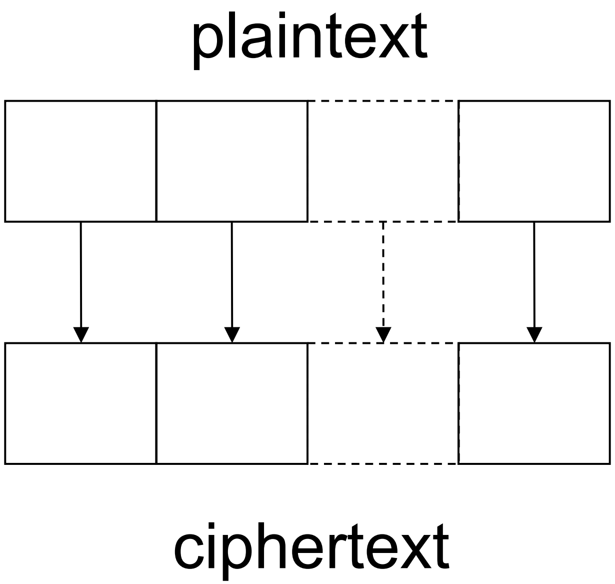
- **Electronic CodeBook**
- **Cipher Block Chaining**

Encryption modes

- A block cipher encrypts plaintext in fixed-size *n*-bit blocks
- When the plaintext exceeds *n* bit, there exist several methods to use a block
  - Electronic codebook (ECB)
  - Cipher-block Chaining (CBC)
  - Cipher-feedback (CFB)
  - Output feedback (OFB)

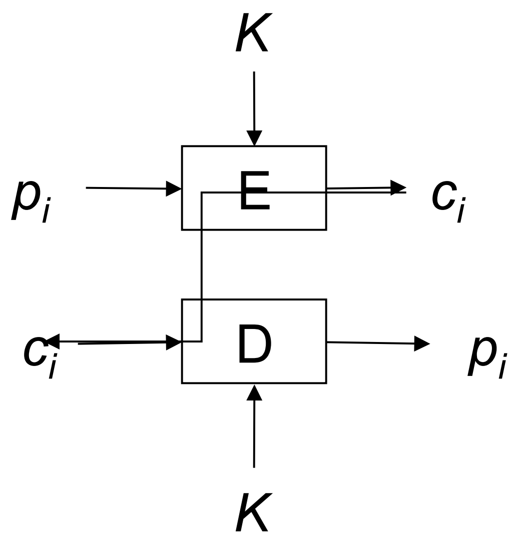
Encryption modes: ECB

- **Electronic Codebook (ECB)**



$$\forall 1 \le i \le t, c_i \leftarrow E_k(p_i)$$

$$\forall 1 \le i \le t, p_i \leftarrow D_k(c_i)$$



plaintext blocks are encrypted separately

Encryption modes: ECB

Properties

- Identical plaintext results in identical ciphertext
  - ECB doesn't hide data patterns
- No chaining dependencies: blocks are enciphered independently of other blocks
  - ECB allows block reordering and substitution
- Error propagation: one or more bit errors in a single ciphertext block affects decipherment of that block only

Encryption modes: ECB

AN EXAMPLE OF BLOCK REPLAY

- A bank transaction transfers a client U's amount of money D from bank B1 to bank B2
  - Bank B1 debits D to U
  - Bank B1 sends the "credit D to U" message to bank B2
  - Upon receiving the message, Bank B2 credits D to U
- Credit message format
  - Src bank: *M* (12 byte)
  - Rcv banck: *R* (12 byte)
  - Client: *C* (48 byte)
  - Bank account: *N* (16 byte)
  - Amount of money: *D* (8 byte)
- Cifrario (n = 64 bit; modalità ECB)

Encryption modes: ECB

AN EXAMPLE OF BLOCK REPLAY

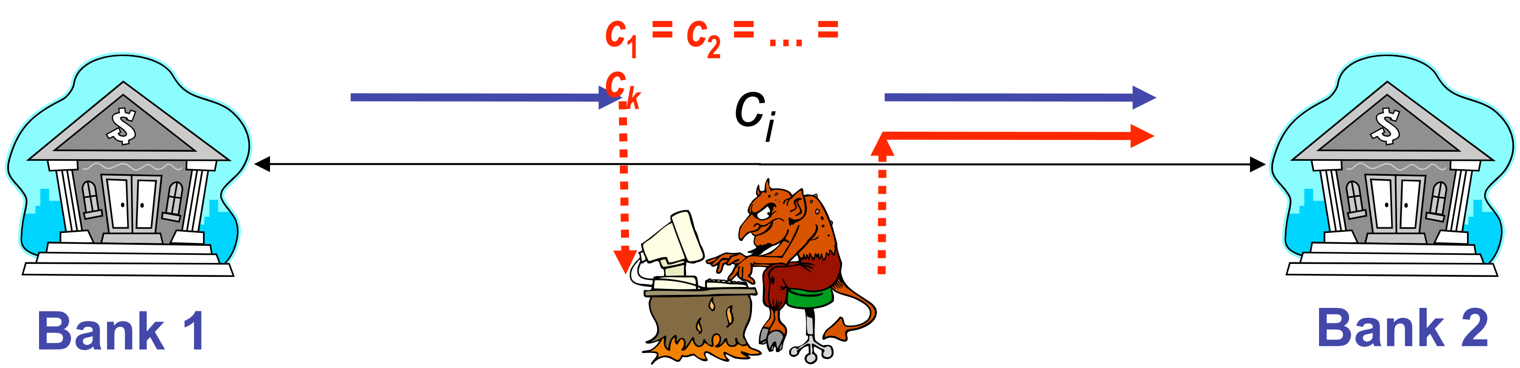
- Mr. Lou Cipher is a client of the banks and wants to make a fraud.
- Lou Cipher is an active adversary and wants to replay a Bank B1's message "credit 100$ to Lou Cipher" many times
- Attack strategy
  - The adversary activates multiple transfers of 100$ so that multiple messages "credit 100$ to Lou Cipher" are sent from B1 to B2
  - The adversary identifies at least one of these messages
  - The adversary replies the message several times

Encryption modes: ECB

AN EXAMPLE OF BLOCK REPLAY

1. The adversary performs **k** equal transfers
   - credit 100$ to Lou Cipher ⇒ c_1
   - credit 100$ to Lou Cipher ⇒ c_2
   - ...
   - credit 100$ to Lou Cipher ⇒ c_k
2. The adversary searches “his own” cryptograms over the network
3. The adversary replies one of these cryptograms

COMMENT. ***k*** is large enough to allow the adversary to identify the cryptograms corresponding to its transfers



Encryption modes: ECB

AN EXAMPLE OF BLOCK REPLAY

- An 8-byte timestamp field ***T*** is added to the message to prevent replay attacks

block no.	1	2	3	4	5	6	7	8	9	10	11	12	13
	T	M		R	C						N		D

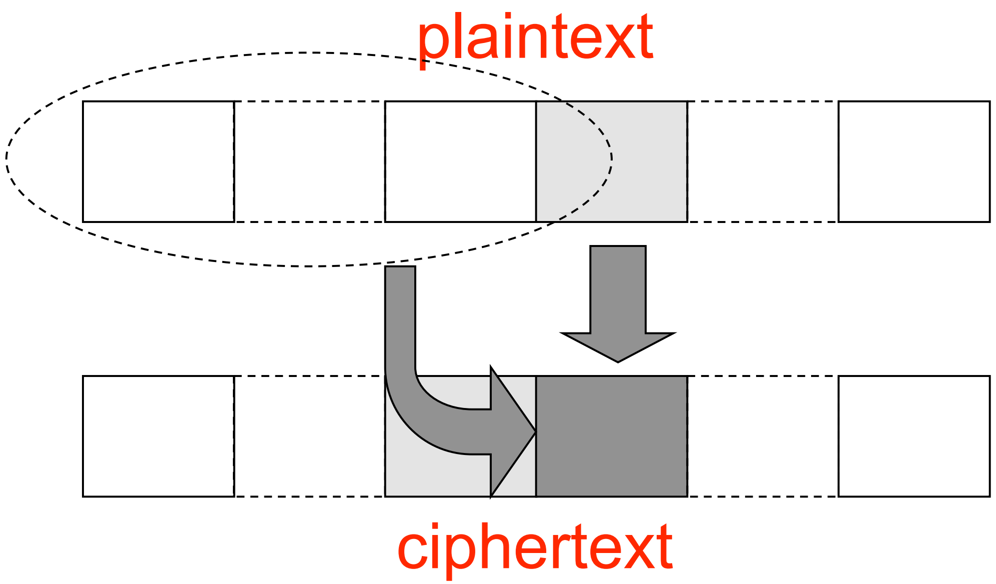
However, the adversary can

1. identify "his own" cryptograms as before by inspecting blocks 2–13;
2. intercept any "fresh" cryptogram;
3. substitute block 1 of "his own" cryptogram with block 1 of the "fresh" cryptogram

Encryption modes: Cipher Block Chaining

- CBC segue il **principio di diffusione** di Shannon introducendo una **dipendenza di posizione** tra il blocco in elaborazione e quelli precedenti
- CBC è un cifrario a blocchi in cui blocchi identici del messaggio vengono cifrati in modo **diverso** eliminando ogni periodicità

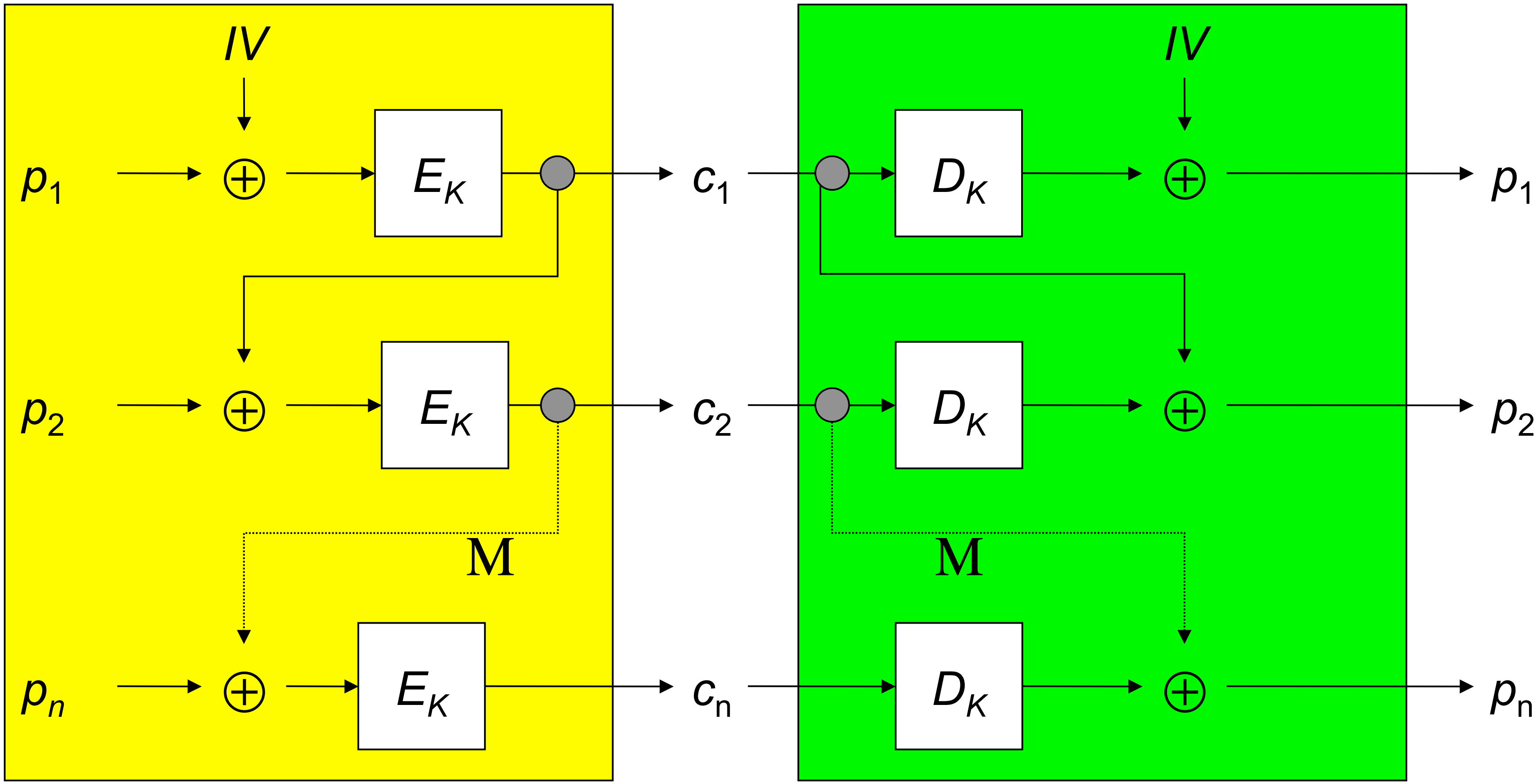
c_i depends on p_i and all preceding plaintext blocks



CBC

$$c_0 \leftarrow IV. \forall 1 \le i \le t, c_i \leftarrow E_k\left(p_i \oplus c_{i-1}\right)$$

$$c_0 \leftarrow IV. \forall 1 \le i \le t, p_i \leftarrow c_{i-1} \oplus D_k\left(c_i\right)$$



CBC: properties

- Identical ciphertext result from the same plaintext under the same key and IV
- IV can be sent in the clear; its integrity must be guaranteed
- Chaining dependencies: c_i depends on p_i and all preceding plaintext blocks
  - Ciphertext block reordering affects decryption
- Error propagation: bit errors in c_i affect decryption of c_i and c_{i+1}
- Error recovery: CBC is self-synchronizing or ciphertext autokey
- Framing errors: CBC does not tolerate “lost” bits

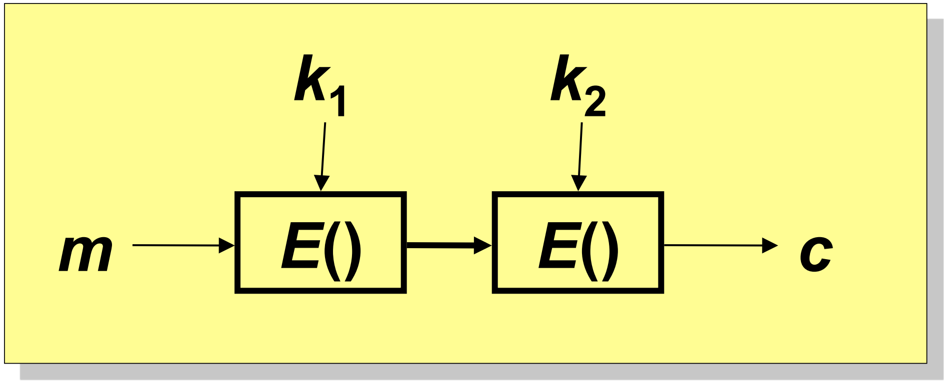
Multiple encryption

- **3DES (EDE, EEE)**

Multiple encryption

- If a cipher is subject to exhaustive key search, encipherment of a message more than once *may* increase security
- Multiple encryption may be extended to messages exceeding one block by using standard modes of operation
- **Cascade cipher** is the concatenation of $L \geq 2$ ciphers, each with independent keys
- **Multiple encryption** is similar to a cascade cipher but the ciphers are identical (either *E* or *D*) and the keys need not be independent

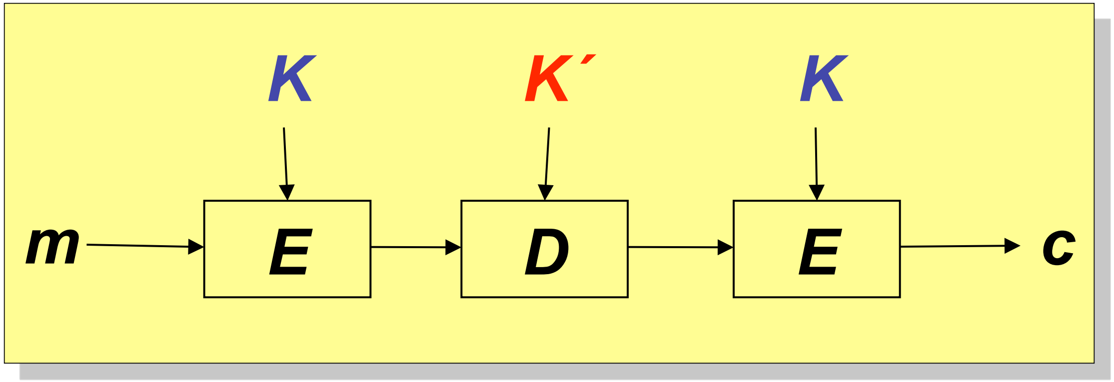
Double encryption



- Double encryption is subject to a known-plaintext attack called "meet-in-the-middle" attack which requires
  2^k operations and
  2^k storage units

Triple encryption

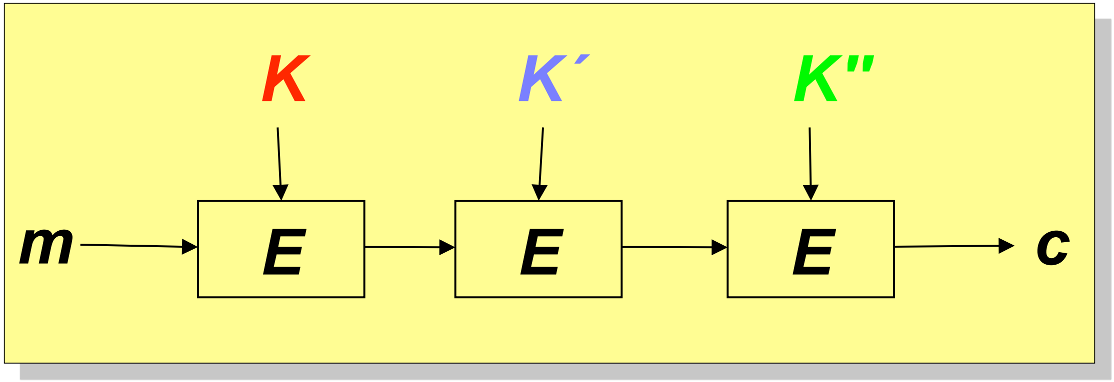
EDE



- Financial applications
- Standard (ANSI X9.17 and ISO 8732)
- A chosen-plaintext attack requires 2^k operations, 2^k data inputs and 2^k storage units
- A known-plaintext attack requires p data inputs, $2^{k+n}/p$ operations, and $O(p)$ storage units
- Backward compatibility with E when $K = K'$

Triple encryption

EEE



- A known-plaintext attack similar to meet-in-the-middle, which requires 2^{2k} operations and 2^k units of storage
- With DES, $k = 56$ (DES), the cipher is practically secure

Cryptographic Libraries and APIs

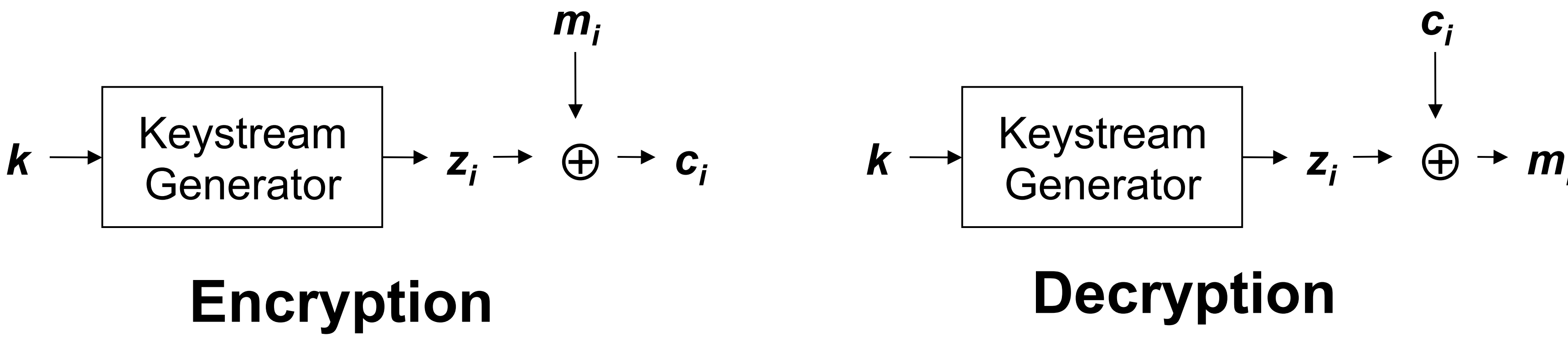
- **Java Cryptography**
- **OpenSSL (ciphers)**

I cifrari a carattere

Stream ciphers

- In **stream ciphers**
  - **a plaintext block is as small as one bit** and
  - **the encryption function may vary as plaintext is processed** (stream ciphers have memory)
- **Stream ciphers are faster than block ciphers in hardware**, and have less complex hardware circuitry
- **Stream ciphers are more appropriate or mandatory**
  - when buffering is limited
  - when characters must be processed as they are received
  - when transmission errors are highly probable since they have limited or no error propagation

Synchronous stream ciphers



Properties

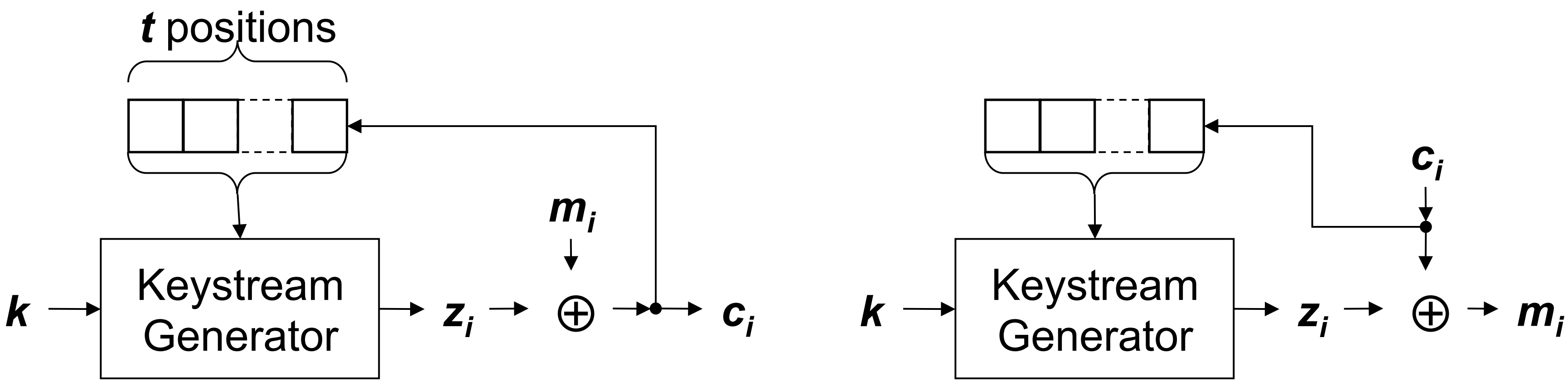
- Sender and receiver must be synchronized.
  If a bit is inserted or deleted, decryption fails.
- No error propagation
- Modifications to cipher text bits may go undetected

Synchronous stream ciphers

Properties

- Sender and receiver must be synchronized.
  - If a bit is inserted or deleted, decryption fails.
- No error propagation.
  - A wrong bit in the ciphertext does not affect the others.
- Some actives attacks may go undetected
  - An adversary that insert/removes one bit can be detected
  - An adversary that changes one bit may be not detected

Self-synchronizing stream ciphers



Encryption

Decryption

Self-synchronizing stream ciphers

Properties

- Self-synchronization.
  - Insertion/removal of one bit in cipher-text causes the loss of *t*-bits
- Limited error propagation
  - The change of a bit in cipher-text changes t-bits
- Active attacks
  - Self-syncronization property makes insertion/removal of a bit more difficult to detect that synchronous ciphers
  - Error propagation property simplifies detection of a bit change w.r.t. synchronous ciphers
- Diffusion of plaintext statistics

Key stream generator

- The key stream must have the following properties:
  - large period
  - unpredictable
  - good statistics
- There are only *necessary conditions* for a KSG to be considered cryptographically secure
- KSGs are computationally secure after public scrutiny (no mathematical proof)

Stream ciphers

- For hardware implementation
  - LFSR-based stream ciphers
- For software implementation
  - SEAL
    - New algorithm (1993) for software implementation on 32-bit processors. It has received not yet much scrutiny
  - RC4
    - commercial products
    - variable key
    - proprietary
  - Output Feedback (OFB), Cipher Feedback (CFB) (modes of block ciphers)

WEP (802.11)

- An example of insecure system made of secure components