An EM-Based Frequency Offset Estimator for OFDM Systems with Unknown Interference

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Abstract—We consider an orthogonal frequency-division multiplexing (OFDM) system and address the problem of carrier frequency estimation in the presence of narrowband interference (NBI) with unknown power. This scenario is encountered in emerging spectrum sharing systems, where coexistence of different wireless services over the same frequency band may result into a remarkable co-channel interference, and also in digital subscriber line transmissions as a consequence of the cross-talk phenomenon.

A possible solution for frequency recovery in OFDM systems plagued by NBI has recently been derived using the maximum-likelihood criterion. Such scheme exhibits good accuracy, but involves a computationally demanding grid-search over the uncertainty frequency range. In the present work, we derive an alternative method that provides frequency estimates in closed-form by resorting to the expectation-maximization algorithm. This makes it possible to achieve some computational saving while maintaining a remarkable robustness against NBI.

Index Terms—OFDM, carrier frequency offset recovery, expectation-maximization algorithm, inverse-gamma power distribution.

I. INTRODUCTION

THE deployment of next generation broadband wireless communication systems may be stifled by the congestion of the frequency spectrum. A promising solution to this problem is the possibility of dynamically redistributing allocated frequency bands according to the spectrum sharing concept [1], where users monitor the radio environment and opportunistically establish a communication link by filling existing gaps in the frequency spectrum. In these applications, orthogonal frequency-division multiplexing (OFDM) is the preferred signaling scheme due to its inherent ability in performing dynamic power and carrier assignment. Spectrum sharing calls for sophisticated signal processing techniques in order to cope with the possible presence of narrowband interference (NBI) within the signal spectrum. A similar situation is expected to occur in future femtocells networks, which represent a promising solution for broadband wireless access in small building areas that cannot be reliably covered by an outdoor base station (BS) due to the high penetration loss from walls [2]. Since femtocells operate on the same frequency band as macro BSs, co-channel interference becomes a primary impairment for such systems. Digital subscriber line (DSL) is another application where OFDM may be affected by NBI due to the inductive coupling between wire pairs in a telephone cable (cross-talk effect).

Despite its appealing features in terms of flexibility in resource management and simplified channel equalization, the use of OFDM in a scenario characterized by potentially interfering signals poses several technical challenges that must be properly addressed to avoid a serious degradation of the system performance. Among them, carrier-frequency offset (CFO) estimation is of primary importance. Frequency recovery in OFDM systems has been extensively studied in the past and some popular approaches can be found in [3] – [6]. These schemes provide good results as long as the received signal is only affected by channel distortions and thermal noise, while significant degradations are expected in the presence of possibly strong NBI [7]. A robust frequency estimation algorithm for jammed OFDM transmissions has recently been derived in [8] using maximum likelihood (ML) methods and assuming that the NBI is Gaussian distributed across the signal spectrum with zero mean and unknown power. Unfortunately, this approach may entail significant computational burden as a complete search is needed to locate the global maximum of the likelihood function.

In the present work, we reconsider the CFO estimation problem of [8] and propose an alternative solution based on the expectation-maximization (EM) algorithm. The latter operates in an iterative fashion and provides frequency estimates in closed form, thereby avoiding any computationally demanding peak search procedure. As we shall see, this approach results into an estimation algorithm of affordable complexity and with an accuracy that is close to the relevant Cramer-Rao bound (CRB).

The remainder of this letter is organized as follows. Next section outlines the investigated system and illustrates the signal model. In Section III we use the EM algorithm to derive the CFO estimator. Numerical results are provided in Section IV, while conclusions are drawn in Section V.

II. SYSTEM DESCRIPTION AND SIGNAL MODEL

We consider an OFDM system employing $N$ subcarriers with indices in the set $\{0, 1, \ldots, N-1\}$ and potentially affected by NBI. The transmission is organized in frames and each frame is preceded by a pilot block, which is exploited for frequency estimation and possibly other synchronization functions. To facilitate the frequency recovery task, the pilot block is divided into $K$ identical segments, each containing $M = N/K$ samples. A practical way to generate such a block is to transmit a pseudonoise sequence $\{a_m; 0 \leq m \leq M-1\}$ onto the subcarriers with indices multiple of $K$ while setting to zero the remaining subcarriers [6]. As is customary in...
OFDM applications, the frequency error between the received signal and the local oscillator (normalized by the subcarrier spacing) is decomposed into a fractional part $\xi$, belonging to the interval $(-K/2, K/2)$, plus a remaining integer part $\eta$ which is a multiple of $K$. In this work, we only concentrate on the estimation of $\xi$ as the integer CFO can be computed with affordable complexity using the technique described in [8]. Furthermore, we consider low mobility applications where the channel does not vary significantly during one OFDM block.

At the receiver side, the samples belonging to the pilot block are partitioned into $K$ adjacent segments, each composed of $M$ elements and corresponding to $M$, with $K = 1, \ldots, K − 1$, and is passed to a $K$-point DFT unit. We denote by $X(m, k)$ the $m$th DFT output of the $k$th segment and arrange all these quantities into $M$ vectors $X(m) = [X(m, 0), X(m, 1), \ldots, X(m, K − 1)]^T$, with $m = 0, \ldots, M − 1$ and $(\cdot)^T$ denoting the transpose operation. Since the repetitive parts of the pilot block remain identical after passing through the channel except for a CFO-induced phase shift, following [8] we may write

$$X(m) = S(m)e_K(\xi) + W(m)$$

where $e_K(\xi) = [1, e^{j2\pi\xi/K}, e^{j4\pi\xi/K}, \ldots, e^{j2\pi\xi(K-1)/K}]^T$ collects the phase shifts introduced by the CFO, $S(m)$ is the signal component and, finally, $W(m) = [W(m, 0), W(m, 1), \ldots, W(m, K − 1)]^T$ accounts for thermal noise plus any other possible interference term. The quantity $S(m)$ is related to the training sequence $\{a_m\}$ by

$$S(m) = \frac{1}{\sqrt{K}} \sum_{\ell=0}^{M-1} H(\ell K) a_\ell f_M(\ell - m + \nu/K)$$

where $H(n)$ is the channel frequency response over the $n$th subcarrier, $\nu = \xi + \eta$ is the normalized CFO and $f_M(x) = e^{j\pi x(M-1)/M} \sin(\pi x/M) / \sin(\pi x/M)$.

Following [9], we model the entries of $W(m)$ as Gaussian random variables with zero mean and unknown variance $\sigma^2(m) = \sigma^2_n + \sigma^2_I(m)$, where $\sigma^2_n$ is the noise power while $\sigma^2_I(m)$ represents the NBI contribution and depends on the frequency index $m$. To ease the derivation, in this study the quantities $W(m, k)$ are treated as statistically independent for different values of $m$ and $k$. This assumption is reasonable when applied to thermal noise, whereas some correlation is expected between the NBI contribution over closely spaced subcarriers. Since such correlation could profitably be exploited to improve the resilience of the system against NBI, the independence assumption may be viewed as a method for describing a worst-case scenario.

Vectors $\{X(m)\}$ can be exploited to obtain the joint ML estimate of the unknown parameters $\xi, \sigma^2 = [\sigma^2(0), \sigma^2(1), \ldots, \sigma^2(M−1)]^T$ and $S = [S(0), S(1), \ldots, S(M−1)]^T$. This approach is adopted in [8] and leads to the maximization of a suitable objective function by means of an exhaustive grid-search. In the next section, we use the EM algorithm to perform such maximization in an iterative fashion, thereby avoiding the need for any complete search.

### III. EM-Based Frequency Estimation

#### A. Derivation of the frequency estimator

In the parlance of the EM algorithm [10], we view $X = [X^T(0), X^T(1), \ldots, X^T(M−1)]^T$ as the incomplete data and define the complete data set as the pair $(\hat{X}, \sigma^2)$. Also, we denote by $\theta = (\xi, S)$ the parameters to be estimated and call $\hat{\theta}_i$ their estimate at the $i$th iteration. Then, the EM algorithm iteratively alternates between an $E$-step, calculating the expectation of the log-likelihood function of the complete data given the observations and the current estimate $\hat{\theta}_i$, and an $M$-step, maximizing that expectation with respect to the unknown parameters. Specifically, at the $i$th iteration the $M$-step operates as follows

$$\hat{\theta}_{i+1} = \arg \max_{\hat{\theta}} \{Q(\hat{\theta}|\hat{\theta}_i)\}$$

where $\hat{\theta} = (\hat{\xi}, \hat{S})$ is a trial value of $\theta$, while function $Q(\hat{\theta}|\hat{\theta}_i)$ is evaluated in the $E$-step and reads

$$Q(\hat{\theta}|\hat{\theta}_i) = E_{\sigma^2} \left\{ \ln p(X|\sigma^2, \hat{\theta}_i) p(\sigma^2, \hat{\theta}_i) \right\}.$$  (5)

In the above equation, $p(X|\sigma^2, \hat{\theta}_i)$ and $p(\sigma^2, \hat{\theta}_i)$ are conditional probability density functions (pdfs) and the expectation is calculated over the statistical distribution of $\sigma^2$. This calls for the adoption of some a-priori pdf for $\sigma^2$. A possible solution is to assume that the entries of $\sigma^2$ are statistically independent and distributed according to an inverse-gamma pdf:

$$p(\sigma^2) = \frac{\lambda}{\sigma^2} \exp \left\{ -\frac{\lambda}{\sigma^2} \right\}, \quad \text{for } \sigma^2 > 0$$  (6)

with $\lambda$ being a design parameter. Compared to the classical gamma distribution, the pdf in (6) has the remarkable advantage of providing closed-form analysis in many, otherwise intractable, mathematical problems. For this reason, its use has been suggested in radar signal processing to statistically describe the power of the clutter originating from a small sea surface area [11], and in the wireless communication literature to model the inter-cell interference in severely fading channels [12]. The inverse-gamma distribution is also adopted in excellent textbooks on estimation theory as a prior pdf for intractable, mathematical problems. For this reason, its use has been suggested in radar signal processing to statistically describe the power of the clutter originating from a small sea surface area [11], and in the wireless communication literature to model the inter-cell interference in severely fading channels [12].
represents a biased estimate of $\sigma^2(m)$ at the $i$th iteration. Maximizing $\Lambda(\hat{\theta} | \hat{\theta}_i)$ with respect to $\hat{\xi}$ and $\hat{S}$ produces

$$\hat{\xi}_{i+1} = \arg \max_{\xi} \left\{ \Gamma_i(\hat{\xi}) \right\}$$  \hspace{1cm} (9)

$$\hat{S}_{i+1}(m) = \frac{1}{K} e^{H} e_{K}(\hat{\xi}_{i+1}) X(m),$$  \hspace{1cm} (10)

with

$$\Gamma_i(\hat{\xi}) = \frac{1}{\sigma^2_i(\hat{\xi})} \left| e^H(\hat{\xi}) X(m) \right|^2.$$  \hspace{1cm} (11)

Unfortunately, the direct maximization of $\Gamma_i(\hat{\xi})$ in (9) requires a grid-search over the set spanned by $\hat{\xi}$, which may be too cumbersome in practice. To overcome this difficulty, we adopt the alternative approach suggested in [14], which allows one to compute $\hat{\xi}_{i+1}$ in closed-form without resorting to any peak search procedure. For this purpose, we rewrite the right-hand-side of (11) in the form

$$\Gamma_i(\hat{\xi}) = \Re \left\{ \sum_{k=0}^{K-1} R_t(k) e^{j2\pi k \hat{\xi} /K} \right\}$$  \hspace{1cm} (12)

where $\Re \{ \cdot \}$ denotes the real part of the enclosed quantity, $R_t(k)$ is defined as

$$R_t(k) = \frac{\gamma(m,k)}{\sigma^2_t(m)}$$  \hspace{1cm} (13)

and $\gamma(m,k)$ is the following $k$-lag sample correlation function over the $m$th subcarrier

$$\gamma(m,k) = \sum_{p=k}^{k=K} X(m,p) X^*(m,p-k).$$  \hspace{1cm} (14)

It is worth observing that the quantities $\{\gamma(m,k)\}$ do not depend on the iteration index $i$ and, accordingly, they can be computed at the beginning of the iterative process and used in all subsequent iterations. The maximization of $\Gamma_i(\hat{\xi})$ is next performed by means of a two step procedure, which operates as follows. A coarse estimate of the CFO is firstly computed as

$$\hat{\xi}^{(c)}_{i+1} = \frac{1}{2\pi} \arg \{ R_t(1) \}$$  \hspace{1cm} (15)

and it is subsequently refined in the second step by looking for an estimate of the residual error $\Delta \xi = \xi - \hat{\xi}^{(c)}_{i+1}$. For this purpose, we let

$$R_t^{(c)}(k) = R_t(k) e^{-j2\pi k \hat{\xi}^{(c)}_{i+1} /K}$$  \hspace{1cm} (16)

and rewrite (12) in the equivalent form

$$\Gamma_i(\Delta \hat{\xi}) = \Re \left\{ \sum_{k=0}^{K-1} R_t^{(c)}(k) e^{-j2\pi k \Delta \hat{\xi} /K} \right\}$$  \hspace{1cm} (17)

with $\Delta \hat{\xi} = \hat{\xi} - \hat{\xi}^{(c)}_{i+1}$. Setting to zero the derivative of (17) with respect to $\Delta \hat{\xi}$ and assuming that $\Delta \hat{\xi}$ is small enough such that

$$e^{-j2\pi k \Delta \hat{\xi} /K} \approx 1 - j2\pi k \Delta \hat{\xi} /K,$$  \hspace{1cm} (18)

an estimate of $\Delta \hat{\xi}$ at the $(i+1)$th iteration is given by

$$\Delta \hat{\xi}_{i+1} = \frac{K}{2\pi} \sum_{k=0}^{K-1} k \cdot \Im \{ R_t^{(c)}(k) \}$$  \hspace{1cm} (19)

with $\Im \{ \cdot \}$ being the imaginary part of the enclosed quantity. The CFO estimate is eventually computed as

$$\hat{\xi}_{i+1} = \Delta \hat{\xi}_{i+1} + \hat{\xi}^{(c)}_{i+1}$$  \hspace{1cm} (20)

and is employed in (10) to get $\hat{S}_{i+1} = [\hat{S}_{i+1}(0), \hat{S}_{i+1}(1), \ldots, \hat{S}_{i+1}(M-1)]^T$. Once $\hat{\xi}_{i+1}$ and $\hat{S}_{i+1}$ have been computed, a new estimate of $\sigma^2$ to be used in the next iteration is obtained from (8) in the form

$$\sigma^2_{i+1}(m) = \frac{1}{K} \left[ \lambda + \left\| X(m) - \hat{S}_{i+1}(m) e_{K}(\hat{\xi}_{i+1}) \right\|^2 \right].$$  \hspace{1cm} (21)

In the sequel, we refer to the above iterative process as the EM-based frequency estimator (EMFE).

It is worth observing that parameter $\lambda$ in (21) is reminiscent of the quantity $\delta_\nu$ characterizing the modified likelihood function given in [8]. However, while $\delta_\nu$ is heuristically introduced in [8] as a practical means to improve the system performance, the quantity $\lambda$ appears naturally in the EMFE as a consequence of the inverse-gamma distribution adopted for $\sigma^2$. Clearly, such parameter must be carefully designed on the basis of the specific operating conditions in order to optimize the accuracy of EMFE.

### B. Initialization

Inspection of (13) reveals that EMFE requires an initial estimate of $\sigma^2$ in order to evaluate the quantities $R_t(k)$ for $k = 0,1,\ldots,K-1$. As it is known, a good initialization is essential to EM-type algorithms. The solution we propose here is based on a subspace approach, which exploits the correlation matrix of the DFT outputs. To see how this comes about, for any given $m$ we define a set of $K-1$ bidimensional vectors $X_k(m) = [X(m,k), X(m,k+1)]^T$, with $k = 0,1,\ldots,K-2$. Following (1), we may write

$$X_k(m) = S(m) e^{j2\pi k \xi/K} e_2(\xi) + W_k(m)$$  \hspace{1cm} (22)

where $e_2(\xi) = [1, e^{j2\pi \xi/K}]^T$, while $W_k(m) = [W(m,k), W(m,k+1)]^T$ is Gaussian distributed with zero mean and covariance matrix $\sigma^2(m) I_2$ (we denote by $I_2$ the identity matrix of order two). An estimate of $\sigma^2(m)$ is now obtained from the eigenvalue decomposition of the correlation matrix $R_X(m) = \{X_k(m) X_k^H(m)\}$. For this purpose, let $\lambda_0(m)$ and $\lambda_1(m)$ be the eigenvalues of $R_X(m)$ arranged in non-increasing order. Then, it can be easily checked that

$$\begin{cases} \lambda_0(m) = 2\sigma_1^2(m) + \sigma^2(m) \\ \lambda_1(m) = \sigma^2(m) \end{cases}$$  \hspace{1cm} (23)

with $\sigma_1^2(m) = |S(m)|^2$ denoting the power of the signal component on the $m$th subcarrier. The above result indicates that $\sigma^2(m)$ coincides with the smallest eigenvalue of $R_X(m)$. Unfortunately, the correlation matrix is not available at the receiver and must be estimated in some manner. One possible
solution is based on the forward-backward principle. Following this approach, \( R_X(m) \) is replaced by

\[
\hat{R}_X(m) = \frac{1}{2} \left[ R_X(m) + J\hat{R}_X(m)J \right]
\]  

(24)

where

\[
\hat{R}_X(m) = \frac{1}{K-1} \sum_{k=0}^{K-2} X_k(m)X_k^H(m)
\]

(25)

is the sample correlation matrix, while \( J \) is the exchange matrix with 1’s on the anti-diagonal and 0’s elsewhere. The initial estimate of \( \sigma^2(m) \), say \( \hat{\sigma}^2_0(m) \), is eventually obtained by looking for the smallest eigenvalue of \( \hat{R}_X(m) \). The latter is found after substituting (25) into (24) and reads

\[
\hat{\sigma}^2_0(m) = \frac{1}{2(K-1)} \sum_{k=0}^{K-2} \left[ |X(m,k)|^2 + |X(m,k+1)|^2 \right] - \frac{1}{K-1} \sum_{k=0}^{K-2} X(m,k+1)X^*(m,k).
\]  

(26)

C. Computational Complexity

In assessing the computational load of EMFE, we assume that the observations \( \{X(m,k)\} \) have already been computed and are available at the receiver. Then, the initialization step involves \( 2MK(2K+1) \) floating point operations (flops) to evaluate the correlations \( \{\gamma(m,k)\} \) in (14) for \( m = 0,1,\ldots,M-1 \) and \( k = 0,1,\ldots,K-1 \), while obtaining \( \hat{\sigma}^2_0(m) \) for \( m = 0,1,\ldots,M-1 \) needs only \( 7M \) further flops as all terms in the right-hand-side of (26) are available from the computation of \( \gamma(m,0) \) and \( \gamma(m,1) \). At any given iteration, evaluating \( S_{i+1}(m) \) in (10) requires \( (8K-2) \) operations for each \( m \), while obtaining \( \hat{\sigma}^2_{i+1} \) as in (21) needs 12\( MK \) flops.

Furthermore, computing \( R_i(k) \) in (13) for \( k = 1,2,\ldots,K-1 \) and \( \Delta\xi_{i+1} \) in (19) involves \( (4M-2)(K-1) \) flops plus 4\( K \) additional operations, respectively. It follows that 4\( MK-4M+2K \) flops are approximately required to determine \( \xi_{i+1} \). The overall complexity of EMFE is summarized in the first line of Table I, where \( N_i \) denotes the number of performed iterations. For comparison, we also report the computational load of the modified ML–based estimator (MMLE) presented in [8]. In writing these figures we have assumed that the MMLE frequency metric is efficiently computed using fast Fourier transform (FFT) techniques with approximately \( M(\gamma N \log_2(N) + 3K) \) flops, where the pruning factor \( \gamma = (N_\log_2 K + 2(N-K))/(N \log_2(N)) \) accounts for the computational saving achievable by skipping the operations on the zeros in the FFT [15].

![IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 8, NO. 8, AUGUST 2009](image)

**Fig. 1.** Accuracy of EMFE vs. \( \lambda \) for SNR = 10 dB.

**IV. SIMULATION RESULTS**

The investigated system has a total of \( N = 1024 \) sub-carriers and operates in the 5 GHz frequency band. The signal bandwidth is 5 MHz, corresponding to a subcarrier distance of 4.9 kHz. The discrete-time channel impulse response is composed by 12 taps collected into a vector \( h = [h(0), h(1), \ldots, h(11)]^T \). They are generated at each simulation run as statistically independent and circularly symmetric Gaussian random variables with zero mean and power \( E\{|h|\} = 1 \). The normalized CFO is uniformly distributed over the interval \((-K/2, K/2]\) and varies at each run.

In addition to background noise with variance \( \sigma_n^2 \), the received signal is affected by NBI, which occupies a bandwidth of 1 MHz covering 200 contiguous subcarriers. The NBI contribution over each jammed subcarrier is modeled as a circularly symmetric Gaussian disturbance term with zero mean and variance \( \sigma_I^2 \). We define the signal-to-noise ratio as \( SNR = 10 \log_{10}(\sigma_s^2/\sigma_n^2) \), with \( \sigma_s^2 \) denoting the average power of the received signal component, while the signal-to-interference ratio is given by \( SIR = 10 \log_{10}(\sigma_s^2/\sigma_I^2) \). For any specific SIR value, the NBI terms over distinct subcarriers are independently generated at each simulation run with the same variance \( \sigma_I^2 \). In this way, EMFE operates in a mismatched mode as \( \sigma_I^2 \) does not follow the inverse-gamma distribution in (6), which is only used to derive the frequency estimator.

We begin by assessing the impact of parameter \( \lambda \) on the accuracy of EMFE. Fig. 1 illustrates the mean square error (MSE) of the CFO estimates obtained after three iterations with \( K = 4 \) or 8. The SNR is fixed to 10 dB, while the SIR is either \(-10\) or \(-5\) dB. We see that the MSE keeps practically constant over the interval \( 0.01 \leq \lambda \leq 1 \), while a rapid increase is observed for \( \lambda > 1 \). Such behavior suggests to choose a value of \( \lambda \) adequately smaller than unity so that EMFE can work in the flat region of the MSE curve with an adequate safety margin. Since \( \lambda = 0.1 \) seems a good choice in all investigated scenarios, this value is adopted in the subsequent
Fig. 2. Accuracy of the frequency estimators vs. SNR for SIR = -5 dB and $K = 4$.

Fig. 3. Accuracy of the frequency estimators vs. SNR for SIR = -5 dB and $K = 8$.

Fig. 4. Accuracy of the frequency estimators vs. SIR for SNR = 10 dB and $K = 8$.

Fig. 5. Complexity of the frequency estimators vs. $N$ for $K = 4$ or 8.

Fig. 2 shows the frequency MSE as a function of SNR when the SIR is fixed to -5 dB. The number of iterations varies from 1 to 3 while $K$ is set to 4. Comparisons are made with the MMLE presented in [8]. In such a case, the parameter $\delta_r$ introduced in the modified likelihood function is set to 0.1. The CRB line provides a benchmark to the estimation accuracy and is given by [8]

$$\text{CRB}(\xi) = \frac{3}{2\pi^2 \zeta N (1 - 1/K^2)}$$  (27)

where

$$\zeta = \frac{1}{M} \sum_{m=0}^{M-1} \frac{|S(m)|^2}{\sigma^2(m)}$$  (28)

is the average signal-to-interference-plus-noise ratio (SINR) at the DFT output. We see that for $N_i = 3$ EMFE has approximately the same accuracy of MMLE and both schemes attain the CRB for SNR $\geq 15$ dB.

The results of Fig. 3 are obtained in the same operating conditions of Fig. 2 except for the value of $K$, which is now set to 8. Compared with Fig. 3, it turns out that the performance of the frequency estimators improves with $K$. In particular, the EMFE converges more rapidly and only marginal improvements are observed in passing from $N_i = 2$ to 3. Furthermore, the accuracy of both schemes keeps close to the CRB at all investigated SNR values.

The accuracy of the frequency estimates as a function of the SIR is shown in Fig. 4 for SNR = 10 dB, $K = 8$ and $N_i = 3$. As is seen, the frequency MSE is virtually independent of the SIR, thereby denoting a remarkable robustness against NBI.

Fig. 5 compares the computational complexity of EMFE and MMLE in terms of millions of flops versus the block length $N$. The curves are obtained from the results in Table
I after letting $K = 4$ or 8. In both cases, we see that the use of EMFE is justified only for large values of $N$. Specifically, the processing load of the two schemes is approximately the same for $N = 64$ or 128, while for $N = 1024$ EMFE allows a substantial computational saving with respect to MMLE.

V. CONCLUSIONS

We have presented an iterative CFO estimator for OFDM systems impaired by NBI. It is based on the EM algorithm and exhibits the same robustness against NBI as a recently proposed algorithm which employs an exhaustive grid-search to locate the maximum of a suitable frequency metric. The main advantage of the proposed scheme is that it provides the frequency estimates in closed-form, thereby avoiding the need for any grid-search. This results into a significant computational saving for a relatively large block length.

APPENDIX

In this Appendix we highlight the major steps leading to the computation of function $Q(\hat{\theta}|\theta_i)$ defined in (5). The latter can be rewritten as

$$Q(\hat{\theta}|\theta_i) = \int \ln[p(X|\sigma^2, \hat{\theta})] \cdot p(X|\sigma^2, \hat{\theta}) p(\sigma^2) d\sigma^2$$

(29)

where

$$p(\sigma^2) = \frac{\lambda}{\sigma_0^2(m)} e^{-\lambda/\sigma^2(m)}$$

(30)

is the a-priori pdf of $\sigma^2$ with support $\Omega = [0, +\infty)^M$. We begin by observing that

$$\ln[p(X|\sigma^2, \hat{\theta})] = -K \sum_{m=0}^{M-1} \ln[\sigma^2(m)] - \sum_{m=0}^{M-1} Y(m, \hat{\theta}_m)$$

(31)

$$p(X|\sigma^2, \hat{\theta}_i) = \prod_{m=0}^{M-1} \frac{1}{\sigma^2(m)} e^{-Y(m, \hat{\theta}_m)/\sigma^2(m)}$$

(32)

where $Y(m, \theta) = \|X(m) - S(m)e_k(\xi)\|^2$ and we have defined $\theta_m = [\xi, S(m)]^T$. Substituting (30)-(32) into (29) and letting $t_m = 1/\sigma^2(m)$, yields

$$Q(\hat{\theta}|\theta_i) = \sum_{m=0}^{M-1} Q_m(\hat{\theta}_m|\theta_m)$$

(33)

with

$$Q_m(\hat{\theta}_m|\theta_m) = -A(m)\frac{\lambda}{\pi K}$$

$$\cdot \int_0^{K} \left[ K \ln(t_m) + t_m Y(m, \hat{\theta}_m) \right] e^{-t_m[\lambda + Y(m, \hat{\theta}_m)]} d\lambda_m$$

(34)

and

$$A(m) = \prod_{\ell=0, \ell \neq m}^{M-1} \frac{\lambda}{\pi K} \int_0^{K} t_\ell e^{-t_\ell[\lambda + Y(\ell, \hat{\theta}_\ell)]} dt_\ell.$$

(35)

We now use the identity

$$\int_0^{\infty} t^\alpha e^{-\lambda t} dt = \frac{L^\alpha}{\lambda^{\alpha+1}}$$

(36)

and rewrite (34) in the equivalent form

$$Q_m(\hat{\theta}_m|\theta_m) = -\rho Y(m, \hat{\theta}_m)$$

(37)

where the quantities

$$\rho = (K + 1) \prod_{\ell=0, \ell \neq m}^{M-1} \frac{K^1}{\pi K} \left[ \lambda + Y(\ell, \hat{\theta}_\ell) \right]^{K+1}$$

(38)

and

$$B(m) = A(m) \frac{K \lambda}{\pi K} \int_0^{\infty} t^K \ln(\pi/t) e^{-\lambda[\lambda + Y(\ell, \hat{\theta}_\ell)]} dt$$

(39)

are independent of $\hat{\theta}_m$. Substituting (37) into (33) yields

$$Q(\hat{\theta}|\theta_i) = -\rho \sum_{m=0}^{M-1} Y(m, \hat{\theta}_m)$$

(40)

which coincides with $A(\hat{\theta}|\theta_0)$ in (7) after skipping the additive terms $B(m)$ and the irrelevant factor $\rho$.

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