Energy-Efficient Contention-Based Synchronization in OFDMA Systems with Discrete Powers and Limited Feedback

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Abstract—In this work, a distributed and iterative algorithm for uplink power control in the initial contention-based synchronization procedure of orthogonal frequency-division multiple access networks is derived. This is achieved by letting the mobile terminals maximize their own energy efficiency in terms of power consumption and average synchronization time exploiting a quantized feedback from the base station. The problem is formulated as a constrained finite noncooperative game in which the transmit powers are chosen from a discrete set. The theoretical solution is investigated and compared to the case of continuous powers. In addition, comparisons with existing alternatives (with or without perfect feedback) are made in terms of power expenditure, average synchronization time, and estimation accuracy.

I. INTRODUCTION

The issue of energy efficiency has attracted a huge interest by the information and telecommunication technology community in the last decade, as is witnessed by the vast literature available in this topic (see for example [1] and references therein). Most of the existing works in this field focus on the data transmission phase (e.g., see [2], [3] and references therein). In [4], the energy-efficient paradigm has been recently applied to the contention-based synchronization procedure employed in the context of orthogonal frequency-division multiple-access (OFDMA)-based technologies (such as the IEEE 802.16m [5] and the 3GPP long term evolution (LTE) [6] standards) to achieve correct synchronization with the base station (BS) references. This procedure, called initial ranging in IEEE 802.16m [7], [8] and random access in LTE [9], takes place over a specified set of subcarriers, in which each terminal notifies its entry request by transmitting a randomly chosen code. Similar operations are performed during the handover mechanism and (on a regular basis) to keep the terminals aligned in time and frequency with the BS.

In [4], the authors propose an energy-efficient and low-complexity algorithm that allows the mobile terminals and the BS to locally choose the transmit powers and the detection strategy so as to obtain a good tradeoff between detection capabilities and power consumption. The proposed method is shown to provide significant gains in terms of reduced synchronization time and parameter estimation accuracy compared to existing alternatives based on a deterministic increase of the transmit powers. However, while these techniques adopt a discrete set of powers and a limited feedback from the BS, the solution illustrated in [4] assumes the mobile terminals to have a continuous set of powers and perfect knowledge of the received signal-to-interference-plus-noise ratio (SINR) at the BS, that makes it unsuited for practical implementation. To overcome this problem, in this work the method proposed in [4] is adapted to a more practical and application-oriented context, using a finite set of transmit powers and a quantized feedback to the mobile terminals. This changes completely the nature of the optimization problem with respect to [4], as assuming discrete values of powers casts the resource allocation problem into the category of finite noncooperative games [10].

The remainder of this work is structured as follows. Section II introduces the system model and formulates the problem. Section III investigates its solution, including a comparison with the continuous case [4], whereas Section IV provides a performance assessment of the proposed algorithm. Finally, Section V concludes the paper and discusses the applicability of this technique to current wireless standards.

II. ENERGY-EFFICIENT SYSTEM DESIGN

A. System model

We consider the uplink of an OFDMA network and assume that there exists \( K \) synchronization terminals (STs) trying to synchronize with the BS references by means of a contention-based synchronization procedure. The transmission takes places over \( N \) orthogonal subcarriers with frequency spacing \( \Delta f \). We assume that \( 2N_s \) null subcarriers are placed at the spectrum edges to avoid aliasing problems, whereas \( N_s \) subcarriers are reserved to STs for the synchronization procedure. The remaining \( N - 2N_s \) subcarriers are used in an exclusive fashion (for data transmission and channel equalization) by terminals already synchronized with the BS.
The terminal already successfully synchronized (in both timing and frequency to the BS) do not generate interference over the $N_s$ subcarriers reserved for synchronization purposes [7].

- The channel frequency response $H_k(m)$ experienced by the $k$th ST over the $m$th tile is nearly flat over each tile and independent across tiles.
- Different STs choose different codes. This assumption is reasonable as long as the number of codes $|C|$ is much larger than the number of STs (i.e., $|C| \gg K$).
- As in [4], low-mobility applications are considered and the downlink frequency estimation errors are neglected.
- A quasi synchronous network in which timing errors $\theta_k$ do not produce any interblock interference (IBI) and only appear as phase shifts at the output of the receive discrete Fourier transform (DFT) unit is considered [12].
- The BS adopts a single-user strategy to detect the presence of the unknown codes $c_\ell$ in the received signal vector $X$. This amounts to saying that for each $\ell = 1, \ldots, |C|$ the following binary hypothesis test is employed: $H_\ell$ $c_\ell$ is present in $X$; and $H_{\ell'}$ $c_\ell$ is not present in $X$.

### B. Problem formulation

As in [4], the energy-efficiency of the network during the synchronization procedure is measured as a tradeoff between achieving good detection capabilities at the BS and prolonging the battery life at the ST side. The optimization problem is formulated as a function of the optimal transmit power $p_k^*$ (at the ST side) and of the optimal detection strategy $L_k$ (at the BS side). Mathematically, we have that [4]:

$$[p_k^*, L_k^*] = \arg \max_{p_k \in P_k, L_k \in \mathcal{L}_k} \ u_k(p_k, L_k) \quad (1)$$

subject to: $\Pi_{\ell_k}(p_k, L_k) = \Pi_{\ell_k}$

$$\text{MSE}(\hat{\theta}_k) \leq \overline{\text{MSE}}_{\theta}$$

for all $k \in \mathcal{K} = \{1, \ldots, K\}$ STs attempting to achieve successful network association.

- The utility function $u_k(p_k, L_k)$ is defined as

$$u_k(p_k, L_k) = \frac{\Pi_{\ell_k}(p_k, L_k)}{L_k} \quad (2)$$

where $\Pi_{\ell_k}(p_k, L_k)$ is the probability of correct detection of code $c_k$, with $p = [p_1, \ldots, p_K]$ and $L_k$ denoting the vector containing the transmit powers $p_k$ of all STs, and the detection strategy adopted at the BS for code $c_k$, respectively. In addition, $L_k = p_k T$ is ST $k$’s transmitted energy per OFDMA symbol where $T = (N + N_G) T_s$ denotes the duration of the cyclically extended OFDMA block and $N_G$ is the length of the cyclic prefix.

- $\mathcal{P}_k$ denotes the set of transmit powers of the $k$th ST. In particular, they are obtained from a minimum power level $p_k^{\text{min}}$ to a maximum one $\overline{p}_k$ using a quantization step $\Delta_k$. Otherwise stated, the power strategy set of the $k$th ST is given by

$$\mathcal{P}_k = \left\{ p_k^{(1)}, p_k^{(2)}, \ldots, p_k^{(Q_k)} \right\}, \quad (3)$$

where the number of power levels $Q_k$ is equal to

$$Q_k = 1 + \frac{\overline{p}_k - p_k^{\text{min}}}{\Delta_k}, \quad (4)$$

with $x_{\text{db}} = 10 \log_{10} x$. Hence, $p_k^{(1)} = p_k^{\text{min}}$, $p_k^{(Q_k)} = \overline{p}_k$, and $p_k^{(\ell)} = p_k^{\text{min}} + \ell \Delta_k$. For simplicity, let us assume $\Delta_k = \Delta = \overline{p}_k - p_k^{\text{min}}$ and $\overline{p}_k = \overline{p}$ for all $k \in \mathcal{K}$. This implies $Q_k = Q$ for all $k$.

- $\mathcal{L}_k$ is the single-user detection strategy set, which groups all possible single-user detection strategies that can be adopted by the BS.

- $\Pi_{\ell_k}(p_k, L_k)$ is the probability of false alarm occurring when the BS chooses $H_k$ when $c_k$ is not actually present in $X$. In addition, $\overline{\Pi_{\ell_k}}$ is the system quality-of-service (QoS) requirement in terms of maximum tolerable probability of false alarm.

- MSE($\hat{\theta}_k$) = $\mathbb{E}\{\|\hat{\theta}_k - \theta_k\|^2\}$ is the mean-square-error (MSE) of the timing estimation $\hat{\theta}_k$, with $\mathbb{E}\{\cdot\}$ denoting expectation whereas $\overline{\text{MSE}}_{\theta}$ is the QoS requirement in terms of maximum MSE that allows us to limit the impact of IBI at the receiver.

Unlike the formulation proposed in [4], imposing finite sets $\mathcal{P}_k$ allows us to meet the technical requirements of practical systems in which transmit powers are usually equally spaced on a logarithmic scale to reduce the complexity of the front-end architecture and to increase the efficiency of power amplifiers. As will be better detailed in Section III, this assumption not only modifies the nature of the optimization problem in (1), but also the properties of its solution.

Note that (1) cannot be solved at the BS in a centralized manner, since it is not known in advance which STs are transmitting. This means that the optimal $p_k^*$ in (1) must be computed by each $k$th ST in a decentralized manner. Moreover, (1) cannot be decoupled into $K$ subproblems since $\Pi_{\ell_k}(p_k, L_k)$ depends not only on $p_k$, but also on all other STs’ transmit powers $p_{k'} = [p_1, \ldots, p_{k-1}, p_{k+1}, \ldots, p_K]$. To overcome these problems, we resort to the analytical tools of noncooperative game theory [10].

### III. Solution of the game

#### A. Analysis of the equilibria

The problem formulation in (1) can be restated as a noncooperative game with complete information [10], defined as $\mathcal{G} = (\mathcal{K}, \{A_k\}, \{u_k\})$, where: $\mathcal{K} = \{1, \ldots, K\}$ is the player set, in which player $k$ is the pair composed by the ST using
code $c_k$, and its receiver at the BS; $A_k = \mathcal{P}_k \times \mathcal{L}_k$ with $\mathcal{P}_k$ and $\mathcal{L}_k$ defined in Section II-B; and $u_k$ is player $k$’s payoff function (2).

As mentioned before, the game $\mathcal{G}$ belongs to the category of finite noncooperative games since the number of possible power strategies $p$ is equal to $Q^K < \infty$. Therefore, all the properties of the continuous version of the game $\mathcal{G}_c$, discussed in [4], do not hold anymore for $\mathcal{G}$ (due to the discrete nature of the action sets [10]).

To proceed with the solution of (1), we first observe that the constraint on the MSE can be met provided that (see [4] for more details)

$$p_k \geq \frac{1}{\nu_k(p_k)} \cdot \frac{3N^2}{2M\pi^2(V^2 - 1)} \cdot \frac{1}{\rho} = \frac{1}{\nu_k(p_k)} \cdot \gamma_{req}. \quad (5)$$

In the above equation, $N$, $M$, and $V$ are the system parameters introduced in Section II-A whereas $\nu_k(p_k)$ is defined as

$$\nu_k(p_k) = \frac{\alpha_k}{\sigma_n^2 + \sum_{\ell \neq k} \alpha_\ell p_\ell} \quad (6)$$

with $\sigma_n^2$ being the additive white Gaussian noise (AWGN) power and

$$\alpha_\ell = \frac{1}{M} \sum_{m=0}^{M-1} |H_\ell(m)|^2 \quad (7)$$

denoting ST $\ell$’s average channel power gain across tiles. In addition, we have that

$$\rho = M\text{SE}_d - \mu^2(\hat{\theta}) \quad (8)$$

with $\mu(\hat{\theta}) = \mathbb{E}\{\hat{\theta}_k\} - \theta_k$ denoting the bias of the timing estimate $\hat{\theta}_k$ with respect to $\theta_k$. Observe that in writing (5) we have approximated the variance of the timing estimate $\text{var}(\hat{\theta}_k) = \mathbb{E}\{|\theta_k - \hat{\theta}_k - \mu(\hat{\theta}_k)|^2\}$ with its Cramér-Rao bound [13].

From (5), it follows that meeting the constraint on the MSE requires to restrict ST $k$’s power set to the following subset:

$$\mathcal{P}_k(p_k) = \left\{ \pi_k^{(q)} \in \mathcal{P}_k : \pi_k^{(q)} \geq \gamma_{req}/\nu_k(p_k) \right\} \subseteq \mathcal{P}_k, \quad (9)$$

where $\gamma_{req}$ represents the minimum received SINR at the BS that guarantees a sufficient accuracy on the timing offset estimation. Since $\mathcal{P}_k(p_k)$ depends on the other STs’ powers $p_{\ell,k}$, the game $\mathcal{G}$ falls in the category of generalized Nash games [10], [14] whose solution is given by the generalized Nash equilibrium (GNE).

Using the subsets $\mathcal{P}_k(p_{\ell,k})$ allows us to decouple the optimization problem in $L_k^*$ and $p_k$, and thus to compute the optimal strategy $L_k^*$ without explicitly solving the game [4]. Using the Neyman-Pearson theorem [13], the optimal $L_k^*$ is found to be the generalized likelihood ratio test (GLRT) for each code $c_k \in \mathcal{C}$ [4]. In these circumstances, the probabilities of false alarm and correct detection are respectively given by:

$$\Pi_{fa,k}(p, L_k^*) = I_{1 - \xi(\gamma_k)}[M(V - 1), M] \quad (10)$$

and

$$\Pi_{d,k}(p, L_k^*) = I_{1 - \xi(\gamma_k)}[M(V - 1), M] \quad (11)$$

where

$$I_z[i,j] = \sum_{n=1}^{i+j-1} \left( \begin{array}{c} i+j-1 \\ n \end{array} \right) x^n(1-x)^{i+j-1-n} \quad (12)$$

denotes the incomplete beta function [15], $\lambda$ is the detection threshold at the BS such that

$$\Pi_{fa,k}(p, L_k^*) = I_{1 - \lambda}[M(V - 1), M] = \Pi_{fa} \quad (13)$$

and

$$\xi(\gamma_k) = \frac{\lambda}{1 + (1 - \lambda)\gamma_k} \quad (14)$$

with $\gamma_k = \nu_k(p_k)p_k$ being ST $k$’s received SINR.

To study the equilibrium of $\mathcal{G}$, let us consider it from another point of view: a vector $p^* = [p_1^*, \ldots, p_K^*]$ is a GNE of $\mathcal{G}$ if each element $p_k^*$ is the best response $r_k(p_{\ell,k}^*)$ to the powers $p_{\ell,k}^*$ chosen by the other players, with $r_k(p_{\ell,k})$ being the map that assigns to each $p_{\ell,k}$ the power level

$$r_k(p_{\ell,k}) = \arg \max_{\pi_k^{(q)} \in \mathcal{P}_k(p_{\ell,k})} \nu_k(\pi_k^{(q)}) \quad (15)$$

where we have omitted the dependence of $u_k$ and $\Pi_{d,k}$ on $L_k^*$ for notational simplicity.

Theorem 1: Let us define the optimal SINR $\gamma^*$ as

$$\gamma^* = \max(\gamma_{req}, \gamma) \quad (17)$$

with $\gamma$ being the solution of

$$\frac{\partial I_{1-\xi(\gamma)}[M(V - 1), M]}{\partial \gamma} \bigg|_{\gamma = \gamma} = I_{1 - \xi(\gamma)}[M(V - 1), M]. \quad (18)$$

The game $\mathcal{G}$ admits pure-strategy GNEs provided that

$$\gamma^*(K - 1) < V, \quad (19)$$

and the number of equilibria is in general $|\mathcal{P}^*| \geq 1$, where $\mathcal{P}^*$ is the set of GNEs (i.e., the GNE may be not unique). The proof is not reported here for the sake of brevity, and can be found in [16]. We limit to observe that it is based on [17] to derive the sufficient condition (19); on the ascending property [18] of the best response, and on the supermodularity [19] of the utility function to prove the existence; and on a counterexample to disprove the uniqueness of the GNE. Note that, unlike $\mathcal{G}_c$ in [4], the GNEs of $\mathcal{G}$ are not unique.

Theorem 2: Among all $p^* \in \mathcal{P}^*$, the smallest component-wise GNE $p^*_\Delta$ is such that

$$p^*_\Delta = \arg \max_{p^* \in \mathcal{P}^*} \sum_{k=1}^{K} u_k(p^*). \quad (20)$$
The proof can be found in [16], and easily follows from the definition of the GNE [18], [20]. The above result states that $p^*_\Delta$ is the best GNE in terms of social welfare (joint optimization), or, equivalently, that it is the most efficient GNE in a social sense [21]. Note that $p^*_\Delta$ is not the social optimum solution, as noncooperative equilibria are generally inefficient [21]. However, improving the equilibrium efficiency is out of the scope of this paper and left for future work.

B. Numerical investigation of the equilibria

Unlike the unique GNE of $G_c$ in [4], the multiple equilibria of $G$ cannot be expressed in a closed form as a function of the network parameters because of the $\max \arg \max$ operator in (15). A numerical analysis is thus conducted to make comparisons and to evaluate the impact of the discretization of the action sets. To this aim, we concentrate on the optimal (in a social sense) $p^*_\Delta$ and resort to the exhaustive search method described in [22] to solve (20). The numerical results are averaged over 20,000 independent realizations of a network with the following parameters: $T_s = 89.28$ ns, $N = 1024$, $M = 4$, $V = 36$, $\overline{\gamma}_n = 10^{-5}$, and $\text{MSE}_{\text{avg}} = 324$, which yield $\rho = 128$, $\lambda = 0.12$, $\gamma_{\text{req}}|_{\text{dB}} = -6.19$, and $\gamma^*|_{\text{dB}} = 7.09$ (see [4] for a detail discussion on this parameter setting). The normalized power constraints are fixed to $p/\sigma_n^2|_{\text{dB}} = -20$ and $\overline{p}/\sigma_n^2|_{\text{dB}} = +30$ for all $k$, whereas the ST distances $d_k$ are randomly chosen from a uniform distribution in $[R/10, R]$ with $R$ being the cell radius. The channel power gains are normalized to a distance $R/2$ and are modeled using the 6-tap ITU modified vehicular-A model [23] with a path loss exponent $\zeta = 2$.

Fig. 1 reports the normalized MSE defined as $\text{NMSE}(p^*_c) = \mathbb{E}\{\|p^*_c - p^*_\Delta\|^2/\|p^*_\Delta\|^2\}$ as a function of $K$ for different quantization steps, where $p^*_c$ is the unique GNE of $G_c$. The maximum number of STs is fixed to $[1 + V/\gamma^*] = 8$. Observe that the condition $K \leq 8$ is required to meet (19).

In addition, it represents a necessary and sufficient condition for the existence of the unique GNE $p^*_c$ [4]. As expected, $\text{NMSE}(p^*_c)$ decreases as $\Delta$ becomes smaller since the discrete action sets in $G$ tend to better approximate the continuous ones in $G_c$. As can be seen, $\text{NMSE}(p^*_c)$ increases as $K$ increases, meaning that the difference between $\|p^*_\Delta\|^2$ and $\|p^*_c\|^2$ becomes larger as the number of STs increases. In particular, we see that $\text{NMSE}(p^*_c)$ is almost constant up to $K = 4$, whereas it rapidly increases for larger values.

To conclude this analysis, it is worth emphasizing that the numerical results, including the investigation on the efficiency of the GNEs (not reported for the sake of brevity), confirm the validity of Theorem 1 and Theorem 2. In particular, note that the average number of GNEs increases as $K$ increases. For instance, when $\Delta|_{\text{dB}} = 1$ the experimental average number of GNEs is $|\mathcal{P}^*| = \{1.0, 1.1, 1.2, 1.3, 1.7, 2.7, 20.9\}$ for $2 \leq K \leq 8$.

IV. PERFORMANCE ASSESSMENT

A. Impact of quantized feedback and discrete powers

In this subsection, we compare the performance of the iterative algorithm described in [4], termed best-response synchronization algorithm (BRSA) and using continuous powers, with that of a modified version, in which discrete power levels are used instead. To this aim, we adapt the BRSA to the discrete case, by using the best response (16) as the update rule. Note that this mechanism, under the hypothesis (19) and initial powers $p_0$ for all $k$, lets each active ST achieve the smallest component-wise equilibrium $p^*_\Delta$ [20], and thus the optimal GNE (in a social sense) (Theorem 2). Interestingly, (16) can be implemented in a distributed and iterative way, as $p_k[n] + 1 = r_k(p_{-k}[n])$ can be computed using

$$p_k(p_{-k}[n]) = \gamma_k[n]/p_k[n],$$

where $n$ is the time step of the algorithm, and $\gamma_k[n]$ is the SINR estimated by the BS at time step $n$ [4], which can be fed back on a downlink common channel.

To further reduce the amount of feedback information, we assume the quantity $\gamma_k[n]$ to be quantized on a logarithmic scale using a uniform B-bit quantizer over the range $[\gamma, \overline{\gamma}]$ of typical expected SINR values, that can be set to $\gamma|_{\text{dB}} = -8$ and $\overline{\gamma}|_{\text{dB}} = +16$, respectively, based on an extensive simulation campaign [16]. Further details on this modified algorithm, termed discrete and limited feedback BRSA (DLF-BRSA) in the sequel, can be found in [16].

The numerical results reported here are obtained by averaging 20,000 independent realizations of a network whose parameters are those listed in Section III-B. The normalized power constraints are fixed to $p/\sigma_n^2|_{\text{dB}} = -20$ and $\overline{p}/\sigma_n^2|_{\text{dB}} = +30$, and the same power initialization $p_k[0] = p$ is used for all STs $k \in K$, which also use a common power quantization step $\Delta|_{\text{dB}} = 1$. In particular, Fig. 2 reports the average normalized power expenditure $p_{\text{avg}}/\sigma_n^2$ (in dB) required by DLF-BRSA for successfully completing the synchronization procedure. The numerical results are plotted as functions of $K$ for $B = \{1, 2, 3, 8\}$. The results obtained with DLF-BRSA
when $B \to \infty$ (i.e., with continuous-SINR feedback) are used as a benchmark. Comparisons are also made with the BRSA illustrated in [4] in which the action sets are continuous and perfect knowledge of the estimated SINRs is available at the STs. To evaluate the impact of the discretization of the action sets in terms of transmit power, let us compare the performance of the DLF-BRSA with continuous feedback (upper triangles) with that provided by the BRSA (square markers). As can be seen, the difference between the two algorithms is negligible. The motivation for this outcome, which apparently is contradictory with the results shown in Section III-B for $K \geq 5$, is due to the time evolution of the transmit powers during the two games, as detailed in [16].

To measure the effects of SINR quantization, we can consider the performance obtained as a function of the quantization bits $B$. As can be seen, the results of Fig. 2 indicate that the quantization of the SINRs has only a marginal effect on the performance of DLF-BRSA. In fact, it has practically the same performance for $B = 3, 8$ and $B \to \infty$, whereas a significant degradation is observed only for $B = 1$. We argue that the quantization of the estimated SINRs marginally impacts the system performance, since it is basically perceived at the STs as an additional estimation error introduced by the BS (which can instead exploit real-valued estimation methods).

As a conclusion, using discrete powers, with $\Delta dB = 1$, and quantized estimated SINRs, with $B = 3$, yields similar performance to the scenario described in [4], while providing relevant gains in terms of complexity of the transmitter (thanks to the discretization of the amplifier power) and of reduced amount of feedback information (thanks to a finite number of bits $B$). Based on the above results, in all subsequent simulations we set $\Delta dB = 1$ and $B = 3$.

**B. Comparisons with existing alternatives**

To evaluate the improvement of the DLF-BRSA in terms of energy efficiency, which translates into power saving and reduced average synchronization time, in this subsection we compare its performance with that achieved by two alternative solutions based on a deterministic increase of the transmit power: the deterministic synchronization algorithm (DSA), in which the update rule is $p_k[n+1]dB = p_k[n]dB + \Delta dB$; and the binary exponential backoff DSA (BEB-DSA), in which $p_k[n+n_b]dB = p_k[n]dB + \Delta dB$, where $n_b$ is an exponentially-distributed backoff counter (see [4] for more details). For a fair comparison, $p_k[0] = p$ and $\Delta dB = 1$ for all algorithms and all STs, with the same parameters introduced in Section III-B.

Figs. 3-5 report the results of the DLF-BRSA with triangular markers, whereas the performance of DSA and BEB-DSA is plotted using circular and square markers, respectively. Fig. 3 shows $p_{avg}/\sigma_n^2$ as a function of the normalized distance $d_k/R$ from the BS to the ST of interest. Similarly, Fig. 4 depicts the average time $T_{avg}$ needed to complete a successful synchronization as a function of $d_k/R$, computed by scaling the average number of algorithm iterations by the frame duration $T_f = 5$ ms, which is the interval between two successive synchronization attempts. Finally, Fig. 5 shows the MSE of the timing estimate $\hat{\theta}_k$ for different distances $d_k/R$, also reporting the constraint on the maximum MSE $\text{MSE}_\theta$. As can be seen, the DLF-BRSA provides roughly the same power consumption of the BEB-DSA, which is significantly lower than the DSA case, while using a synchronization time which is shorter compared to DSA, and much shorter compared to BEB-DSA, especially as $d_k/R$ increases, and achieving higher estimation accuracy (similar results, not reported for the sake of brevity, are obtained for the MSE of the receiver power estimate).

Hence, the DLF-BRSA provides better results in terms of both energy efficiency, fairness, and parameter estimation accuracy, at the expense of a minimal increase in the feedback rate at the BS (whose applicability is discussed in the following subsection): in detail, during each synchronization frame the BS needs to broadcast, for each $c_r \in C$, the outcome of the GLRT (1 bit) and the quantized SINR ($B = 3$ bits), for a total of $|C| \cdot (B + 1)$ bits per frame time $T_f$. 

**Fig. 2.** Average power consumption as a function of the number of STs.

**Fig. 3.** Average power consumption as a function of the normalized distance.
V. DISCUSSION

In this paper, we have focused on the energy-efficient contention-based synchronization framework proposed in [4] and we have assumed sets of discrete powers at the mobile terminals and a limited feedback from the BS. We have studied the resulting resource allocation problem as a finite noncooperative game and we have derived the analytical properties of its equilibrium points. The performance of the above solution has been evaluated and compared with alternatives by means of numerical simulations. Using realistic system parameters and widely agreed-upon channel models, we have shown that the proposed solution incurs only a negligible degradation with respect to the scheme illustrated in [4], while a significant gain is achieved with respect to deterministic-based power allocation approaches (both with and without contention resolution methods). This achieved by exploiting a feedback on the downlink on the order of a few tens of kb/s, which can be easily accommodated in current IEEE 802.16m [5] and LTE [6] standards.

REFERENCES