# *Target Signal Models*

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# The Doppler Shift-Sinusoidal waveform

- If the radar and target are not at rest with respect to one another, the frequency $f_R$ of the received echo will differ from the transmitted frequency $f_0$ due to the Doppler effect. A correct description of the Doppler shift for the electromagnetic waves requires the theory of special relativity.

- Consider a monostatic radar that is not moving and suppose that the target is moving with a radial velocity *v* toward the radar. The relativity theory predict that, for a sinusoid of infinite duration and a constant velocity v, the received frequency will be:

$$f_R = \left( \frac{1 + v/c}{1 - v/c} \right) f_0$$

- An approaching target causes an increase in the received frequency, a receding target a decrease.

# The Doppler Shift

- Previous equation can be simplified without loss of precision because the velocity of actual targets is a small fraction of *c*. For example, the value of *v/c* for a supersonic aircraft traveling at Mach 2 (about 660 m/s) is only 2.2e-6. Then expanding the denominator of the equation:

$$f_R = \left(\frac{1+v/c}{1-v/c}\right) f_0 = (1+v/c)\left(1+v/c+(v/c)^2+\ldots\right) f_0$$
$$= \left(1+2v/c+2(v/c)^2+\ldots\right) f_0$$

- Discarding all second-order and higher terms in *v/c* we get

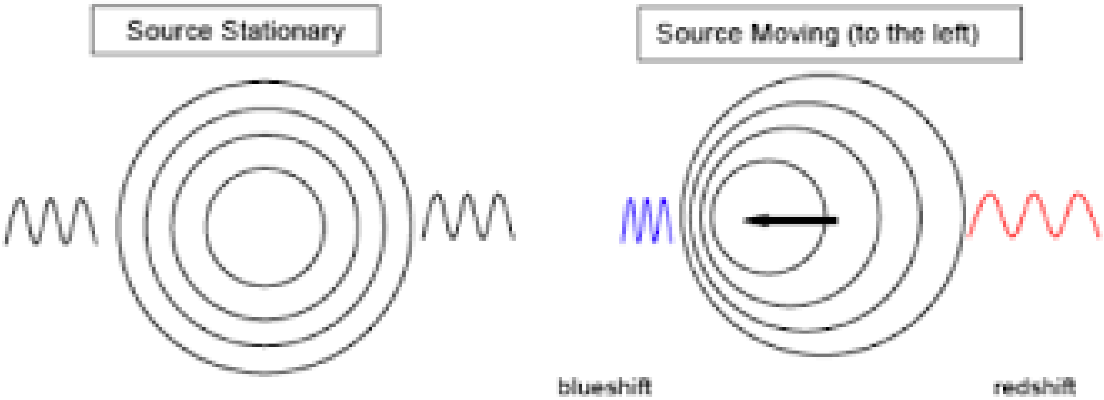
$$f_R \cong (1+2v/c)\, f_0$$

- The difference $f_D$ between the transmitted and received frequencies is called the Doppler frequency or Doppler shift. For an **approaching target** it is:

$$f_D = 2\frac{v}{c} f_0 = 2\frac{v}{\lambda}$$

# The Doppler Shift

**Example 1:** An airplane moving at Mach 1 along the antenna boresight of a 10 GHz radar creates a Doppler shift of 22.87 kHz.
**Example 2:** The SCR-270 radar in use at Pearl Harbor during the Japanese attack on December 7, 1941, operated at 106 MHz and an A6M Zero attack aircraft had a diving speed of around 400 mi/hr. That corresponds to a Doppler shift of a mere 633 Hz.




$$f_D = 2\frac{v}{c} f_0 = 2\frac{v}{\lambda}$$

# The Doppler Shift-Arbitrary waveform

- Let's now generalize the Doppler shift formula for arbitrary motion between the radar and the target and for any waveform using a classical physics approach.

- The radar echo from a moving target with varying time delay $\tau(t)$ can we written as (ignoring amplitude scale factors)

$$y(t) = x\left[t - \tau(t)\right]$$

- For example, suppose that the signal $x(t)$ is a simple pulse of the form

$$x(t) = \mathrm{Re}\left\{s(t)\exp\left(j2\pi f_0 t\right)\right\}$$

then

$$y(t) = \mathrm{Re}\left\{s\left[t - \tau(t)\right]\exp\left[j2\pi f_0\left(t - \tau(t)\right)\right]\right\}$$

# The Doppler Shift-Arbitrary waveform

- A point on waveform $y(t)$ received at time t was transmitted at [t-τ(t)]; this point was incident on the target at time [t-τ(t)/2], at which time the target range was R[t-τ(t)/2]. The round-trip distance traveled by this point on the waveform is 2R[t-τ(t)/2]; this distance, from the definition of τ(t), is also equal to cτ(t); that is (receding target)

$$2R\left[t-\tau(t)/2\right]=c\tau(t)\Rightarrow R_0+v\left[t-\tau(t)/2\right]=c\tau(t)/2$$

$$\Rightarrow \tau(t)=\frac{2R_0/c}{1+v/c}+\frac{2vt/c}{1+v/c}$$

$$y(t)=\mathrm{Re}\left\{s\left[\left(\frac{c-v}{c+v}\right)t-\frac{2R_0/c}{1+v/c}\right]\exp\left[j2\pi f_0\left(\left(\frac{c-v}{c+v}\right)t-\frac{2R_0/c}{1+v/c}\right)\right]\right\}$$

$$=\mathrm{Re}\left\{s\left[\left(\frac{c-v}{c+v}\right)\left(t-\tau_0^{'}\right)\right]\exp\left[j2\pi f_0\left(\frac{c-v}{c+v}\right)\left(t-\tau_0^{'}\right)\right]\right\}$$

$$\tau_0^{'}=\frac{2R_0/c}{1-v/c}$$

# The Doppler Shift-Arbitrary waveform

$$y(t) = \mathrm{Re}\left\{ s\left[\left(\frac{c-v}{c+v}\right)\left(t-\tau_0^{'}\right)\right]\exp\left[ j2\pi f_0\left(\frac{c-v}{c+v}\right)\left(t-\tau_0^{'}\right)\right]\right\}$$

Stretching or compression factor, depending on the sign of v. For narrow-band signals this effect is negligible.

For many applications $\tau_0^{'} \cong \tau_0 = \dfrac{2R_0}{c}$

$$y(t) \cong \mathrm{Re}\left\{ s\left(t-\tau_0\right)\exp\left[ j2\pi f_0\left(\frac{c-v}{c+v}\right)\left(t-\tau_0\right)\right]\right\}$$

$$\frac{c-v}{c+v} = \frac{1-v/c}{1+v/c} \cong \left(1-\frac{v}{c}\right)^2 \cong 1-\frac{2v}{c}$$

$$y(t) \cong \mathrm{Re}\left\{ s\left(t-\tau_0\right)\exp\left[ j2\pi\left(f_0+f_D\right)\left(t-\tau_0\right)\right]\right\}$$

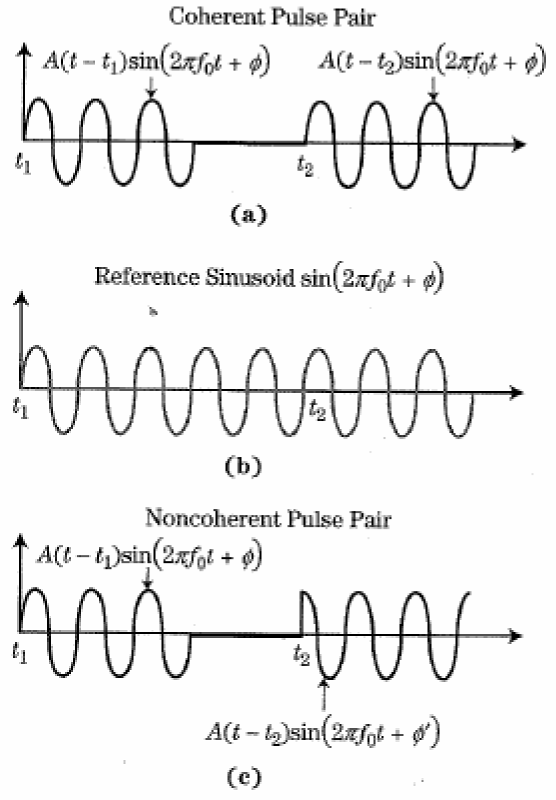
where $f_D = -\dfrac{2v}{c} f_0$ , for a receding target

# The Doppler Shift-Arbitrary waveform

- Now let's consider only the complex envelope of the target signal and sample it at time $t=nT+\tau_0$. The sampled echo is

$$\tilde{y}(nT+\tau_0) = A(n)e^{j\theta(n)}\exp\left[j2\pi f_D nT\right]$$

- To measure the Doppler shift using multiple pulses, a deterministic phase relationship from pulse to pulse must be maintained over the CPI so that phase shifts measured by the coherent radar are due to relative radar-target motion only. In this figure two pulse pairs are shown, one coherent and one not.



# Swerling target models

- Each Swerling model is a combination of a probability density function and a decorrelation time for the target RCS. They are formed from the four combinations of two choices for the pdf and two for the decorrelation time.

- Swerling considered two extreme cases for the correlation properties of this block of N measurement of the RCS. The first assumes they are all perfectly correlated, so that all N pulses collected on one sweep have the same value. The N new pulses collected on the next antenna sweep all have the same value as one another also, but their value is independent of the value measured on the first sweep. This case is referred to as **scan-to-scan decorrelation.**

- The second case assumes that each individual pulse on each sweep results in an independent value for the RCS. This case is referred to as **pulse-to-pulse decorrelation**.

# Swerling target models

- The model called **Swerling I** assumes that the **amplitude** of an entire pulse train is a single random variable with a **Rayleigh pdf**. In addition the initial **phase** of each pulse is assumed to be the same and statistically independent random variable with a **uniform** pdf.

$$\underset{N\times 1}{\mathbf{s}_t} = \beta\,\mathbf{p}(f_d) = |\beta| e^{j\theta}\mathbf{p}(f_d) = Ae^{j\theta}\mathbf{p}(f_d)$$

$$p_A(a) = \frac{a}{A_0^2}\exp\left(-\frac{a^2}{2A_0^2}\right)u(a) \qquad \theta \in U(0, 2\pi)$$

- **Swerling II** differs from Swerling I in that the **amplitude** of each pulse in the train is a **statistically independent random variable,** with the same Rayleigh pdf. The phases of each pulse in the train are assumed to be independent with uniform probability

$$s_t(n) = A(n)e^{j\theta(n)}$$

# Swerling target models

**Swerling III** is similar to Swerling I in that each pulse in the train has the same amplitude. In the Swerling III model, however, pulse-train amplitude is assumed to be a random variable with a **one-dominant-plus-Rayleigh** pdf function given by

$$\underset{N\times 1}{\mathbf{s}_t} = \beta\,\mathbf{p}(f_d) = |\beta| e^{j\theta}\mathbf{p}(f_d) = A e^{j\theta}\mathbf{p}(f_d)$$

$$p_A(a) = \frac{9a^3}{2A_0^4}\exp\left(-\frac{3a^2}{2A_0^2}\right)u(a) \qquad \theta \in U(0, 2\pi)$$

**Swerling IV** is similar to Swerling II, however, pulse-train amplitude is assumed to be a random variable with a **one-dominant-plus-Rayleigh** pdf.

$$s_t(n) = A(n)e^{j\theta(n)}$$