Computer Engineering
Electronics and Communications Systems

**MIMO & 5G Cellular Communications**

Marco Luise
marco.luise@unipi.it

Dip. Ingegneria dell’Informazione, Univ. Pisa, Pisa, Italy
Multiantenna Systems 1: Reception Diversity

At time $mT$...

\[ r_1^{(m)} = s^{(m)} + n_1^{(m)} \]

\[ r_2^{(m)} = s^{(m)} + n_2^{(m)} \]

\[ z^{(m)} = \frac{r_1^{(m)} + r_2^{(m)}}{2} = s^{(m)} + \frac{n_1^{(m)} + n_2^{(m)}}{2} = s^{(m)} + n^{(m)} \]

- 1 transmitting antenna
- 2 receiving antennas with combining
- No increase in bit-rate
- DIVERSITY Gain = 3 dB

The noise variance is reduced by a factor 2
Multiantenna Systems 2: Independent Channels

- 2 transmitting antennas
- 2 receiving antennas
- No mutual interference
- No diversity gain
- MULTIPLEXING GAIN x 2

The bit-rate is DOUBLED
The terms $h_{ij}$ are just complex coefficients that modify the amplitude/phase of the transmitted symbols because the channel is non-frequency-selective!

\[
\begin{align*}
    r_1^{(m)} &= h_{11}s_1^{(m)} + h_{12}s_2^{(m)} \\
    r_2^{(m)} &= h_{21}s_1^{(m)} + h_{22}s_2^{(m)}
\end{align*}
\]
How can we possibly have a narrowband channel at the high signaling rates of modern communications?

\[
\begin{align*}
    r_1^{(m)} &= h_{11}s_1^{(m)} + h_{12}s_2^{(m)} \\
    r_2^{(m)} &= h_{21}s_1^{(m)} + h_{22}s_2^{(m)}
\end{align*}
\]
The flat MIMO channel is actually experienced on each single frequency \( k/T_{MC} \) of the subcarriers raster in OFDM communications!

\[
\begin{align*}
    r_{1,k}^{(m)} &= H_{11} \left( \frac{k}{T_{MC}} \right) s_{1,k}^{(m)} + H_{12} \left( \frac{k}{T_{MC}} \right) s_{2,k}^{(m)} \\
    r_{2,k}^{(m)} &= H_{21} \left( \frac{k}{T_{MC}} \right) s_{1,k}^{(m)} + H_{22} \left( \frac{k}{T_{MC}} \right) s_{2,k}^{(m)}
\end{align*}
\]

\( k = 0, \ldots, N - 1 \)
General MIMO (Multiantenna) System

- $N_T$ transmitting antennas
- $N_R$ receiving antennas
- low-rate channels with flat-fading
- $N_R \times N_T$ channel matrix $H$

The channels are “different” in SPACE
The complex-valued MIMO channel at time $m$

- **Independent (equi-power Rayleigh) Channels**
- **Spatially White equi-power noise**
- **Stationary channel (long coherence time)**

\[
\mathbf{r}_m = \mathbf{H}\mathbf{s}_m + \mathbf{w}_m
\]

\[
\mathbf{w}_m = \begin{bmatrix}
W_1^{(m)} \\
W_2^{(m)} \\
\vdots \\
W_{N_R}^{(m)}
\end{bmatrix}
\]

\[
\mathbf{r}_m = \begin{bmatrix}
r_1^{(m)} \\
r_2^{(m)} \\
\vdots \\
r_{N_R}^{(m)}
\end{bmatrix}
\]

- **Transmitted symbols vector**
- **Channel Matrix**
- **Received (soft) symbols vector**
Simple Examples

- Diagonal matrix ($N_R = N_T$): independent channels, max. multiplexing gain, no diversity gain
- $N_R \times 1$ matrix: conventional diversity reception, max. diversity gain, no multiplexing gain
**MIMO Reception (spatial MUX)**

\[ r_m = Hs_m + w_m \]

\[ z_m = \hat{H}^{-1}r_m = \hat{H}^{-1}(Hs_m + w_m) = s_m + \hat{H}^{-1}w_m = s_m + w'_m \]

- Need to know the channel matrix \( H \)
- \( H \) has to be non-singular
- Possible noise enhancement \( \rightarrow \) need to reduce constellation size
The MIMO Channel

- **DIVERSITY** gain by combination of multiple RX copies of the same signal with *independent* noise components (improves SNR)

- **MULTIPLEXING** gain by using multiple *independent* channels with multiple TX/RX antennas (improves rate) but increases noise and increases bit-rate less than expected

**SNR/Rate relation through capacity theorem**
Compute the SVD of $H$: \[ H = UDV^\dagger \]

\[ H = \begin{bmatrix} N_T \\ N_R \end{bmatrix} \]

\[ UU^\dagger = I_{N_R} \]

\[ VV^\dagger = I_{N_T} \]

Unitary Matrix

Unitary Matrix

$N_R > N_T$, uplink-like

\[ \sqrt{\text{eigen}(HH^\dagger)} \]

and the channels are DECOUPLED
\[
\mathbf{r}_m = \mathbf{H}s_m + \mathbf{w}_m = \mathbf{U}\mathbf{D}\mathbf{V}^\dagger\mathbf{s}_m + \mathbf{w}_m = \mathbf{U}\mathbf{D}(\mathbf{V}^\dagger\mathbf{s}_m) + \mathbf{w}_m = \mathbf{U}\mathbf{D}\mathbf{s}_m' + \mathbf{w}_m
\]

and also

\[
\mathbf{r}_m' = \mathbf{U}^\dagger\mathbf{r}_m = \mathbf{U}^\dagger(\mathbf{U}\mathbf{D}\mathbf{s}_m' + \mathbf{w}_m) = \mathbf{D}\mathbf{s}_m' + \mathbf{w}_m'
\]

\[
r_i''(m) = d_is_i''(m) + w_i''(m), \quad i = 1, \ldots, N_T
\]

I am left with an equivalent set of \(N_T\) independent (parallel) channels whose capacity is evaluated via the Shannon formula for the AWGN channel.
If we want to actually “exploit” such parallel channels, we have to pre-code $s_m$ by $V$ before transmitting, and then post-process $r_m$ by $U^\dagger$ disregarding the excess $N_R-N_T$ useless channels.

$$D = \begin{bmatrix} d_1 & 0 \\ \vdots & \ddots \\ 0 & d_{N_T} \end{bmatrix}$$
Now, if the TX has knowledge of $\mathbf{D}$ then it can i) allocate different power on the different “virtual channels”, and ii) allocate it in an optimal fashion by WATER FILLING! (independent channels)

\[
C = \sum_{k=1}^{N_T} \log_2 \left( 1 + \frac{d_k^2 P_k}{\sigma^2} \right), \quad P_k + \frac{\sigma^2}{d_k^2} = \text{const.}
\]

**Constraint:**

\[
\sum_{k=1}^{N_T} P_k = P_{TOT}
\]
Cannot do water-filling – I just distribute power evenly, $P_k = \frac{P_{TOT}}{N_T}$:

$$C = \sum_{k=1}^{N_T} \log_2 \left( 1 + \frac{d_k^2 P_{TOT}}{\sigma^2 N_T} \right) = \log_2 \prod_{k=1}^{N_T} \left( 1 + \frac{d_k^2 P_{TOT}}{\sigma^2 N_T} \right)$$

**BUT**

$$d_k^2 = \text{eigen}_k (HH^\dagger) \Rightarrow 1 + \frac{d_k^2 P_{TOT}}{\sigma^2 N_T} = \text{eigen}_k \left( I_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T} HH^\dagger \right)$$

$$C = \log_2 \prod_{k=1}^{N_T} \text{eigen}_k \left( I_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T} HH^\dagger \right) = \log_2 \det \left( I_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T} HH^\dagger \right)$$
Ergodic MIMO Capacity (Telatar)

\[ C = \log_2 \det \left( \mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T} \mathbf{H} \mathbf{H}^\dagger \right) \]

The entries \( h_{ij} \) of channel matrix are actually ZERO-MEAN GAUSSIAN (Rayleigh Fading) independent of each other.

What we have computed up to now has to be averaged over the channel realizations:

\[ C_E = E_{\mathbf{H}} \left\{ \log_2 \det \left( \mathbf{I}_{\min(N_R,N_T)} + \frac{P}{\sigma^2 N_T} \mathbf{H} \mathbf{H}^\dagger \right) \right\} \]
Ergodic MIMO Capacity (Telatar)

\[
C = E_H \log_2 \det \left( \mathbf{I}_{\min(N_R, N_T)} + \frac{P}{\sigma^2 N_T} \mathbf{HH}^\dagger \right)
\]

Leads to the notion of

MASSIVE MIMO (very many antennas at the BTS or AP)

\(N_T \to \infty\) (Downlink): \[C = N_R \log_2 \left( 1 + \frac{P}{\sigma^2} \right)\] MUX gain only

\(N_R \to \infty\) (Uplink): \[C = N_T \log_2 \left( 1 + \frac{P}{\sigma^2 / (N_R / N_T)} \right)\] MUX and DIV gain
• $v_{mobile} = 50 \text{ km/h}$ - mobile receiver speed
• $T_s = 1 \mu s$ – sampling time
• $R = 50 \text{ m}$ - radius of circle containing the scatterers
• $L = 2 \text{ km}$ – separation of $TX$ and $RX$
• $d = 5\lambda$ - separation of antenna elements
• $f_0 = 2.4\text{GHz}$ - carrier frequency
• $E_s/N_0 = 10\text{dB}$ - symbol signal to noise ratio
The block MIMO channel

Just stack in a row $M$ time-consecutive TX, RX, and noise vectors, then:

$$\mathbf{R} = \mathbf{HS} + \mathbf{W} \ , \ \mathbf{R} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \cdots & \mathbf{r}_M \end{bmatrix} \text{ etc.}$$
Redundant (parity-check symbols) can be added in time (interleaved with information symbols), in space, by multiplexing them on the TX antennas with information symbols, or BOTH: **space-time codes**.
Space-Time Block Codes

\[ E \{ |s_i^{(m)}|^2 \} = 1, \quad E \{ |w_i^{(m)}|^2 \} = 2\sigma^2 \]

\[
R = HS + W
\]

**Space-Time Codeword**

\[
S = \begin{bmatrix}
S_1^{(1)} & S_1^{(2)} & \cdots & S_1^{(M)} \\
S_2^{(1)} & S_2^{(2)} & \cdots & S_2^{(M)} \\
\vdots & \vdots & \ddots & \vdots \\
S_{N_T}^{(1)} & S_{N_T}^{(2)} & \cdots & S_{N_T}^{(M)}
\end{bmatrix}
\]
The mother-of-all ST Block Codes (Alamouti)

$S = \begin{bmatrix}
  s_1^{(1)} = d_1^{(1)} & s_1^{(2)} = -d_2^{(1)*} \\
  s_2^{(1)} = d_2^{(1)} & s_2^{(2)} = d_1^{(1)*}
\end{bmatrix}

|h_1|^2 + |h_2|^2 = 2

Unit-power Constellation

Designed for a $N_R=1 \times N_T=2$ MIMO system (single RX antenna !!!)

Coding rate: 1 in space, $\frac{1}{2}$ in time, overall $\frac{1}{2}$

BUT no bandwidth increase wrt 1 x 1 system with no coding

$$R = \begin{bmatrix}
  r^{(1)} \\
  r^{(2)}
\end{bmatrix} = \sqrt{\frac{P_s}{2}} \begin{bmatrix}
  h_1 \\
  h_2
\end{bmatrix} \begin{bmatrix}
  d_1^{(1)} & -d_2^{(1)*} \\
  d_2^{(1)} & d_1^{(1)*}
\end{bmatrix} + \begin{bmatrix}
  w_1 \\
  w_2
\end{bmatrix}

P_s is the TOTAL transmitted power from the two antennas

Variance of noise terms: $\sigma_1^2 = \sigma_2^2 = \sigma_i^2 + \sigma_Q^2 = 2N_0 / T_s$
The mother-of-all ST Block Codes (Alamouti)

Received code block:

\[ \mathbf{R} = \begin{bmatrix} r^{(1)} \\ r^{(2)} \end{bmatrix} = \sqrt{\frac{P_S}{2}} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \begin{bmatrix} d_1^{(1)} \\ -d_2^{(1)*} \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \]

\[ = \begin{bmatrix} \sqrt{\frac{P_S}{2}} \left( h_1 d_1^{(1)} + h_2 d_2^{(1)} \right) + w_1 \\ \sqrt{\frac{P_S}{2}} \left( -h_1 d_2^{(1)*} + h_2 d_1^{(1)*} \right) + w_2 \end{bmatrix} \]
The decoder needs (perfect) CSI to compute the sufficient statistics (soft symbols) $Z$ for data detection

$$Z = \begin{bmatrix} z_1^{(1)} \\ z_2^{(1)} \end{bmatrix} = \begin{bmatrix} r_1^{(1)} & -r_2^{(1)*} \end{bmatrix} \begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix} = \begin{bmatrix} r_1^{(1)}h_1^* + h_2r_2^{(1)*} \\ r_1^{(1)}h_2^* - h_1r_2^{(1)*} \end{bmatrix}$$

$$= \sqrt{\frac{P_s}{2}} \left( |h_1|^2 + |h_2|^2 \right) d_1^{(1)} + w'_1 + \sqrt{\frac{P_s}{2}} \left( |h_1|^2 + |h_2|^2 \right) d_2^{(1)} + w'_2$$

where

$$w'_1 = w_1h_1^* + w_2^*h_2$$
$$w'_2 = w_1h_2^* - w_2^*h_1$$
Decoding the Alamouti code

The decoder needs (perfect) CSI to compute the sufficient statistics (soft symbols) for data detection

\[
Z = \begin{bmatrix}
z_1^{(1)} \\
z_2^{(1)}
\end{bmatrix} = \begin{bmatrix}
r_1^{(1)} \\
-r_2^{(1)*}
\end{bmatrix} \begin{bmatrix}
h_1^* \\
h_2^*
\end{bmatrix} = \begin{bmatrix}
r_1^{(1)}h_1^* + h_2r_2^{(1)*} \\
r_1^{(1)}h_2^* - h_1r_2^{(1)*}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
2\sqrt{\frac{P_s}{2}} \cdot d_1^{(1)} + w_1' \\
2\sqrt{\frac{P_s}{2}} \cdot d_2^{(1)} + w_2'
\end{bmatrix}
\]

Where the new noise variances are

\[
\sigma_1'^2 = \sigma_2'^2 = \left( |h_1|^2 + |h_2|^2 \right) \sigma_1^2 = 2 \cdot 2N_0 / T_s
\]
Where is the gain?

- The signal power increases by
  \[(|h_1|^2 + |h_2|^2)^2 = 4\]

- The noise power increases by
  \[|h_1|^2 + |h_2|^2 = 2\]

- There are no CROSS TERMS between channels!

- TWO-FOLD GAIN DIVERSITY (3dB gain) as in a conventional receive-diversity 2 x 1 system (requires CSI!)

Can we GENERALIZE?
With two RX antennas (2 x 2 code)

\[
R = \begin{bmatrix}
  r_1^{(1)} & r_1^{(2)} \\
  r_2^{(1)} & r_2^{(2)}
\end{bmatrix} = \sqrt{\frac{P_s}{2}} \begin{bmatrix}
  h_{11} & h_{12} \\
  h_{21} & h_{22}
\end{bmatrix} \begin{bmatrix}
  d_1^{(1)} & -d_2^{(1)*} \\
  d_2^{(1)} & d_1^{(1)*}
\end{bmatrix} + W
\]

\[
Z = \begin{bmatrix}
  z_1^{(1)} & z_2^{(1)}
\end{bmatrix} = \begin{bmatrix}
  r_1^{(1)} & -r_1^{(2)*}
\end{bmatrix} \begin{bmatrix}
  h_{11}^* & h_{12}^* \\
  -h_{12} & h_{11}
\end{bmatrix} + \begin{bmatrix}
  r_2^{(1)} & -r_2^{(2)*}
\end{bmatrix} \begin{bmatrix}
  h_{21}^* & h_{22}^* \\
  -h_{22} & h_{21}
\end{bmatrix}
\]

Show that we have FOUR-FOLD diversity gain (but no mux gain)
Do we really need 5G?

- over 50%/year growth in data traffic
- at least, a 1000× increase every decade
- in 2018, global traffic will reach more than 1/100 of a zettabyte (1 ZB=10^{21} B)

Source: Cisco VNI Mobile, 2014
According to CISCO, mobile data traffic will reach the following milestones within the next five years:

- Monthly global mobile data traffic will surpass 15 exabytes by 2018
- The number of mobile-connected devices will exceed the world’s population by 2014
- The average mobile connection speed will surpass 2 Mb/s by 2016
- Due to increased usage on smartphones, smartphones will reach 66% of mobile data traffic by 2018
- Tablets will exceed 15% of global mobile data traffic by 2016
- 4G traffic will be more than half of the total mobile traffic by 2018
Basics of beyond-4G technologies

Beyond-4G technologies (3/3)

Enhanced Mobile Broadband (eMBB)
- 10-20 Gbps peak
- 100 Mbps whenever needed
- 10000x more traffic
- Macro and small cells
- Support for high mobility (500 km/h)
- Network energy saving by 100 times

Massive Machine Communication (mMTC)
- High density of devices ($2 \times 10^5 - 10^6$/km$^2$)
- Long range
- Low data rate (1 - 100 kbps)
- M2M ultra low cost
- 10 years battery
- Asynchronous access

Ultra Reliability and Low Latency (URLLC)
- Ultra responsive
  - $<1$ ms air interface latency
  - 5 ms E2E latency
- Ultra reliable and available (99.9999%)
- Low to medium data rates (50 kbps - 10 Mbps)
- High speed mobility
Technology drivers

- network densification
- massive MIMO
- mm-wave technology

and many more: full-duplex antennas, spectrum sharing, advanced PHY and interference management, device-to-device (D2D) communications, etc.
Cooper’s “law”: the wireless capacity has doubled every 30 months over the last century in the last fifty years, capacity has increased about a million times!

The right path to pursue is network densification: very dense deployment of BTSs
Network densification (2/3)

Shannon’s law:

$$C \propto N \cdot B \cdot \log_2 (1 + \gamma) \quad [\text{b/s/Hz}]$$

With extreme network densification, we can reuse Shannon’s law everywhere, thus increasing the area spectral efficiency (in b/s/Hz/m^2).

heterogeneous, multi-tier dense networks: small cells, relays, etc.
Open challenges include:

- **self-organization**, exacerbated by random/unplanned deployment of small cells.

- **coverage and performance prediction**: stochastic geometry and random matrix theory could serve as powerful tools.

- **interference management and resource allocation**, also considering the presence of multiple tiers.
Massive MIMO (a.k.a. as multiuser MIMO) technology:

$$N_t \gg 1 \text{ antennas (hundreds)}$$

$$K \gg 1 \text{ single-antenna terminals}$$

The massive MIMO concept relies on the law of large numbers, to average out frequency selectivity and thermal noise:

- **spatial multiplexing gain** ∝ \( K \)
- **array gain** ∝ \( N_t \)
In the **uplink**, the BTS:
- acquires CSI from pilot symbols
- detects the information symbols
- since \( N_t \gg K \), adopts linear processing techniques (MRC, ZF, MMSE), which are **nearly optimal**

In the **downlink**, the BTS:
- uses CSI obtained in the uplink
- applies multiuser MIMO precoding, using **low-complexity precoders**

- **Thanks to the** \( N_t - K \) **unused degrees of freedom, massive MIMO can obtain:**
  - **hardware-friendly** waveform shaping
  - **PAPR reduction** due to multiuser precoding
- **The major challenge is getting** **accurate CSI estimation**
Increasing the carrier frequency increases the path loss

However, network densification includes small cells with coverage radius $R \approx 20\, \text{m}$, path loss becomes acceptable.
mm-wave technology (2/2)

- mm-waves can provide a brand-new, very wide frequency band, with very high-gain steerable antennas at both the MS and the BTS sides.

- We can augment the currently saturated 700 MHz – 2.5 GHz radio spectrum, moving to 60 GHz.

- Due to very low wavelengths, we can accommodate the antennas required by massive MIMO.

Wireless channel modeling is not valid anymore: research challenges include new propagation models for mm-wave communications.
MMIMO Antennas

Dip. Ingegneria dell’Informazione
University of Pisa, Pisa, Italy

Marco Luise
MIMO & 5G Cellular Communications
When the number of antennas is much larger than the number of users in the cell, MMIMO also implements Multiple Access/Multiplexing by «synthesizing» multiple beams directed to spatially separated users.
# Summary of LTE PHY Parameters

<table>
<thead>
<tr>
<th>Nominal BW</th>
<th>1.25 MHz</th>
<th>2.5 MHz</th>
<th>5 MHz</th>
<th>10 MHz</th>
<th>15 MHz</th>
<th>20 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FFT Size</strong></td>
<td>128</td>
<td>256</td>
<td>512</td>
<td>1024</td>
<td>1536</td>
<td>2048</td>
</tr>
<tr>
<td><strong>Sampling Frequency</strong></td>
<td>1.92 MHz</td>
<td>3.84 MHz</td>
<td>7.68 MHz</td>
<td>15.36 MHz</td>
<td>23.04 MHz</td>
<td>30.72 MHz</td>
</tr>
<tr>
<td><strong>Samples per Slot</strong></td>
<td>960</td>
<td>1920</td>
<td>3840</td>
<td>7680</td>
<td>11520</td>
<td>15360</td>
</tr>
<tr>
<td><strong>Samples per Normal CP</strong></td>
<td>10 (#0)</td>
<td>20 (#0)</td>
<td>40 (#0)</td>
<td>80 (#0)</td>
<td>120 (#0)</td>
<td>160 (#0)</td>
</tr>
<tr>
<td><strong>Samples per Long CP</strong></td>
<td>9 (#1-6)</td>
<td>18 (#1-6)</td>
<td>36 (#1-6)</td>
<td>72 (#1-6)</td>
<td>108 (#1-6)</td>
<td>144 (#1-6)</td>
</tr>
<tr>
<td><strong>Number of PRBs</strong></td>
<td>6</td>
<td>12</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td><strong>Occupied Subcarriers</strong></td>
<td>76</td>
<td>151</td>
<td>301</td>
<td>601</td>
<td>901</td>
<td>1201</td>
</tr>
<tr>
<td><strong>Zero Padded Carriers</strong></td>
<td>52</td>
<td>105</td>
<td>211</td>
<td>423</td>
<td>635</td>
<td>847</td>
</tr>
<tr>
<td><strong>Actual Bandwidth</strong></td>
<td>1140 KHz</td>
<td>2265 kHz</td>
<td>4515 kHz</td>
<td>9015 kHz</td>
<td>13515 kHz</td>
<td>18015 kHz</td>
</tr>
</tbody>
</table>
New Radio: the Radio Interface of 5G

- CP-OFDM format, both UL and DL, plus SC-FDMA for UL
- QAM constellations up to 256-QAM
- Multiple carrier spacing: $\Delta f = 2^\mu \times 15$ kHz, where $\mu = 0, 1, 2, 3, 4$ is the so-called numerology
- 1 slot=14 OFDM symbols (twice as many as in LTE)

- (I)FFT size 2048 or 4096