Ingegneria delle Telecomunicazioni
Comunicazioni Digitali

Basics of Wireless Propagation

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Received contributions at the receiver (uplink):

- Thermal noise (e.g., due to electromagnetic radiations, electronics impairments, etc.)
- Multiple access interference (MAI)

The wireless channel between the transmitter and the receiver fluctuates randomly for a number of causes.
To sum up, the received signal is a linear combination of a number of different propagation paths, each having its own attenuation, phase rotation, and time delay:

$$y(t) = \sum_{i=1}^{N(t)} \rho_i(t) e^{j\varphi_i(t)} x(t - \tau_i(t)) e^{-j2\pi f_0 \tau_i(t)}$$

$$= \sum_{i=1}^{N(t)} \rho_i(t) e^{j\theta_i(t)} x(t - \tau_i(t))$$

- $N(t)$: number of propagation paths
- $\rho_i(t)$: attenuation of the $i$-th path
- $\theta_i(t)$: phase delay of the $i$-th path
- $\tau_i(t)$: time delay of the $i$-th path
(Preliminary) classification of the wireless channel

Time domain:
- **static** (time-invariant): its statistics change very slowly with respect to signaling time
- **time-varying**: its statistics are a function of time

Frequency domain:
- **frequency-flat**: its behavior is similar across the frequency components of the signal
- **frequency-selective**: each frequency component of the signal is distorted in a different way by the wireless channel
A **static** channel means: the random processes $N(t)$, $\{\rho_i(t)\}$, $\{\theta_i(t)\}$, and $\{\tau_i(t)\}$ are **not** functions of time

$$y(t) = \sum_{i=1}^{N} \rho_i e^{j\theta_i} x(t - \tau_i)$$

For the sake of simplicity, let’s consider the **two-ray channel**, i.e., $N=2$:

$$y(t) = \rho_1 e^{j\theta_1} x(t - \tau_1) + \rho_2 e^{j\theta_2} x(t - \tau_2)$$
Static frequency-selective channels (2/4)

To simplify the notation, let’s take:

\[ \rho_1 = 1, \quad \theta_1 = 0, \quad \tau_1 = 0 \]
\[ \rho_2 = \rho, \quad \theta_2 = \theta, \quad \tau_2 = \tau \]

Received signal:
\[ y(t) = x(t) + \rho e^{j\theta} x(t - \tau) \]

Fourier transform:
\[ Y(f) = X(f) \cdot \left[ 1 + \rho e^{j\theta} e^{-j2\pi f \tau} \right] \]

The frequency response of the channel is
\[ H(f) = \frac{Y(f)}{X(f)} = 1 - \rho e^{-j2\pi (f - f_n) \tau} \]

where \[ f_n = \frac{1}{2\tau} + \frac{\theta}{2\pi \tau} \] is the notch frequency of the channel.
The amplitude response of the two-ray channel is

\[ |H(f)| = \sqrt{1 + \rho^2 - 2\rho \cos (2\pi (f - f_n)\tau)} \]

\[ \tau = 1 \mu s \]
\[ f_n = 0.25 \text{ MHz} \]
\[ \theta = -\pi/2 \]

period: \( 1/\tau \)

\[ f - f_0 \text{ [MHz]} \]
When extending the calculations to the $N$-ray channel, we get

$$B_c = \frac{1}{\tau_N - \tau_1} \left( \frac{1}{10} \div \frac{1}{100} \right) = \frac{1}{\Delta \tau} \left( \frac{1}{10} \div \frac{1}{100} \right)$$

$B_c$ is called the **coherence bandwidth** of the channel.
The concept of frequency selectivity (1/2)

We know that the bandwidth of a signal \( x(t) \) is

\[
B \propto \frac{1}{T_s}
\]

where

- \( B \) is the bandwidth of the signal,
- \( T_s \) is the symbol period.

In a frequency-selective channel, the frequency response \( |H(f)| \) of the channel is not flat across the bandwidth, whereas in a frequency-flat channel, the response is approximately flat.

- For a frequency-selective channel, \( T_s \gg \Delta \tau \)
- For a frequency-flat channel, \( T_s \approx \Delta \tau \)

\( \Delta \tau \) is the delay spread of the channel.
The concept of frequency selectivity (2/2)

The frequency selectivity depends on the statistics of the channel and of the input signal.

There is a practical way to assess the frequency selectivity of a channel:

- \( B \leq B_c \) : frequency-flat channel
- \( B > B_c \) : frequency-selective channel

Example:

- urban scenarios: \( \Delta \tau \approx 1 \mu s \), \( B_c \approx 1 \text{ MHz/100} = 10 \text{ kHz} \)
- 3G and 4G signals: \( B \geq 3.5 \text{ MHz} \)

Some form of equalization is needed to combat the frequency selectivity.
Time-varying frequency-flat channels (1/5)

Due to the relative motion between the transmitter and the receiver, the communication medium (the wireless channel) evolves through time:

\[ y(t) = \sum_{i=1}^{N(t)} \rho_i(t) e^{j\theta_i(t)} x(t - \tau_i(t)) \]

For simplicity, assume a frequency-flat channel: \( B \leq B_c \Rightarrow T_s \geq 100\Delta \tau \Rightarrow \Delta \tau \ll T_s \)

And so \( \Delta \tau \ll T_s \Rightarrow \tau_i \approx \bar{\tau} \ \forall i \)

\[ y(t) \approx x(t - \bar{\tau}) \cdot \sum_{i=1}^{N(t)} \rho_i(t) e^{j\theta_i(t)} \]

\[ = \bar{\rho}(t) \cdot e^{j\bar{\theta}(t)} \cdot x(t - \bar{\tau}) \]

\[ = A(t) \cdot x(t - \bar{\tau}) \]

fading process
To study time and frequency characteristics of $A(t)$, let’s use the **kinematic model** for the MS:

$$y(t) = x(t) \exp(j2\pi\Delta f t)$$

Due to the **Doppler effect**, each frequency component is shifted at the receiver side by its **Doppler shift** $\Delta f$:

$$\Delta f = \frac{v}{c} \cdot f \cdot \cos(\alpha_i)$$
Time-varying frequency-flat channels (3/5)

In reality, when we have rich multipath, the incident wave comes from a variety of different angles, and the Doppler shift has not just one single value:

The values of the Doppler effect, are spread around a certain interval of Doppler frequencies, according to the different $\alpha_i$'s. The received signal is

$$y(t) = \left[ \rho_1 e^{j\theta_1} \exp(j 2\pi \Delta f_1 t) + \rho_2 e^{j\theta_2} \exp(j 2\pi \Delta f_2 t) + \rho_3 e^{j\theta_3} \exp(j 2\pi \Delta f_3 t) + \ldots \right] x(t) = A(t) x(t)$$
When the «scatterers» (i.e., the objects) in the scenario are very many, the Doppler shifts are distributed with continuity in the interval \([-f_0v/c; f_0v/c]\).

The decomposition of \(A(t)\) into many Doppler components becomes a continuos Doppler spectrum, confined within the interval \([-f_0v/c; f_0v/c]\).
The behavior of $A(t)$ is given by the impact of the Doppler effect over the received signal.

A key parameter is the maximum Doppler shift at the carrier frequency $f_0$, called the **Doppler spread** $f_D$:

$$f_D \triangleq \max_{\alpha_i} |\Delta f_0| = \frac{v}{c} \cdot f_0$$

Using the Clarke’s model, we can compute the power spectral density (PSD) of the random process $A(t)$:

$$P_a(f) = \frac{\sigma^2}{2\pi f_D} \cdot \frac{1}{\sqrt{1 - (f/f_D)^2}}$$

![Graph](image)
Example of Flat Fading

Signal amplitude $|A(t)|$ received at $f_0 = 1$ GHz by a mobile terminal travelling at $v = 20$ km/h in an urban area.

$\sim 30$ msec

$\sim 300$ bits @ 9.6 kb/sec
The concept of time selectivity 1/2

Selectivity is quantified through the Coherence Time $T_c$:

$$ T_c = \frac{1}{10 \div 100} \frac{1}{f_D} $$

- $T_s \leq T_c$: static channel (also means $f_D << B$)
- $T_s > T_c$: time-selective channel

The time selectivity depends on the statistics of the channel and of the input signal.
The concept of time selectivity 2/2

\[ T_c = \frac{1}{10 \div 100} \frac{1}{f_D} \]

Example (GSM on Frecciarossa): \( v = 324 \text{ km/h} = 90 \text{ m/s}, f_0 = 1.8 \text{ GHz} \)

\( f_D = 540 \text{ Hz} \quad T_C = 18.5 \mu s \)

\( R_s = 270.833 \text{ kbaud} \quad T_s = 3.7 \mu s \)

BUT the burst time is \( T_B = 546.5 \mu s \)
Bibliography


