



MSc Course in *Cybersecurity*

Digital Signals

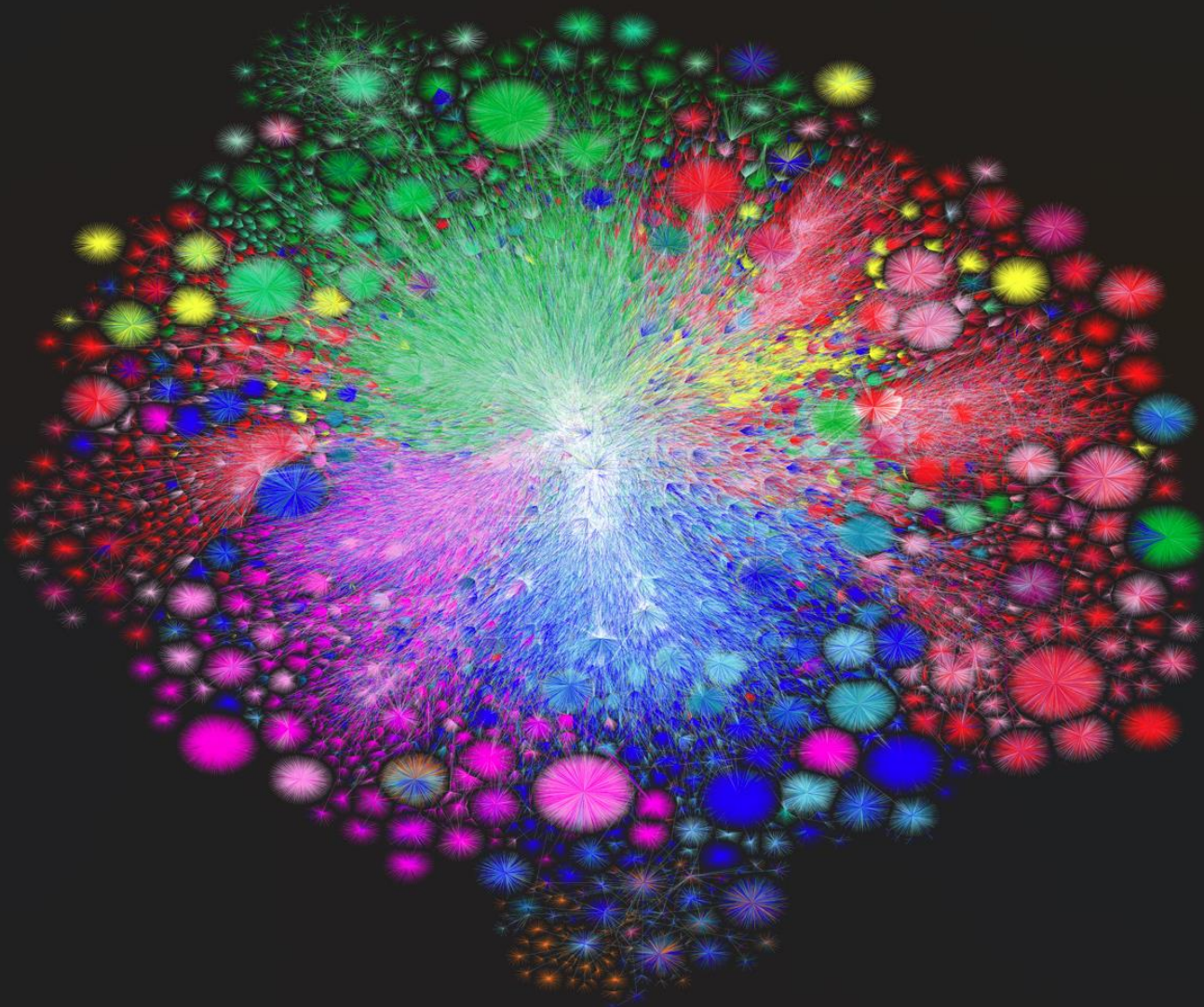
Marco Luise

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<http://www.iet.unipi.it/m.luise/>

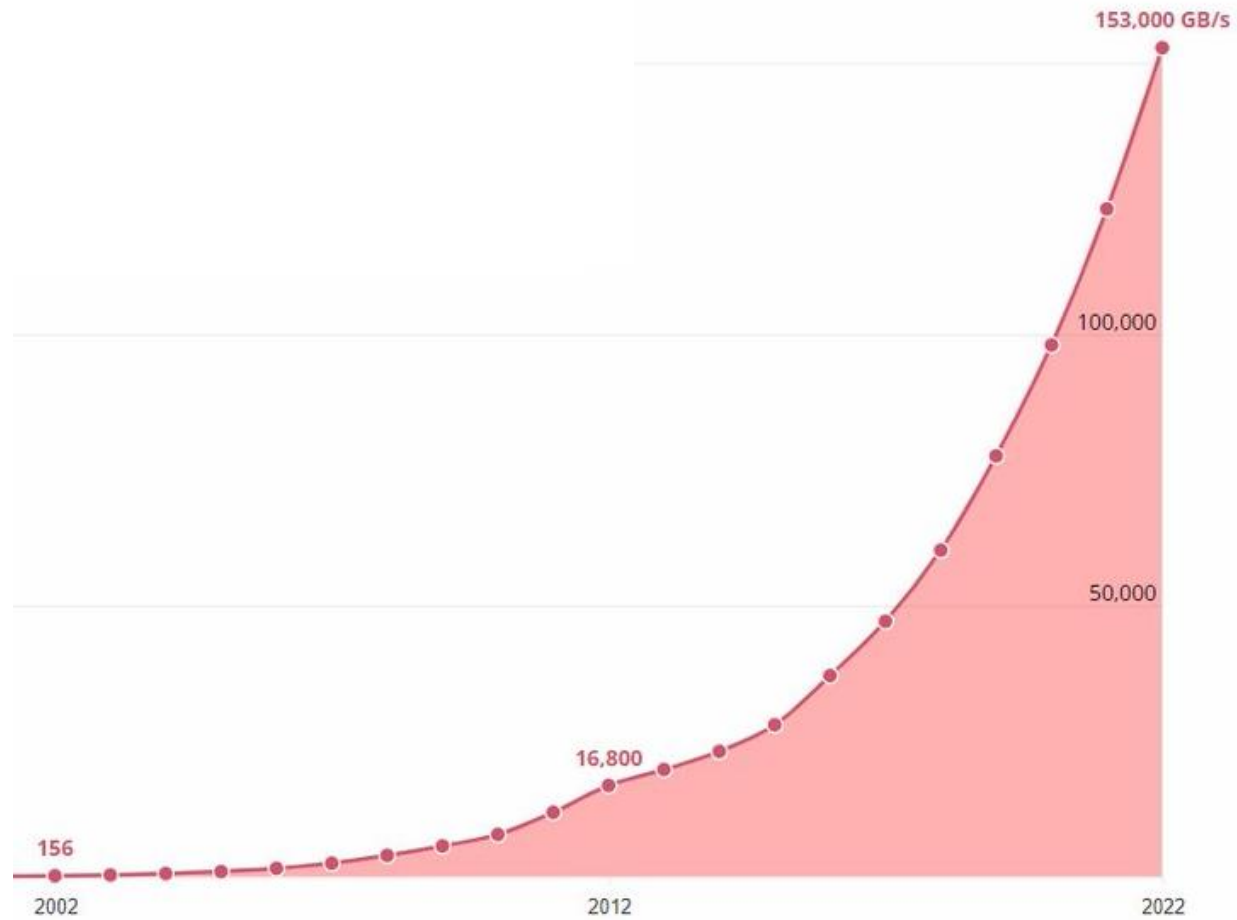


Dip. Ingegneria dell'Informazione, University of Pisa, Italy

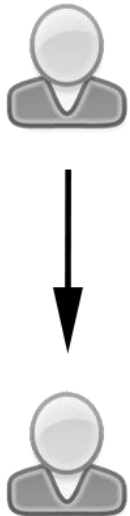


<https://www.opte.org/>

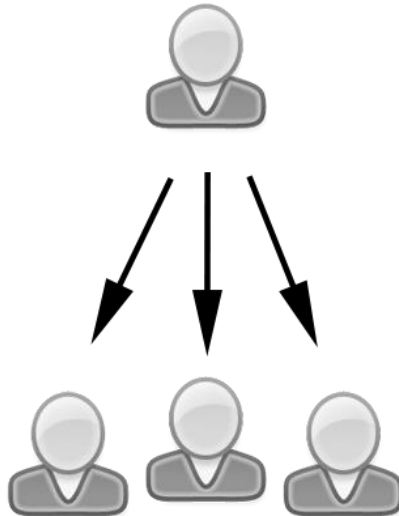
Global Internet Traffic



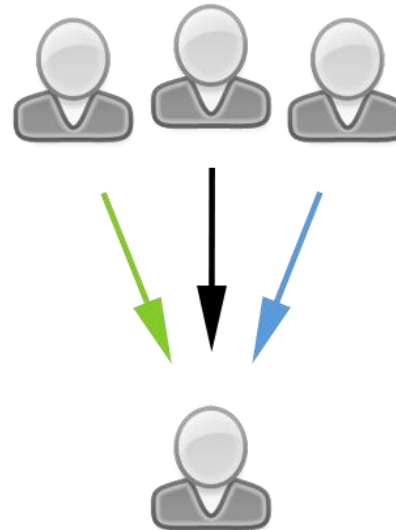
ONE TO ONE



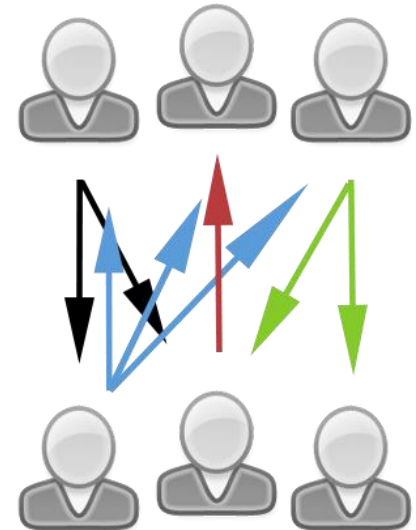
ONE TO MANY



MANY TO ONE

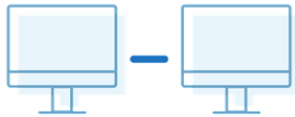


MANY TO MANY

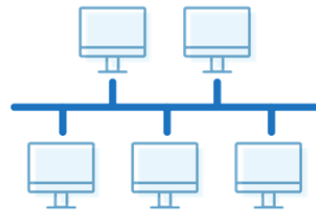


Communication Networks Architecture

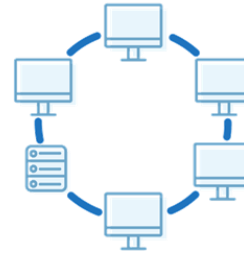
1 Point to point



2 Bus



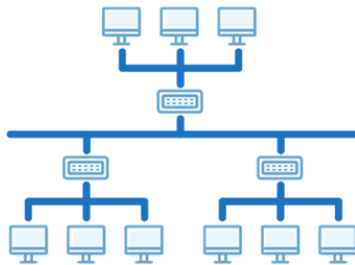
3 Ring



4 Star



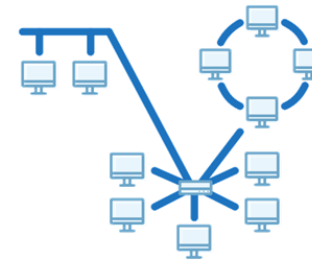
5 Tree



6 Mesh

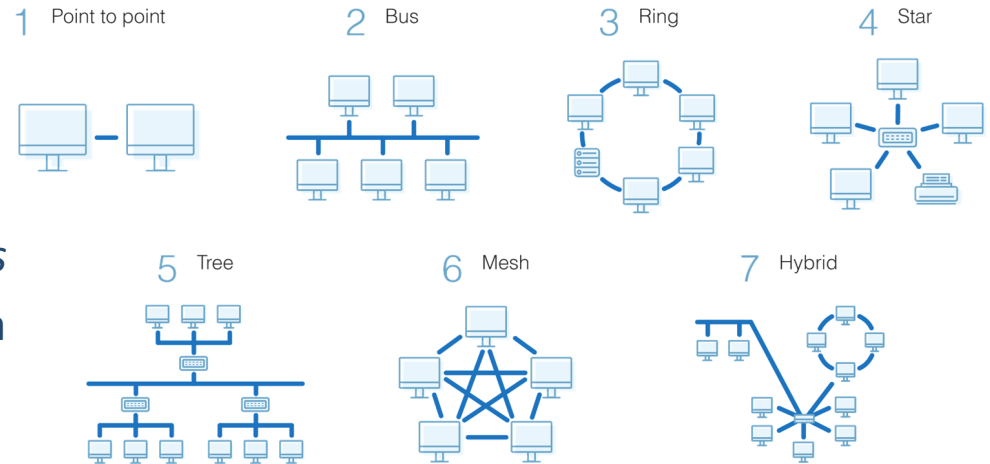


7 Hybrid

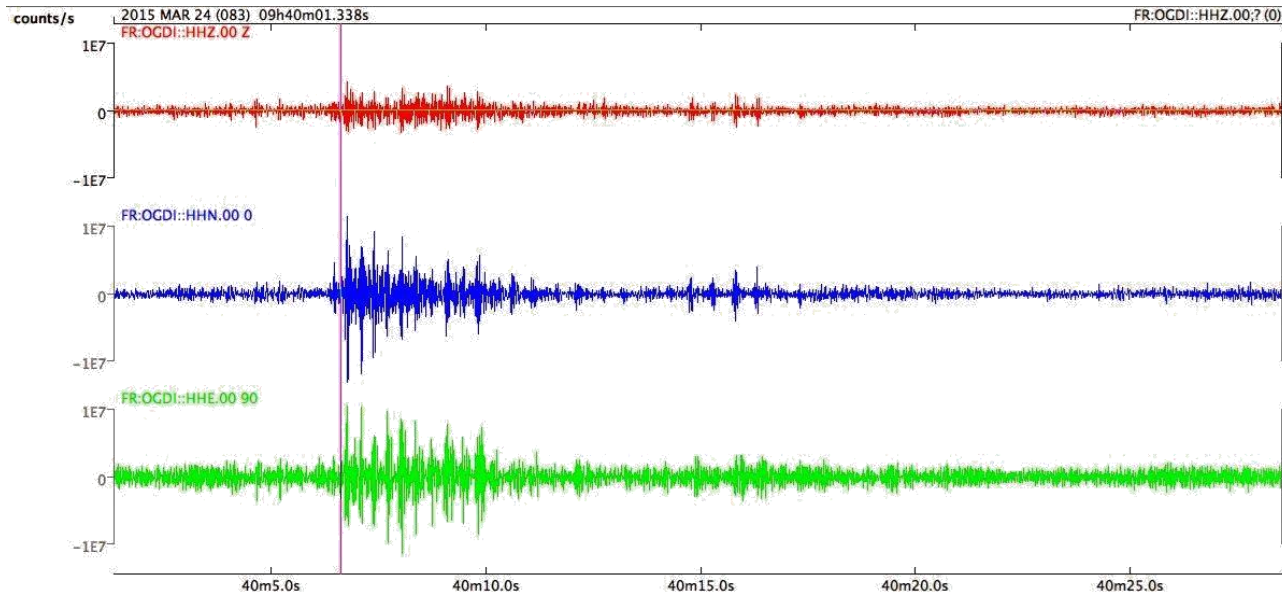
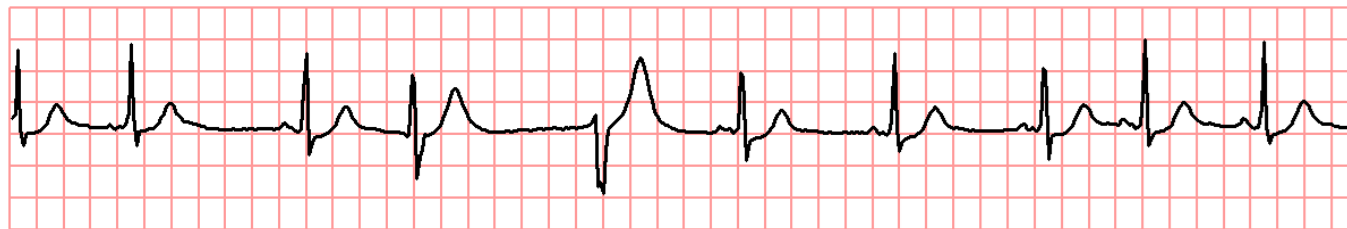


Point-to-Point Multimedia Digital Communications

- Any network topology is made of a collection of point-to-point digital *links*
- The links are implemented exploiting a *physical medium*
- Different physical media require different technologies
 - Wireless
 - Copper
 - Fibers
- Whatever the medium, any link transfers *digital information* carrying *multimedia messages* (audio, images, video, text, pure data)
- How can everything be reduced to the same *digital format* ?



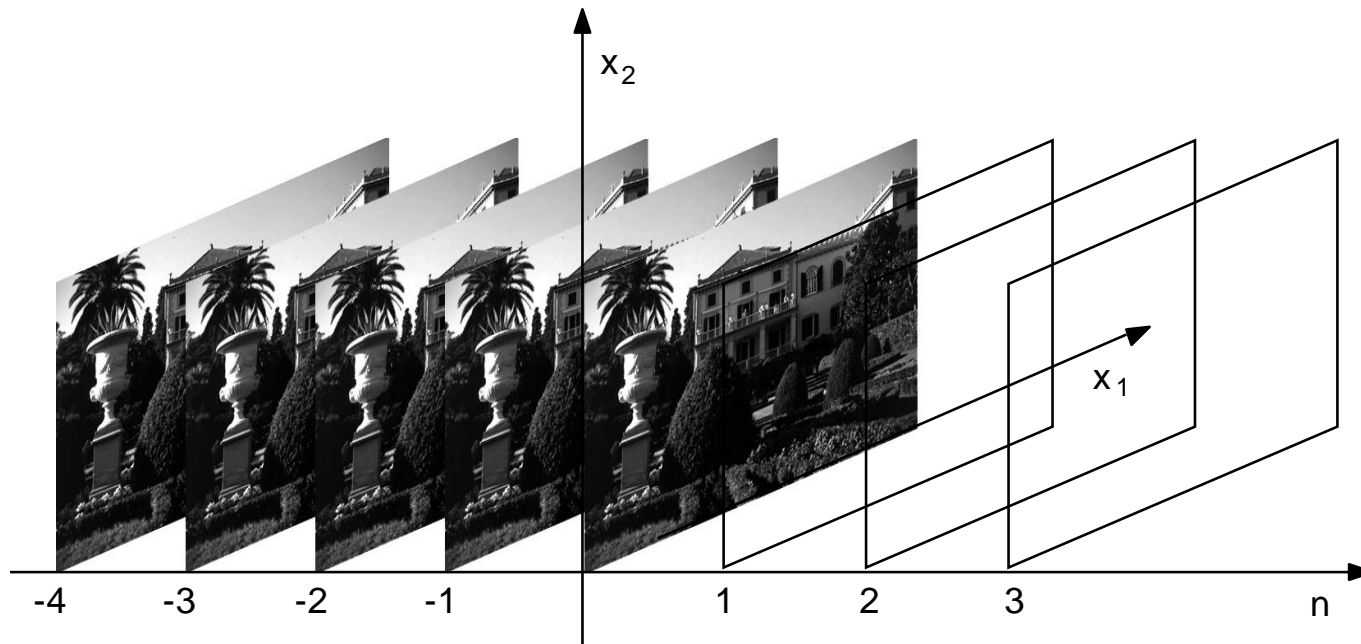
A signal is *any variable physical quantity* that carries a certain amount of *information*, that makes the signal itself «interesting»

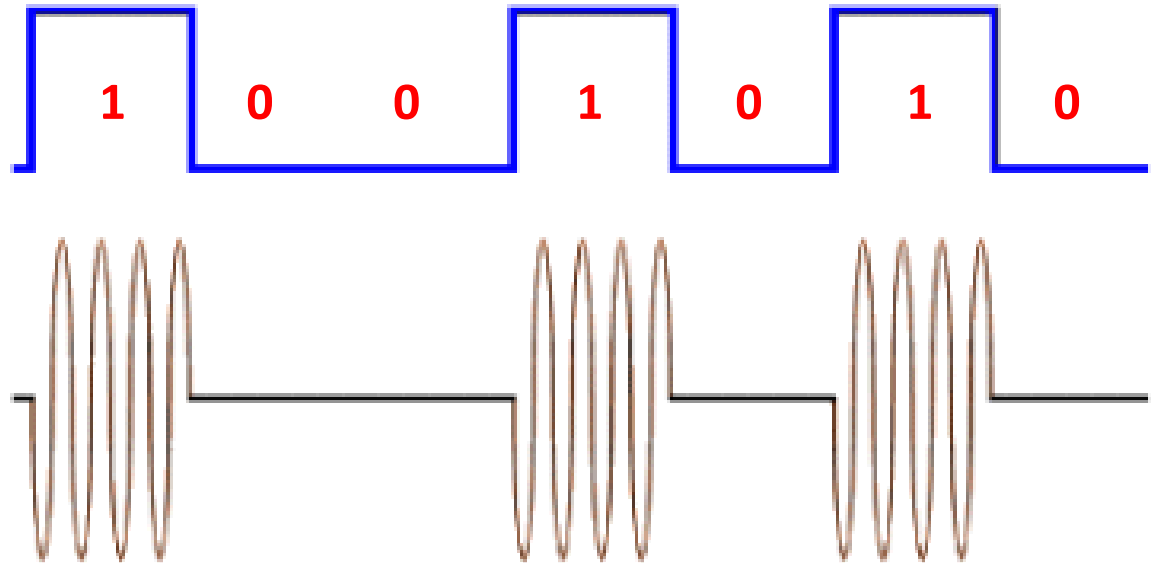


A signal is *any variable physical quantity* that carries a certain amount of *information*, that makes the signal itself «interesting»

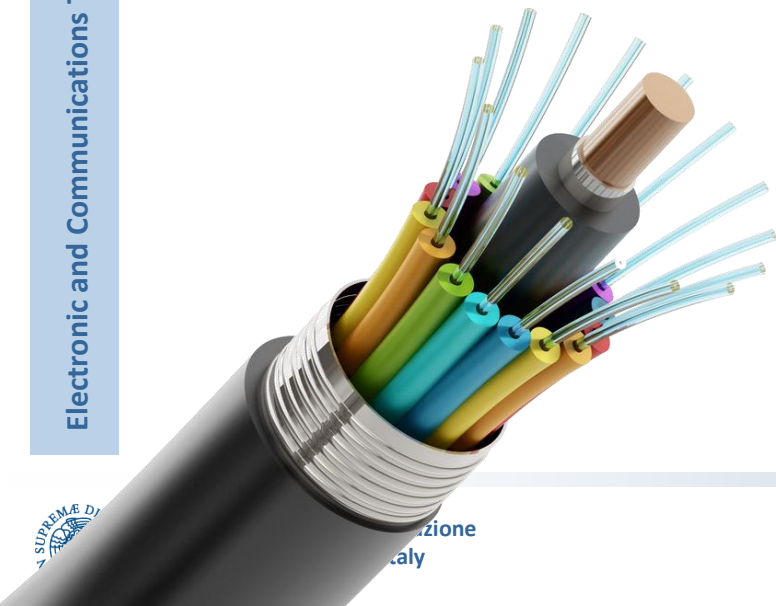


A signal is *any variable physical quantity* that carries a certain amount of *information*, that makes the signal itself «interesting»

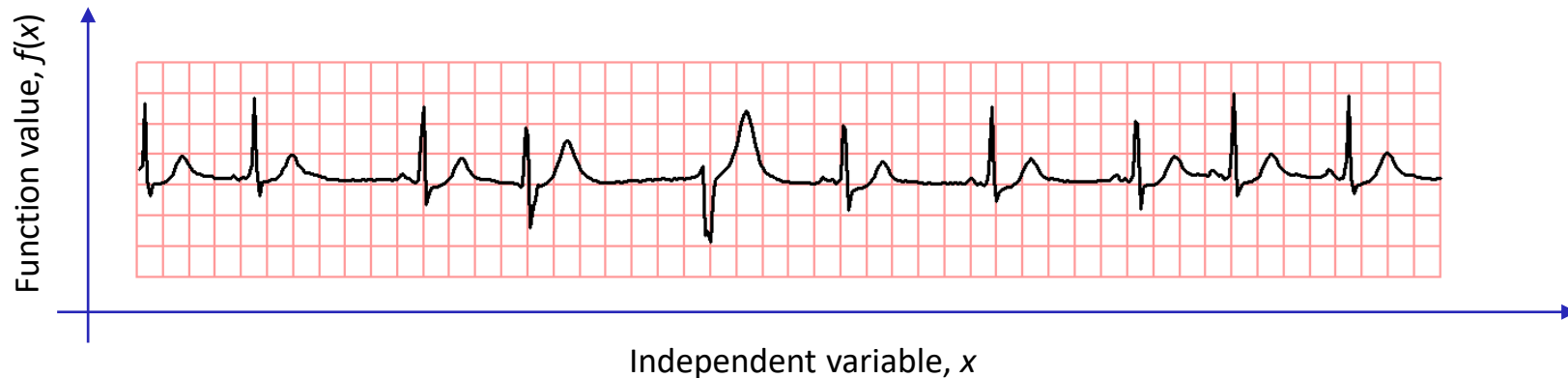




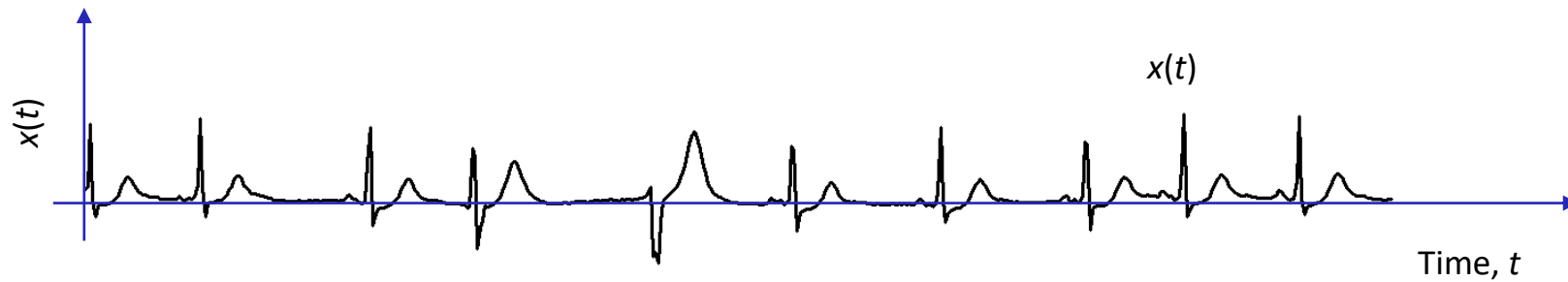
194 THz Frequency – 1.55 μm wavelength



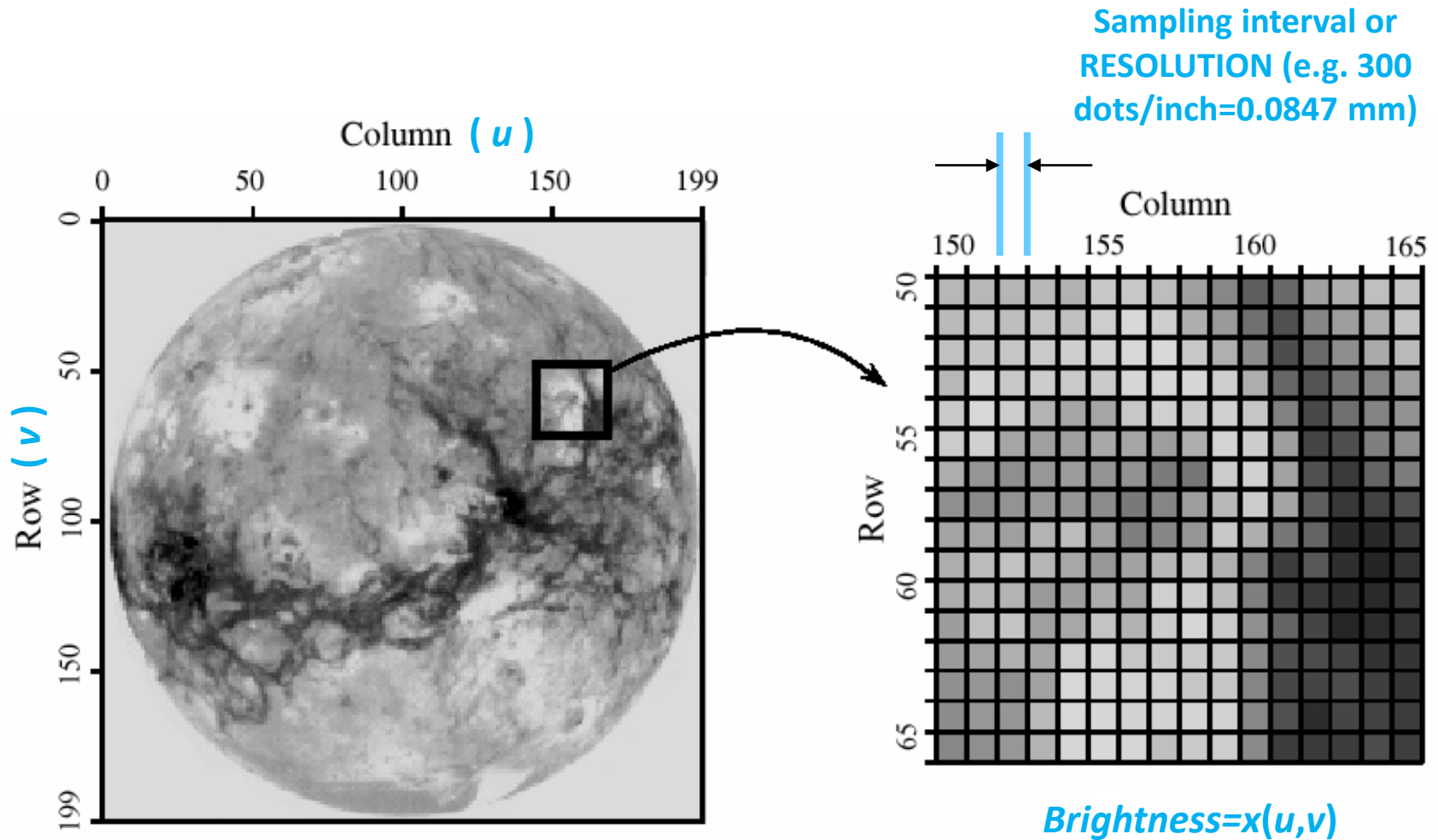
Mathematical representation of a signal



- The “signal” is the value of a mathematical function of a certain independent variable
- Here (ECG), the independent variable is *time*
- We will reserve the letter f for the *frequency*, and we will call the signal x, y, z



B/W Satellite Image for Weather Forecast

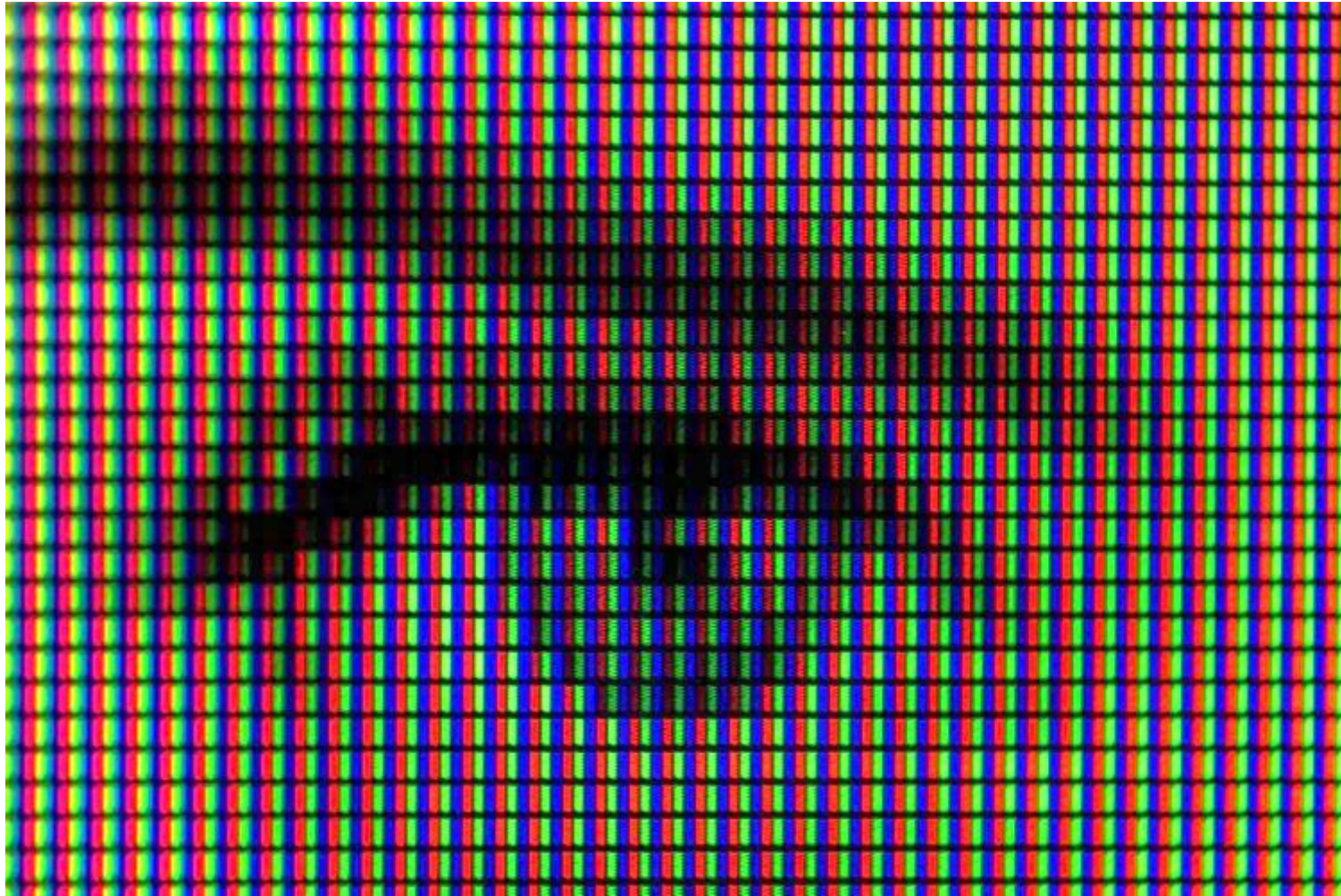


Creating PIXELS: «Sampling» the values of the image along the two u - v SPACE dimensions

Color Image...



... = 3 x monochromatic RGB images



B/W Original image: digital, but you cannot perceive it



Sampling: (too) Low resolution



Quantization: Different numbers of bits/sample

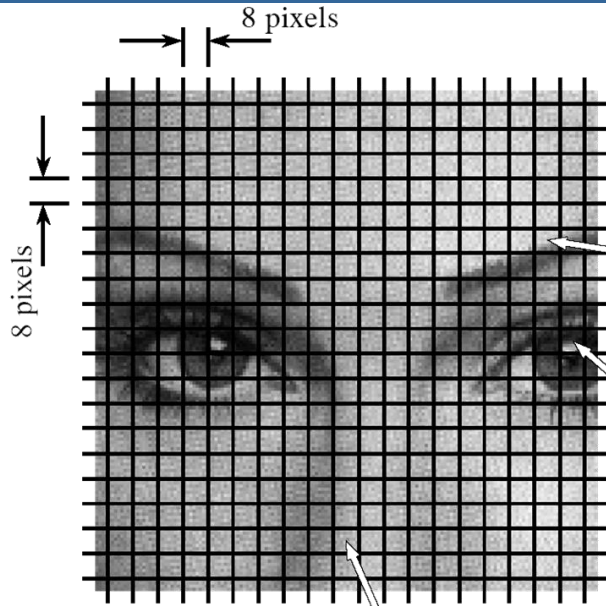


4 bits/pixel



2 bits/pixel

Image segments (JPEG)

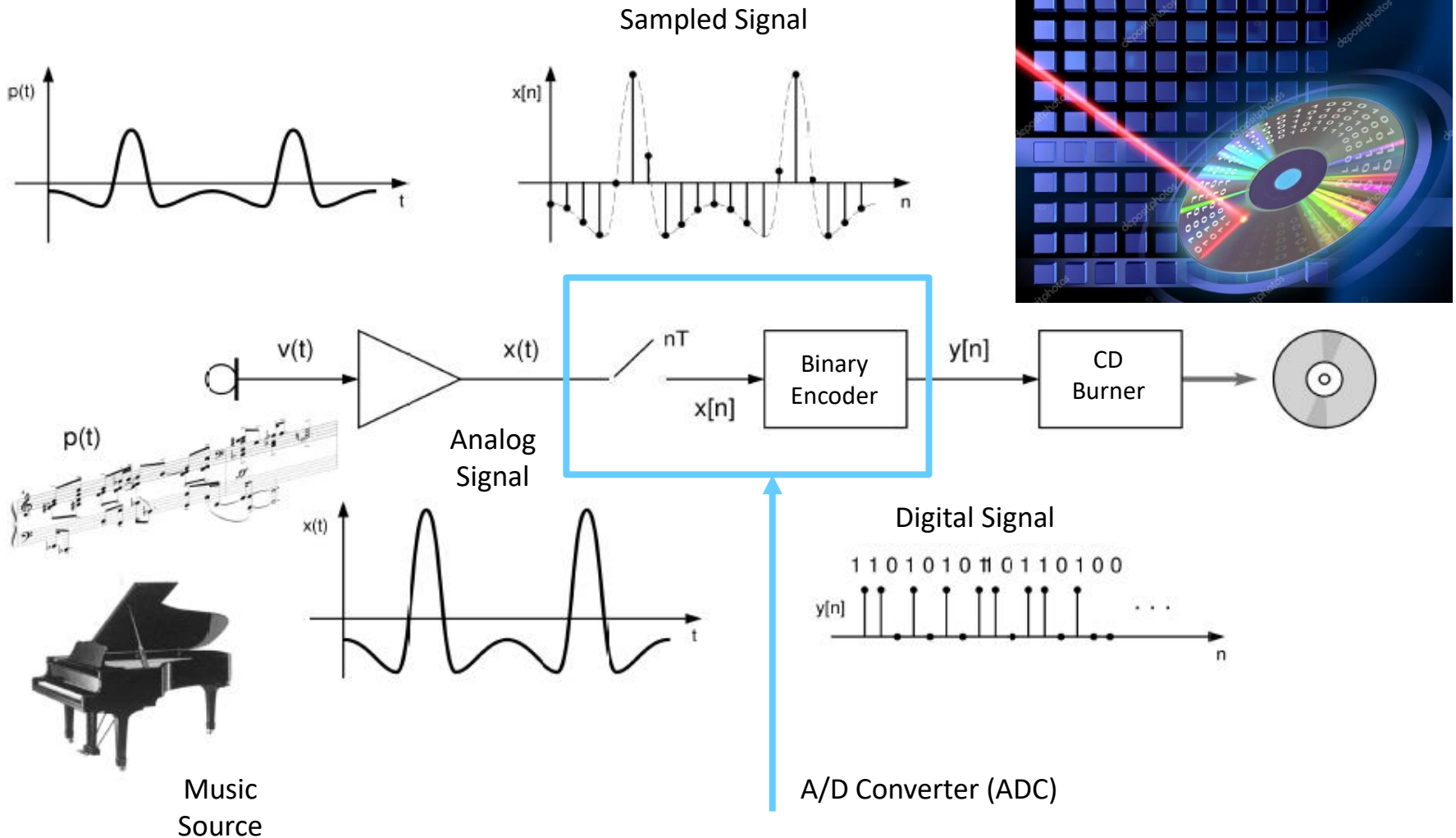


231	224	224	217	217	203	189	196
210	217	203	189	203	224	217	224
196	217	210	224	203	203	196	189
210	203	196	203	182	203	182	189
203	224	203	217	196	175	154	140
182	189	168	161	154	126	119	112
175	154	126	105	140	105	119	84
154	98	105	98	105	63	112	84

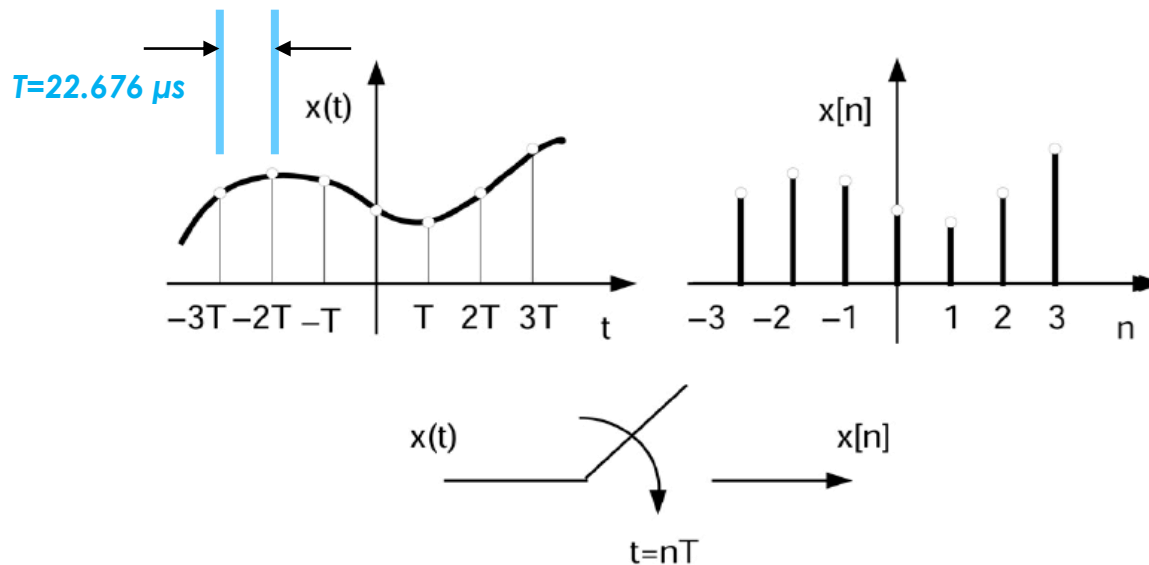
42	28	35	28	42	49	35	42
49	49	35	28	35	35	35	42
42	21	21	28	42	35	42	28
21	35	35	42	42	28	28	14
56	70	77	84	91	28	28	21
70	126	133	147	161	91	35	14
126	203	189	182	175	175	35	21
49	189	245	210	182	84	21	35

154	154	175	182	189	168	217	175
154	147	168	154	168	168	196	175
175	154	203	175	189	182	196	182
175	168	168	168	140	175	168	203
133	168	154	196	175	189	203	154
168	161	161	168	154	154	189	189
147	161	175	182	189	175	217	175
175	175	203	175	189	175	175	182

Digital Music Recording



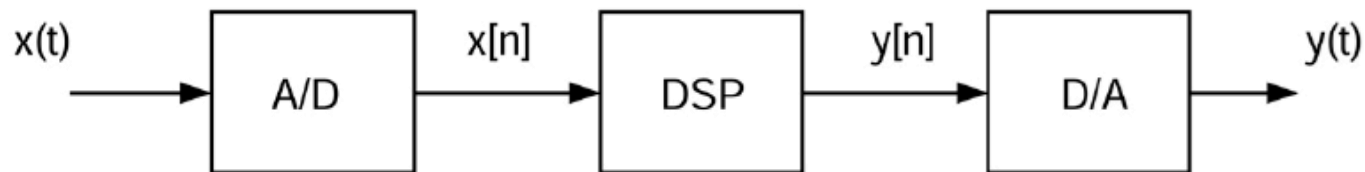
Sampling an analog signal



Sampling an analog signal $x(t)$ means collecting one after the other the sequence $x[n]$ of values of that signal at the time instants nT , i.e, taken at the rate $f_c = 1/T$ that is called the *sampling frequency* (Hz or samples/s)

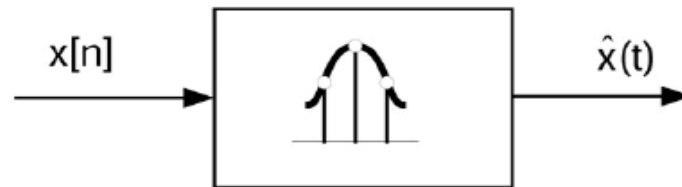
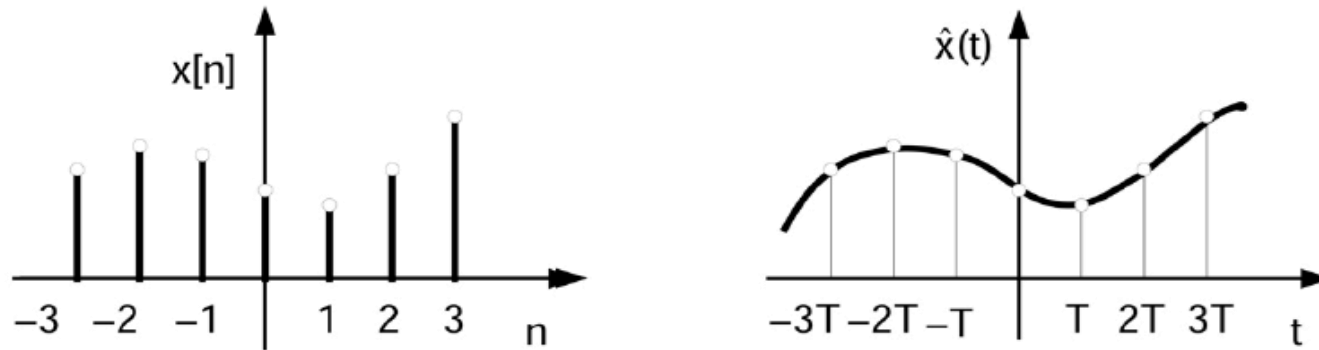
Audio recording: $f_c = 44.1 \text{ kHz}$, $T = 22.676 \mu s$

Signal processing is performed on the digital values extracted from the signal itself after A / D conversion, and results in the execution of an appropriate program by a digital processor. This structure is extremely flexible, in the sense that different processing functions can be realized simply by changing the processing program (software) without having to modify the physical structure (hardware) of the system.

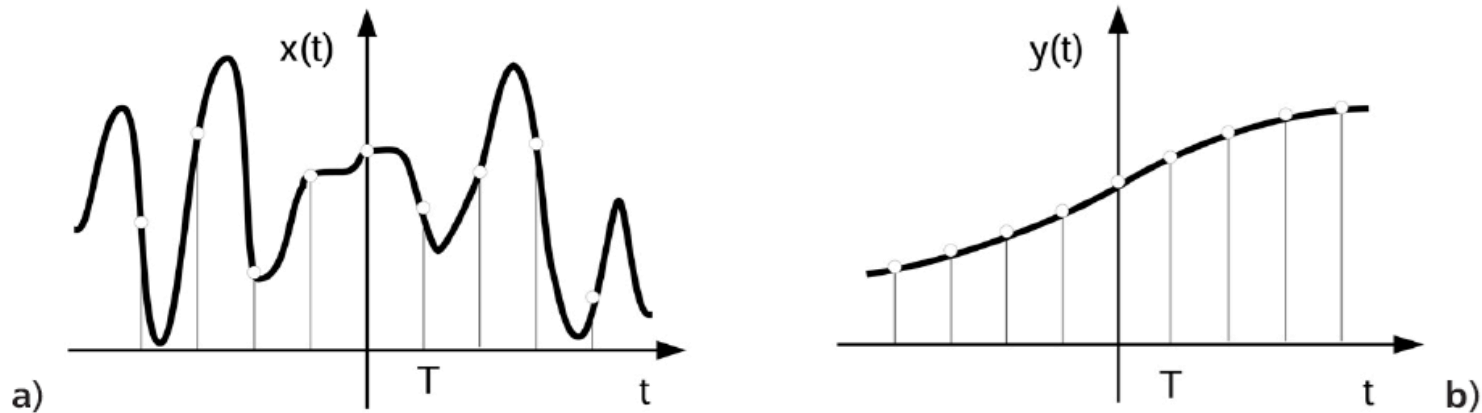


Sampling (ADC) & Interpolation (DAC)

(Time continuous) interpolation of a time-discrete signal (sequence)



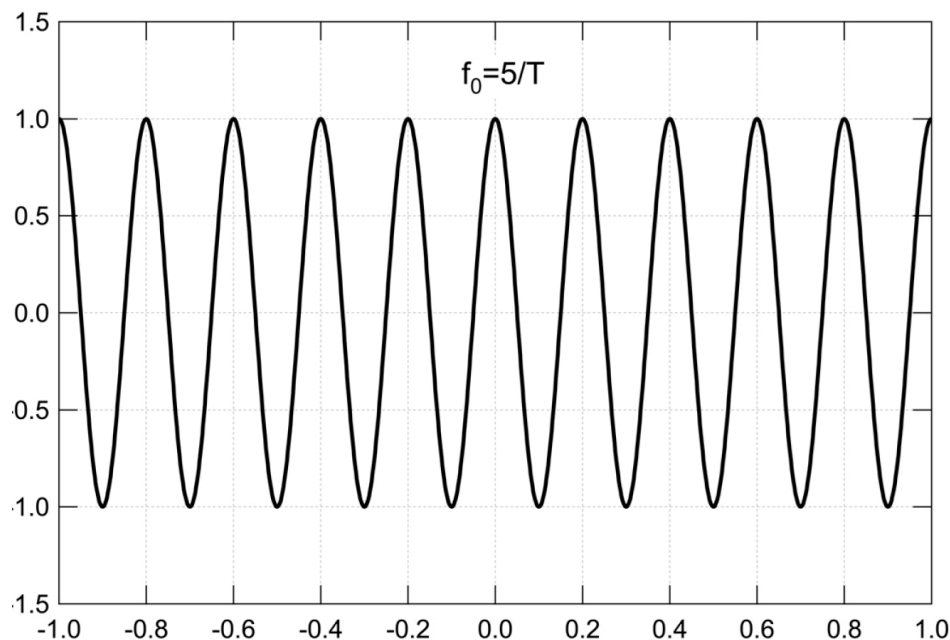
«Fast» and «Slow» Signals



The sampling period is adequate for $y(t)$, but it is clearly too large for $x(t)$. In this sense, the sampling frequency must be commensurate with the “speed” (i.e., with the rate of change) of the signal – how can we quantify this feature?

ONE (Simple) Sinusoidal Signal

$$x(t) = \cos\left(2\pi t \cdot \frac{5}{T}\right)$$



PERIODIC signal: the repetition period is

$$T_0 = 5/T$$

The oscillation frequency f_0 is the inverse of the period:

$$f_0 = 1/T_0$$

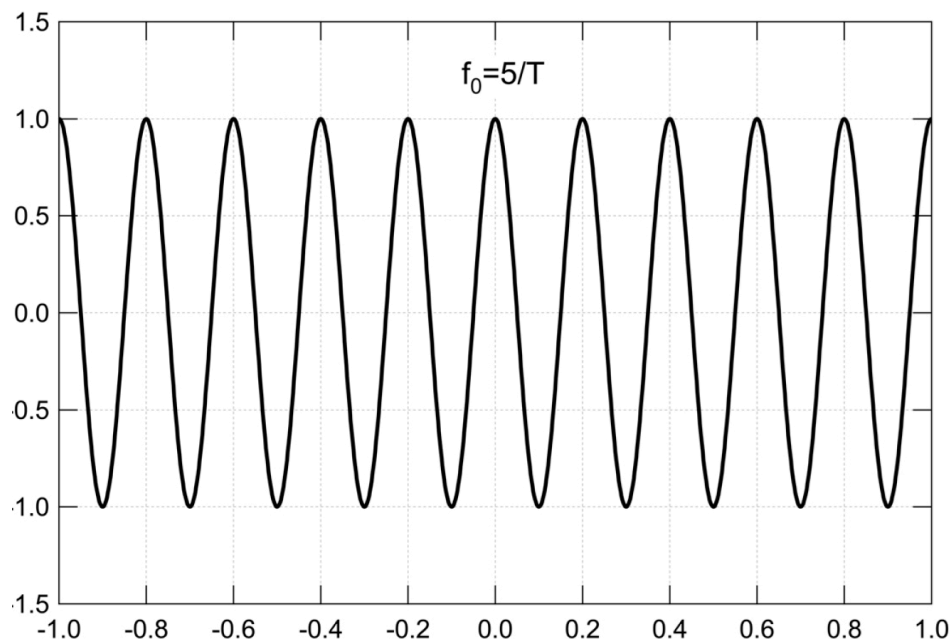
The amplitude A is the peak value (maximum value) of the signal, in the example $A=1$

In general,

$$x(t) = A \cos(2\pi f_0 t + \mathcal{G})$$

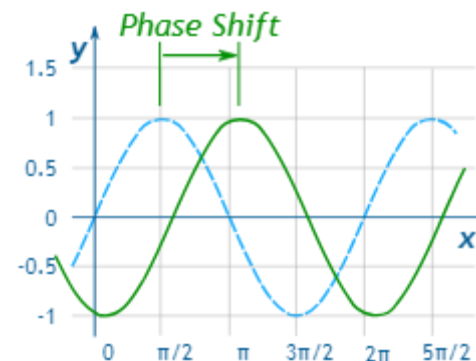
ONE (Simple) Sinusoidal Signal

$$x(t) = \cos\left(2\pi t \cdot \frac{5}{T}\right)$$



The phase-shift ϑ indicates the initial point or initial value of the waveform at $t=0$:

$$x(0) = A \cos(\vartheta)$$

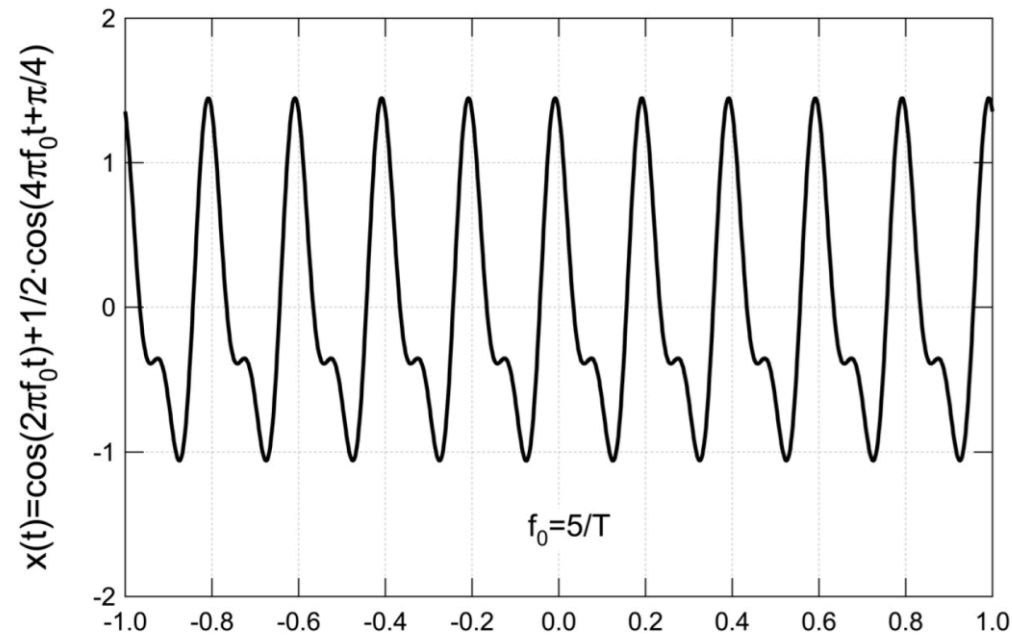


In general,

$$x(t) = A \cos(2\pi f_0 t + \vartheta)$$

TWO (Simple) Sinusoidal Signals

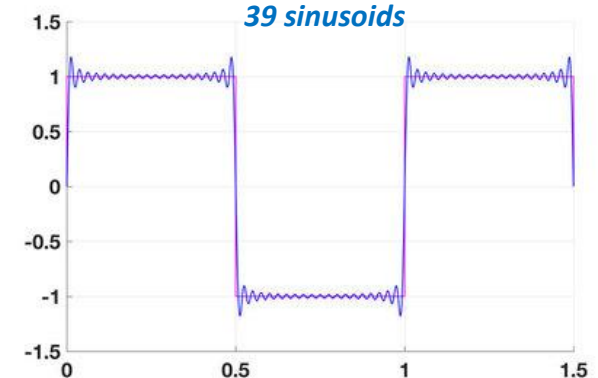
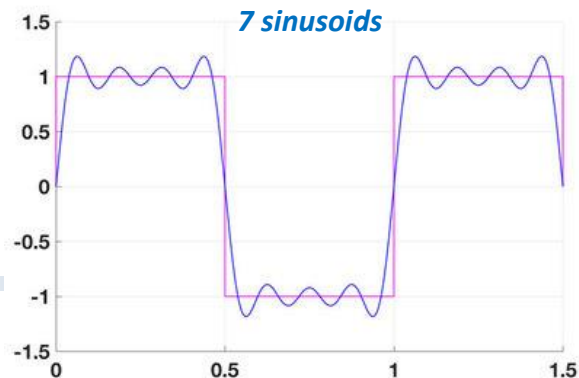
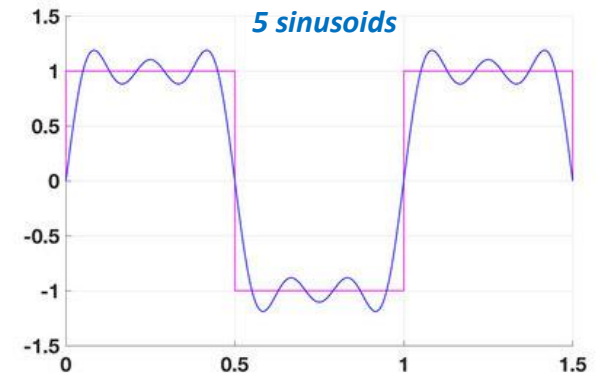
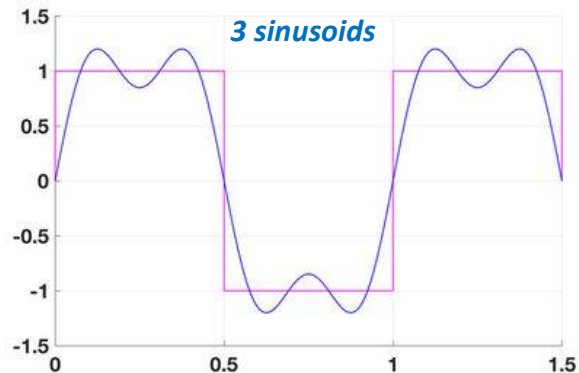
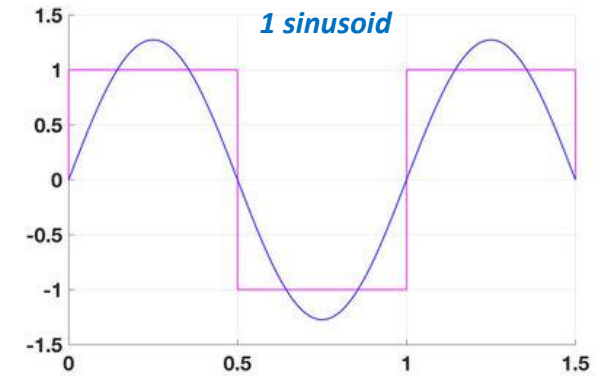
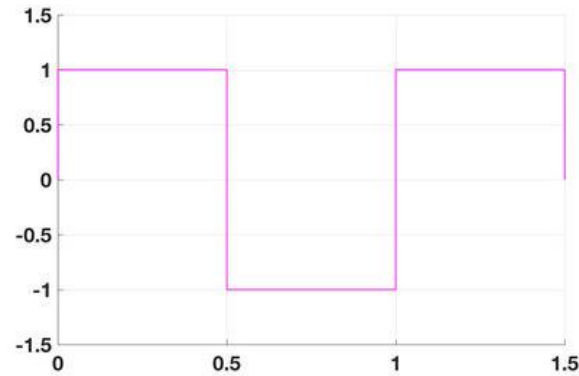
$$x(t) = \cos(2\pi 5t / T) + 0.5 \cos(2\pi 10t / T + \pi / 4)$$



$$x(t) = A_1 \cos(2\pi f_0 t) + A_2 \cos(2\pi 2 f_0 t + \pi / 4)$$

VERY MANY (Simple) Sinusoidal Signals...

... can make a complicated (periodic) signal: the square wave – or: Fourier analysis for dummies



General Fourier Series

$$x(t) = A_1 \cos(2\pi f_0 t + \theta_1) + A_2 \cos(2\pi 2 f_0 t + \theta_2) + A_3 \cos(2\pi 3 f_0 t + \theta_3) + \dots + A_k \cos(2\pi k f_0 t + \theta_k) + \dots$$

SYNTHESIS equation



General Fourier Series

$$x(t) = A_1 \cos(2\pi f_0 t + \theta_1) + A_2 \cos(2\pi 2 f_0 t + \theta_2) + A_3 \cos(2\pi 3 f_0 t + \theta_3) + \dots + A_k \cos(2\pi k f_0 t + \theta_k) + \dots$$

SYNTHESIS equation



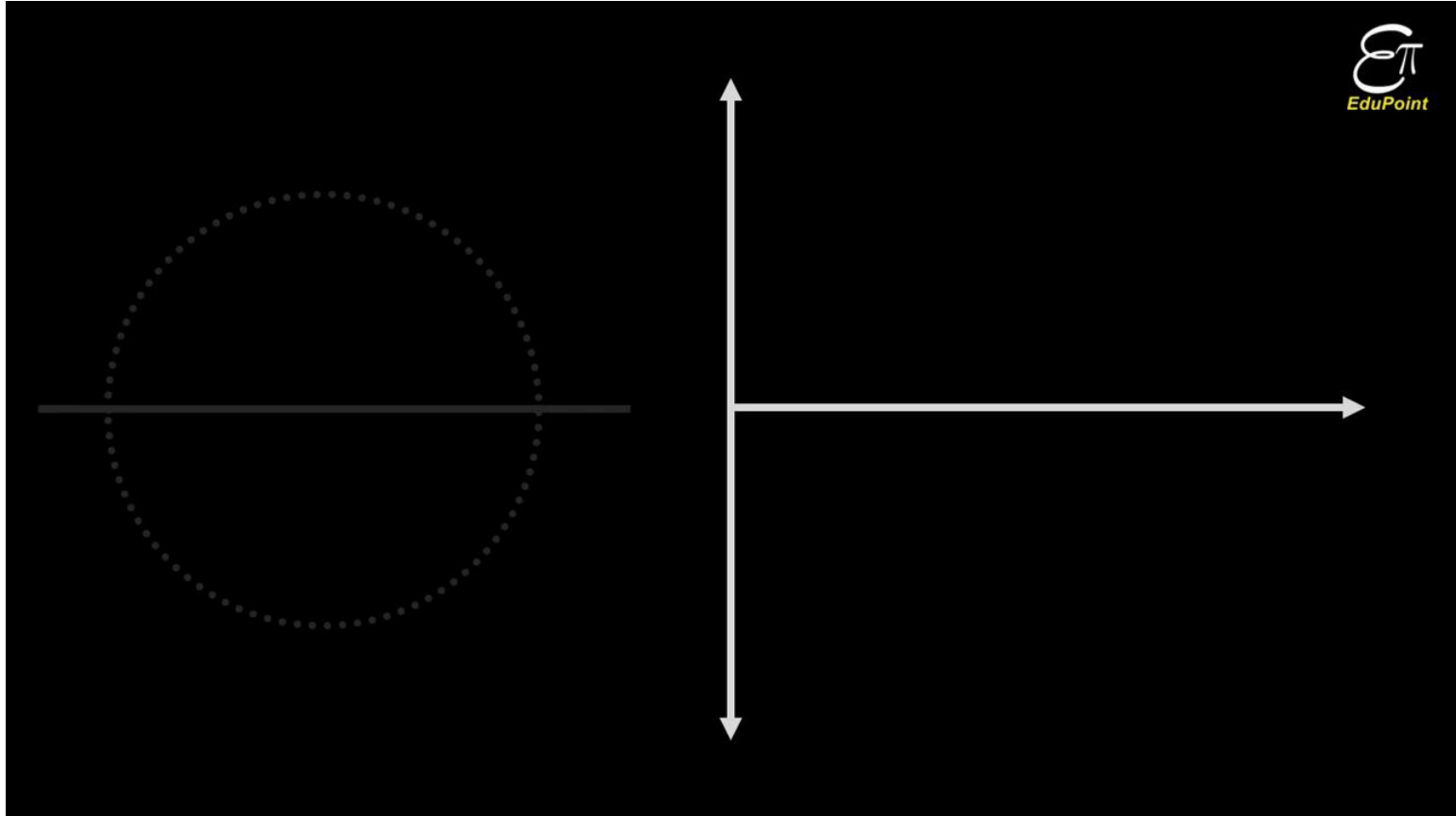
General Fourier Series

$$A_k e^{j\theta_k} = \frac{1}{T_0} \int_0^{T_0} x(t) [\cos(2\pi k f_0 t) - j \sin(2\pi k f_0 t)] dt$$

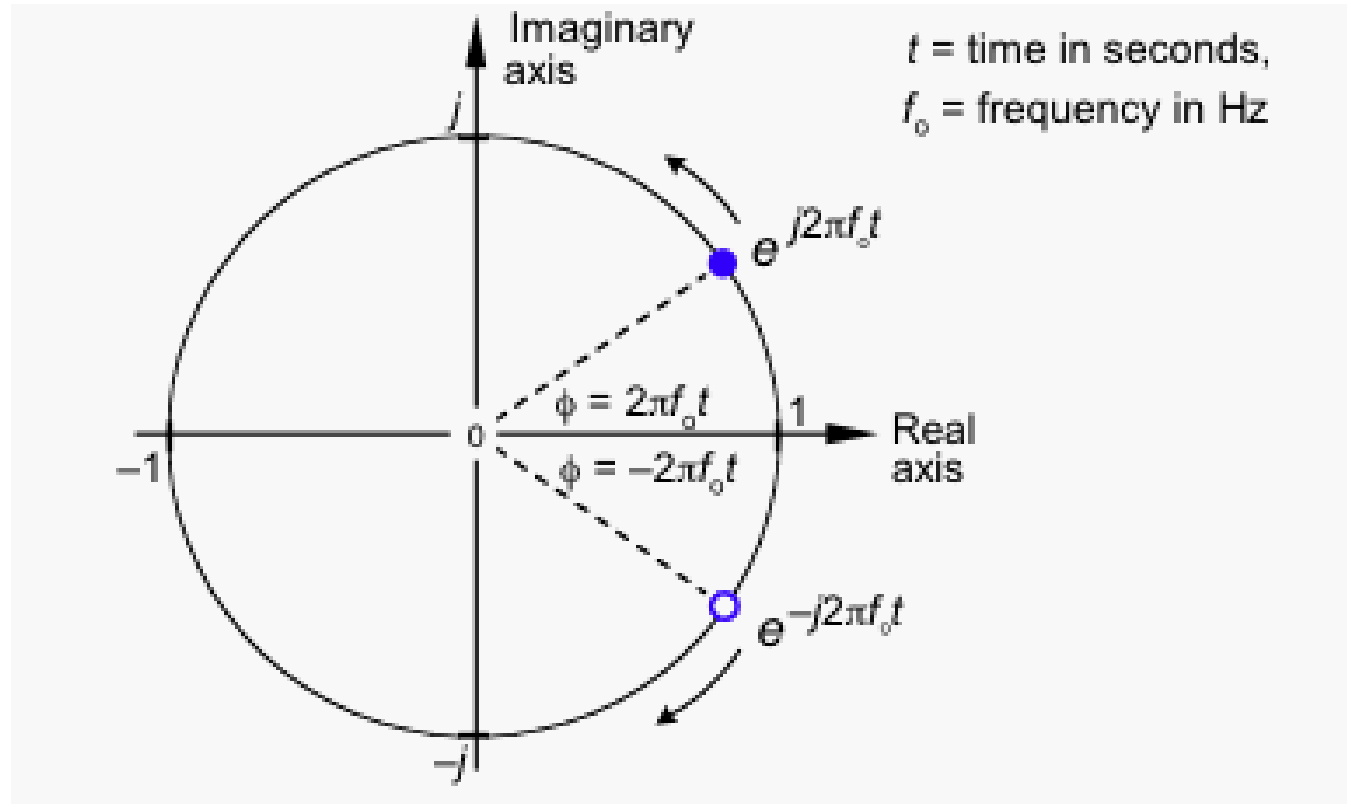
ANALYSIS equation



Positive and Negative (?) Frequencies 1/2

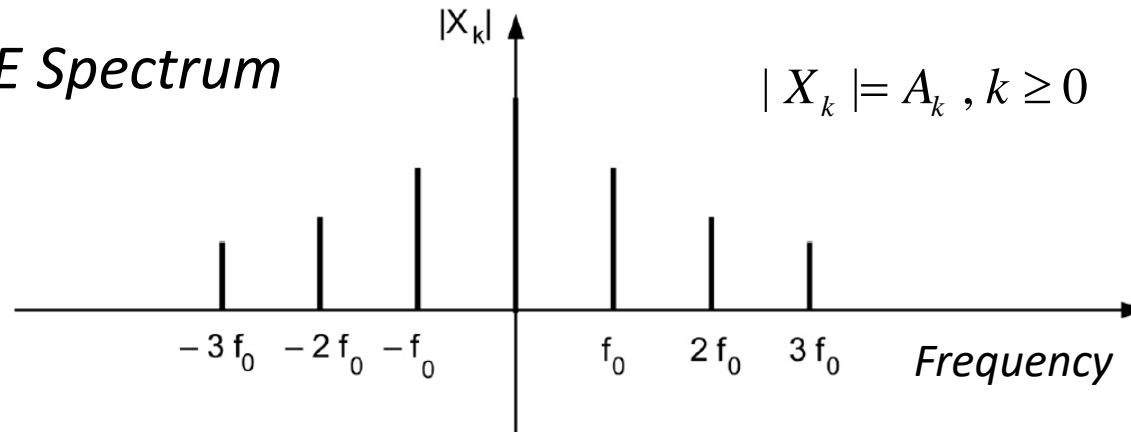


Positive and Negative (?) Frequencies 2/2

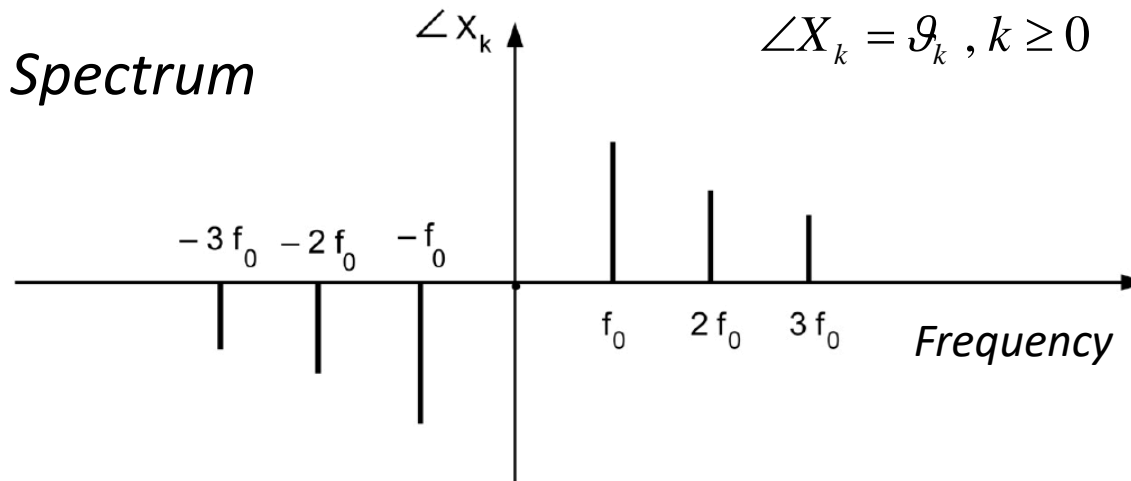


The frequency spectrum: frequency contents of a signal

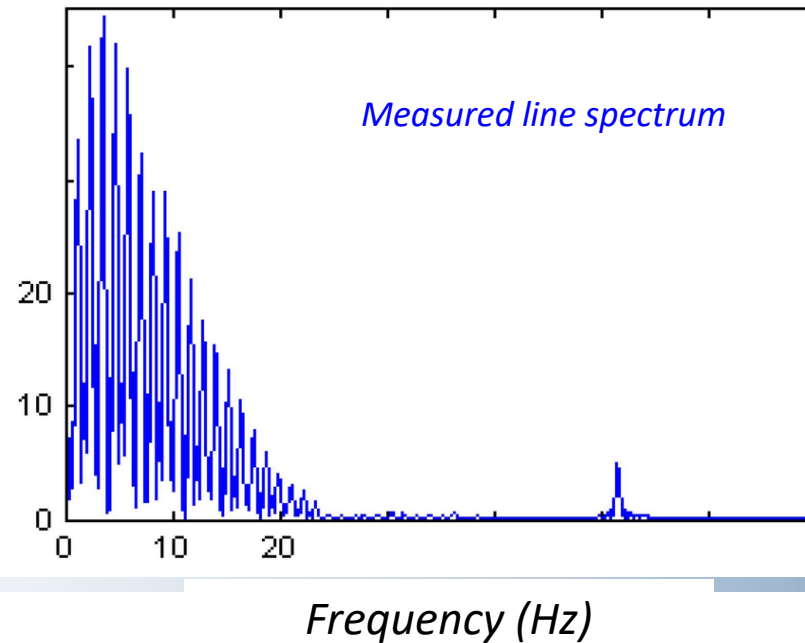
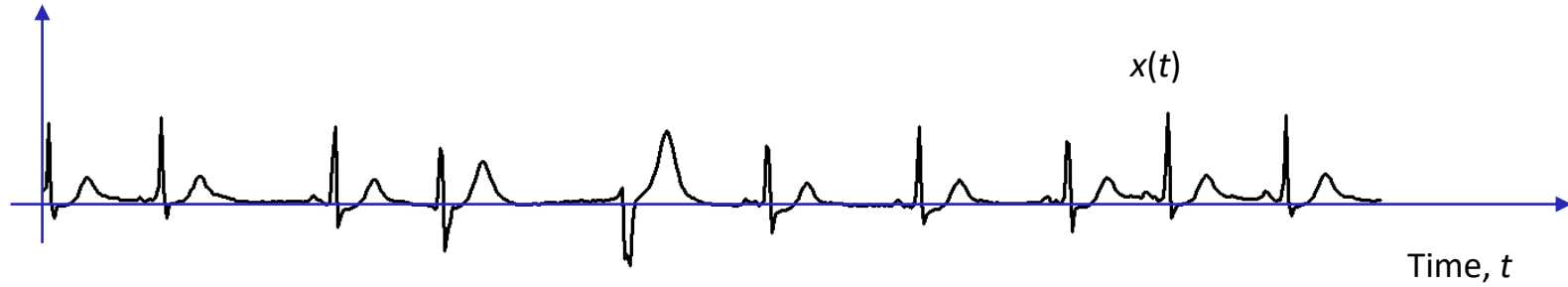
AMPLITUDE Spectrum



PHASE Spectrum

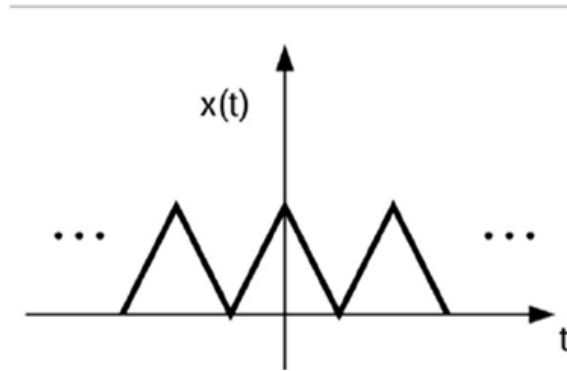


To sum-up: PERIODIC SIGNAL



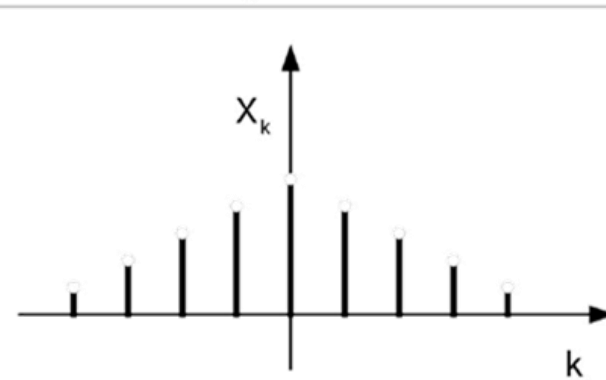
Signals and Spectra

TIME

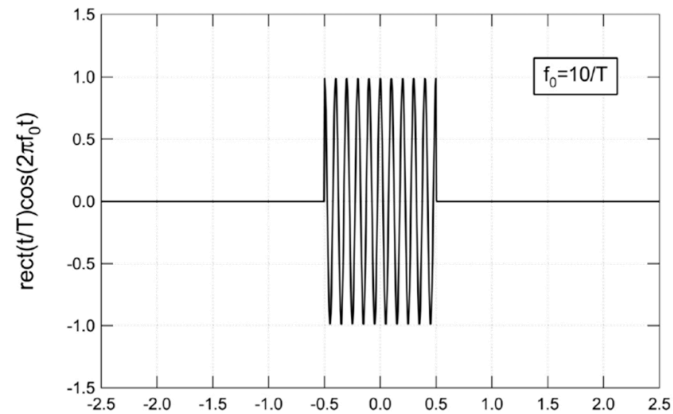


Periodic Signal

FREQUENCY



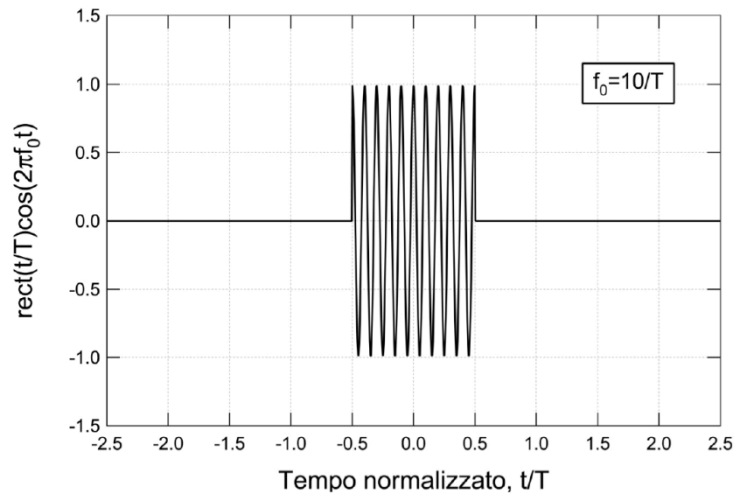
Line Spectrum



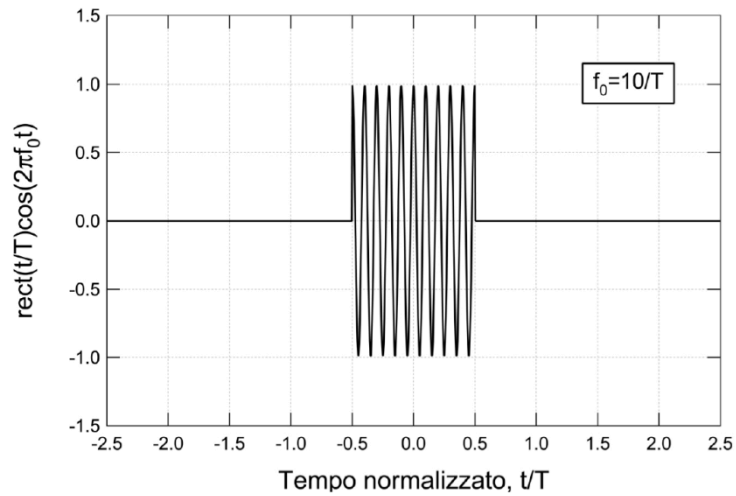
Aperiodic Signal

?

Time and Frequency



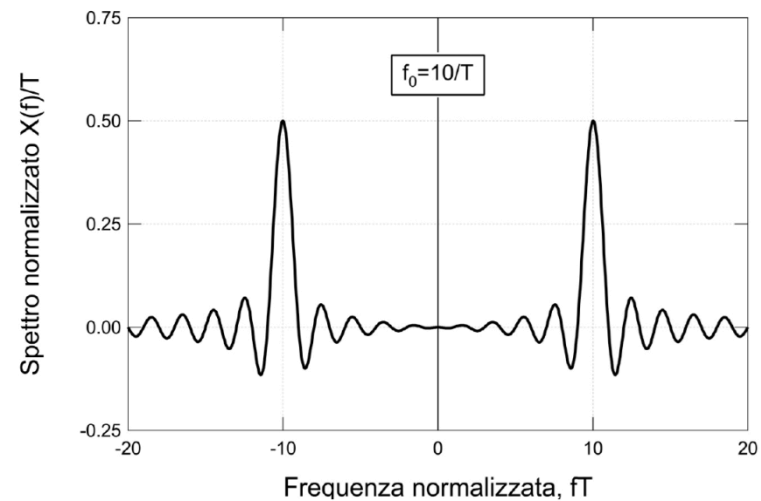
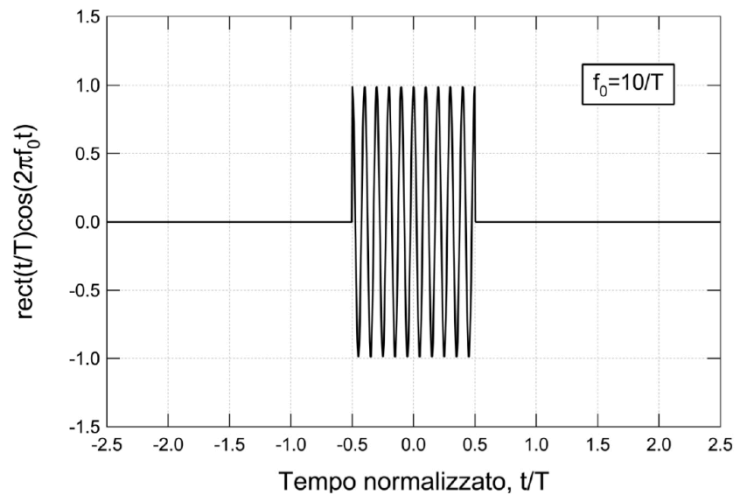
Time and Frequency



$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

ANALISYS Equation

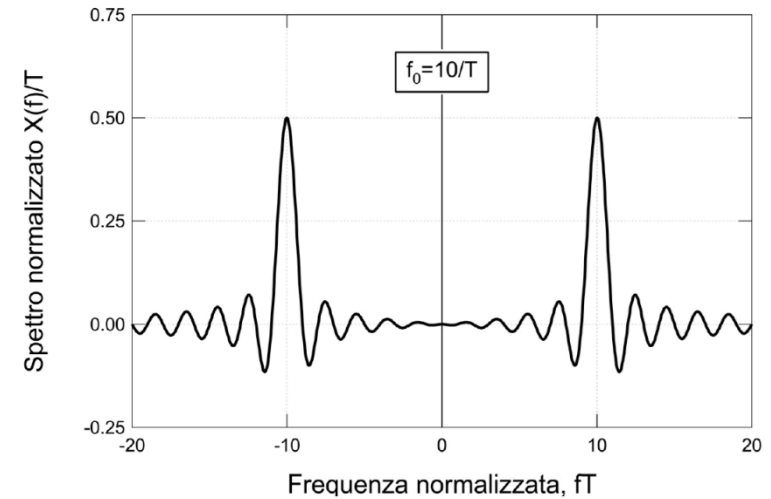
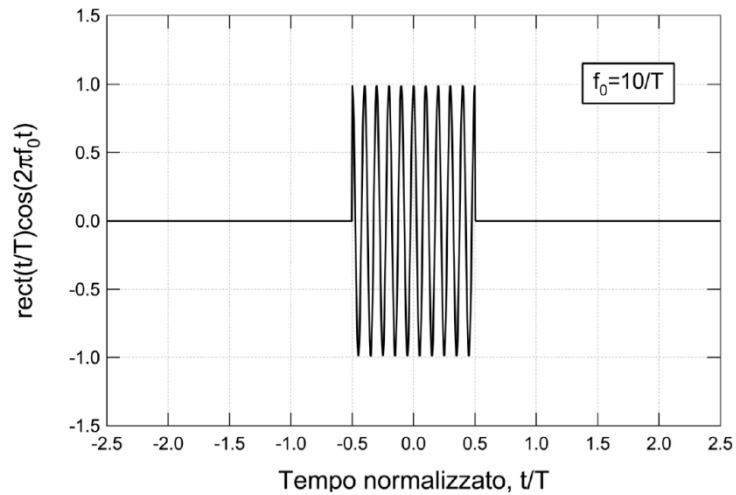
Time and Frequency



$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

ANALISYS Equation

Time and Frequency



$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} df$$

SYNTHESIS Equation

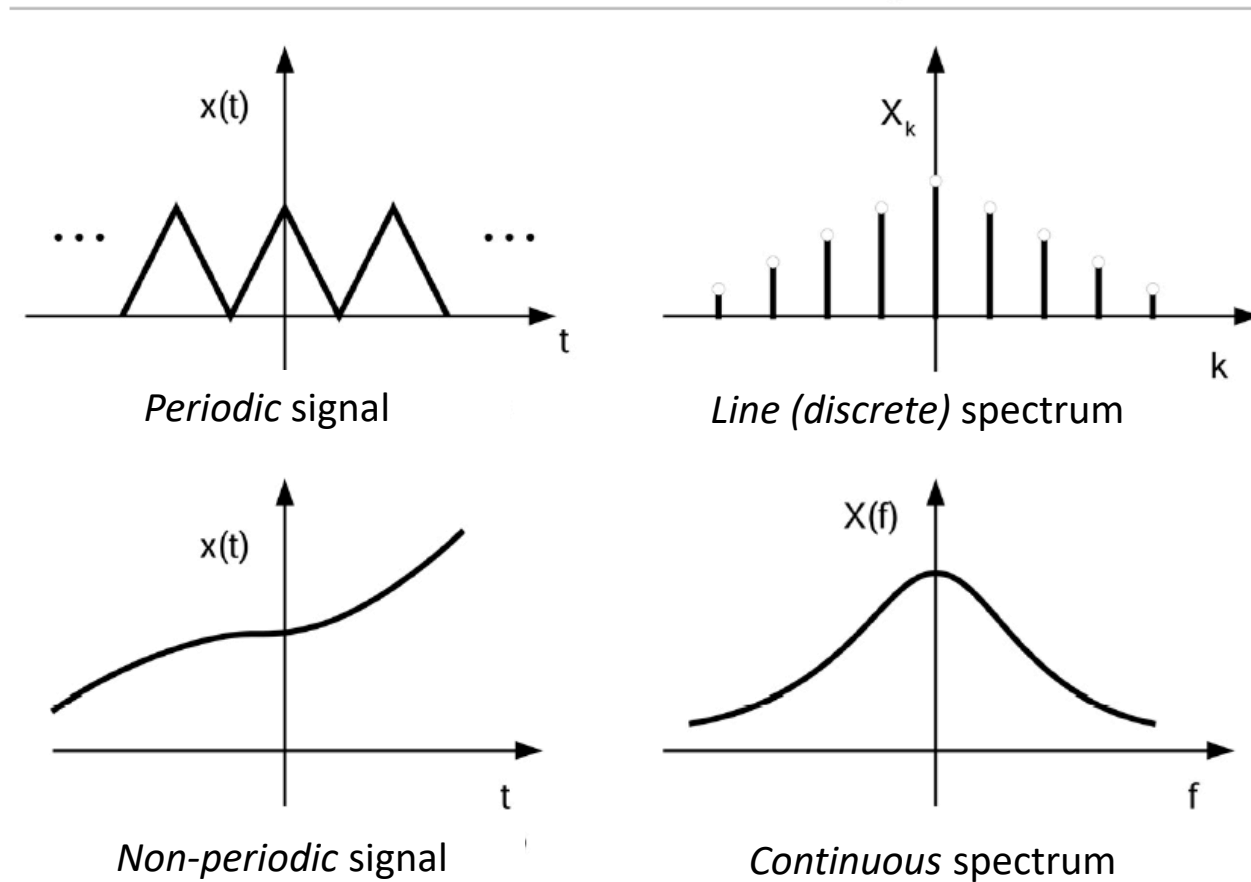
$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

ANALYSIS Equation

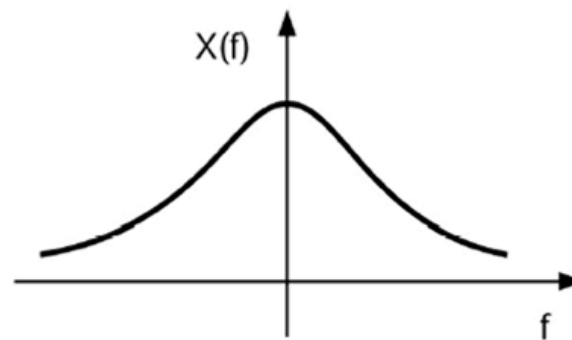
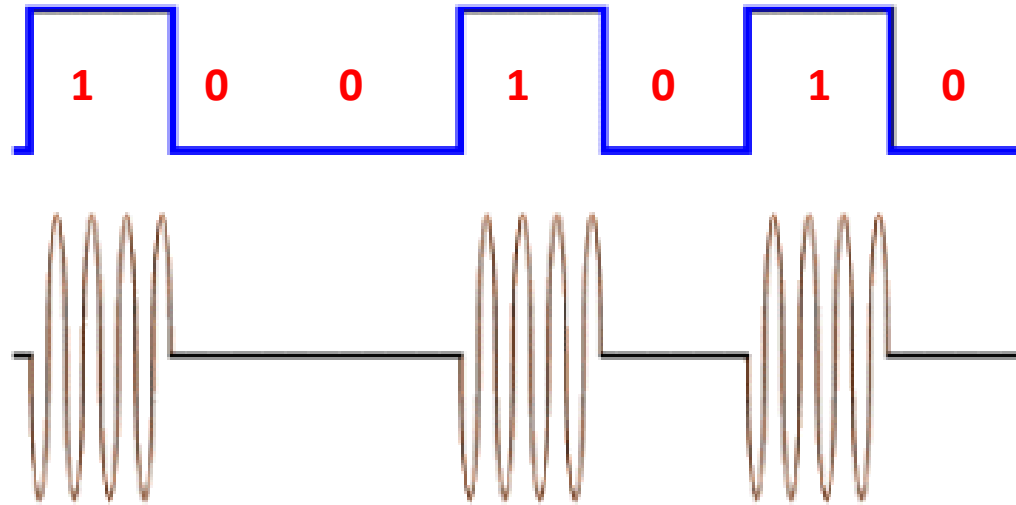
Different kind of Spectra

Time

Frequency



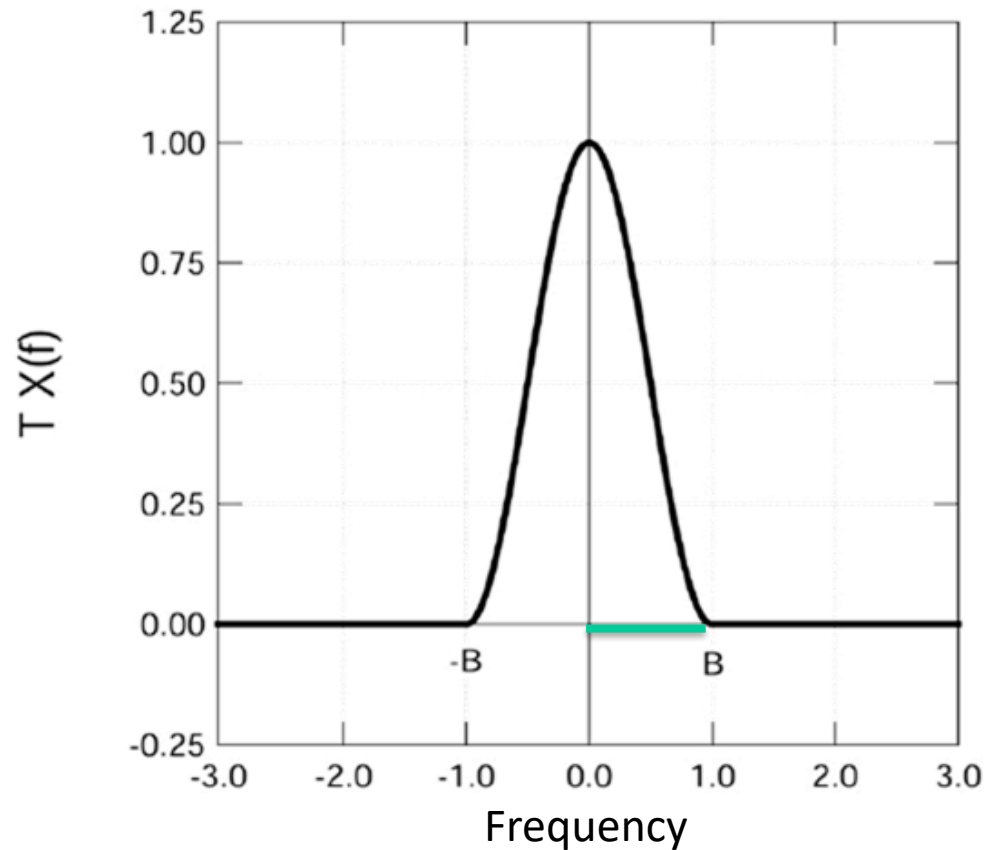
Non.-periodic, RANDOM Signals



Continuous POWER spectrum as well !

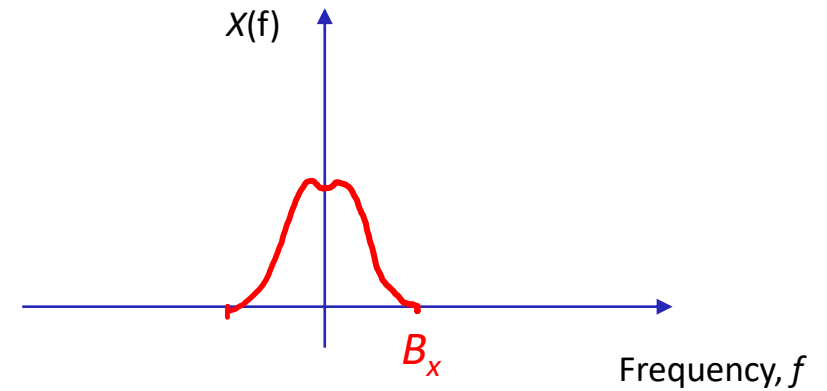
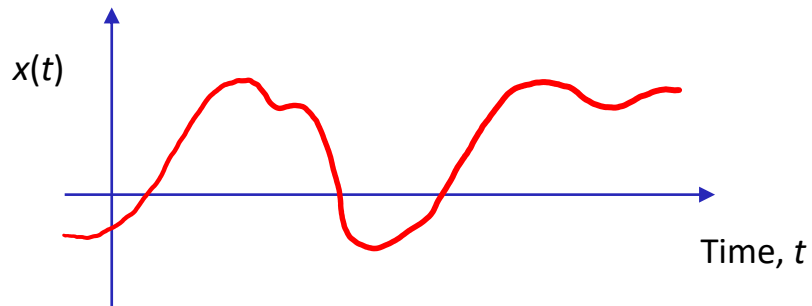
The BANDWIDTH of a signal

The bandwidth B is the width of the frequency interval (on positive frequencies) on which the signal spectrum is different from 0

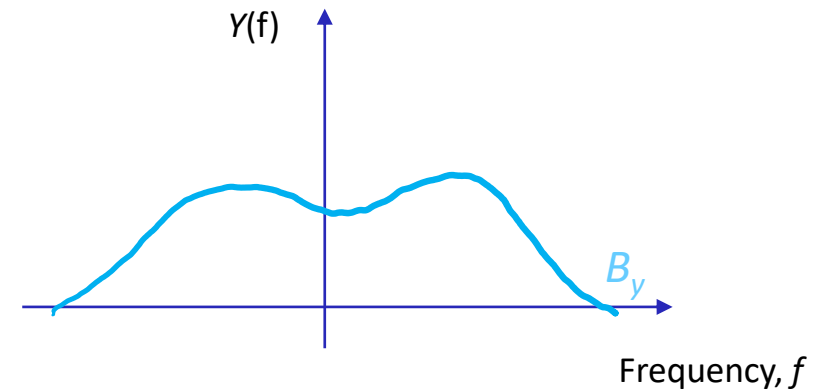
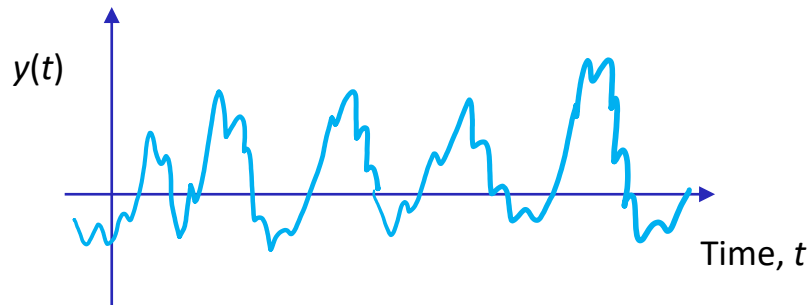


Wideband and Narrowband

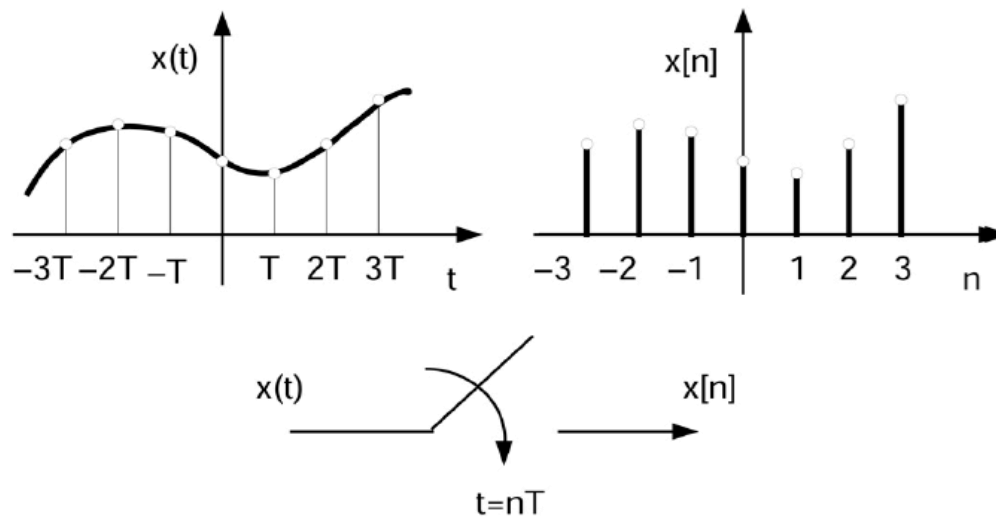
“Slow”, narrowband signal and its spectrum



“Fast”, wideband signal and its spectrum



$$B_x < B_y$$

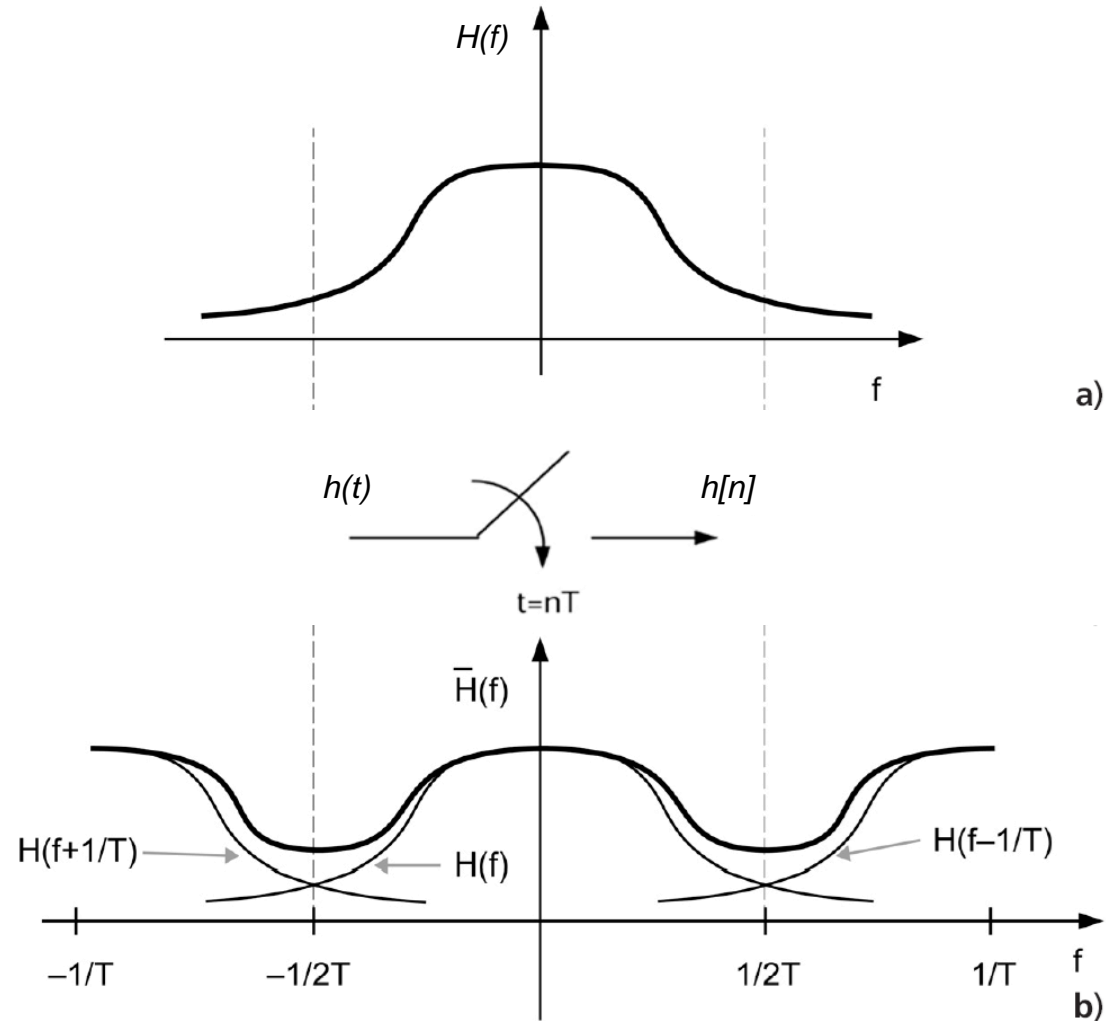


$$x[n] = x(nT)$$

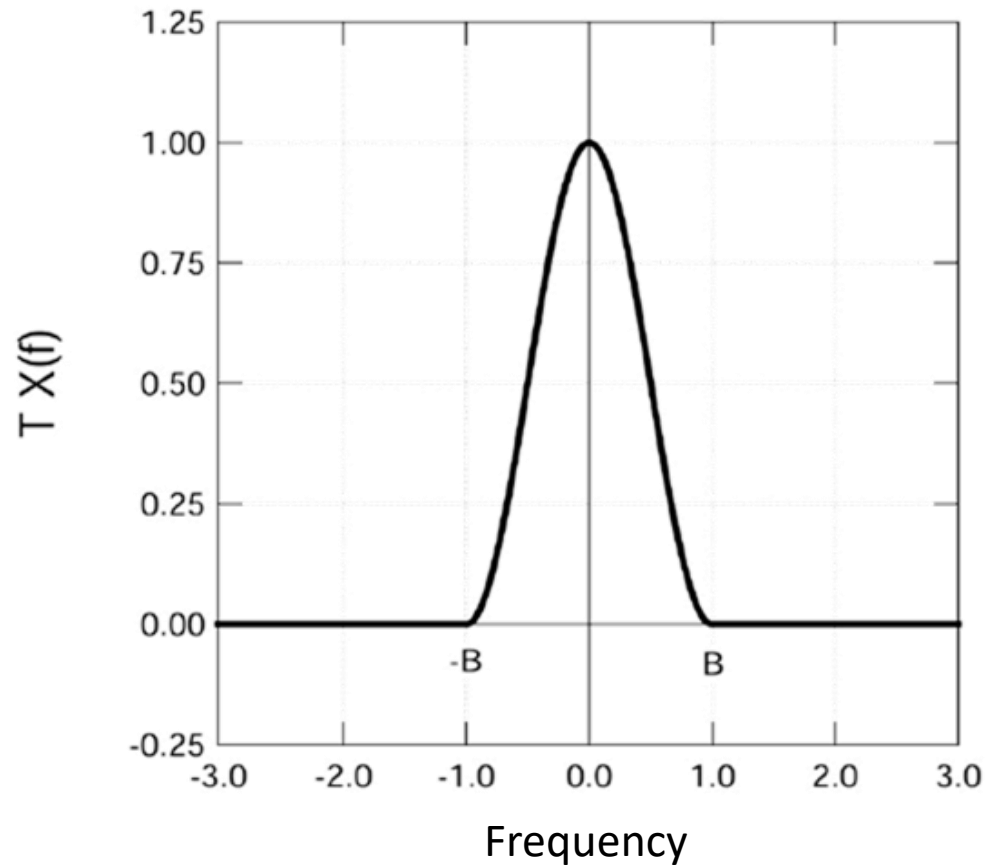
Poisson's relation that gives the spectrum of the digital signal

$$\bar{X}(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{T}\right)$$

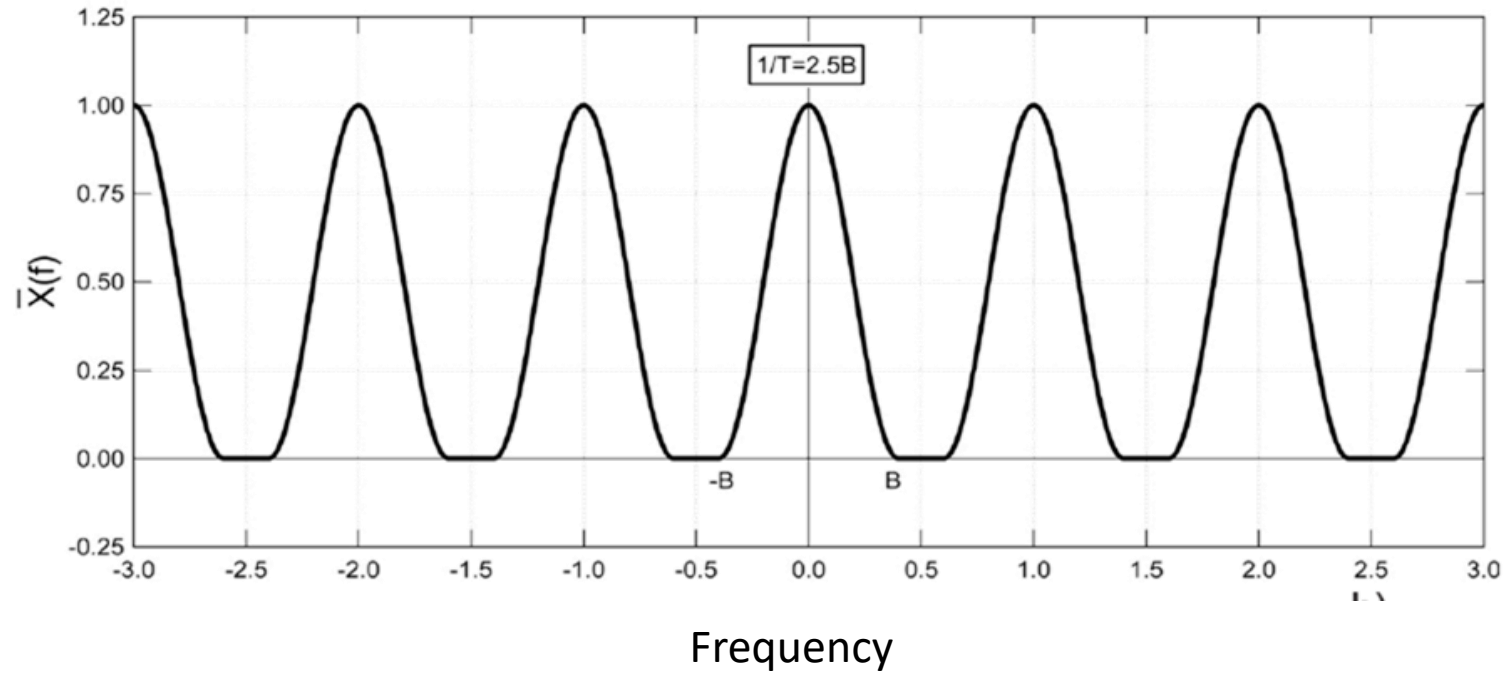
Periodic spectrum of a sampled signal



An example

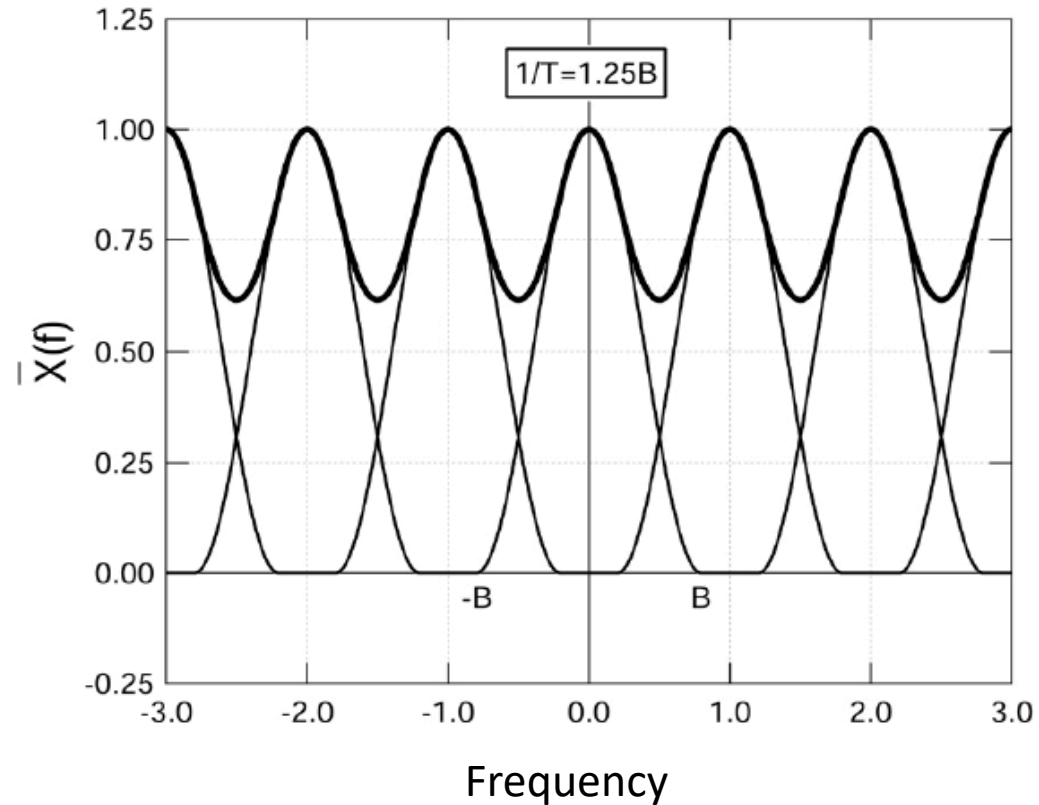


No Aliasing



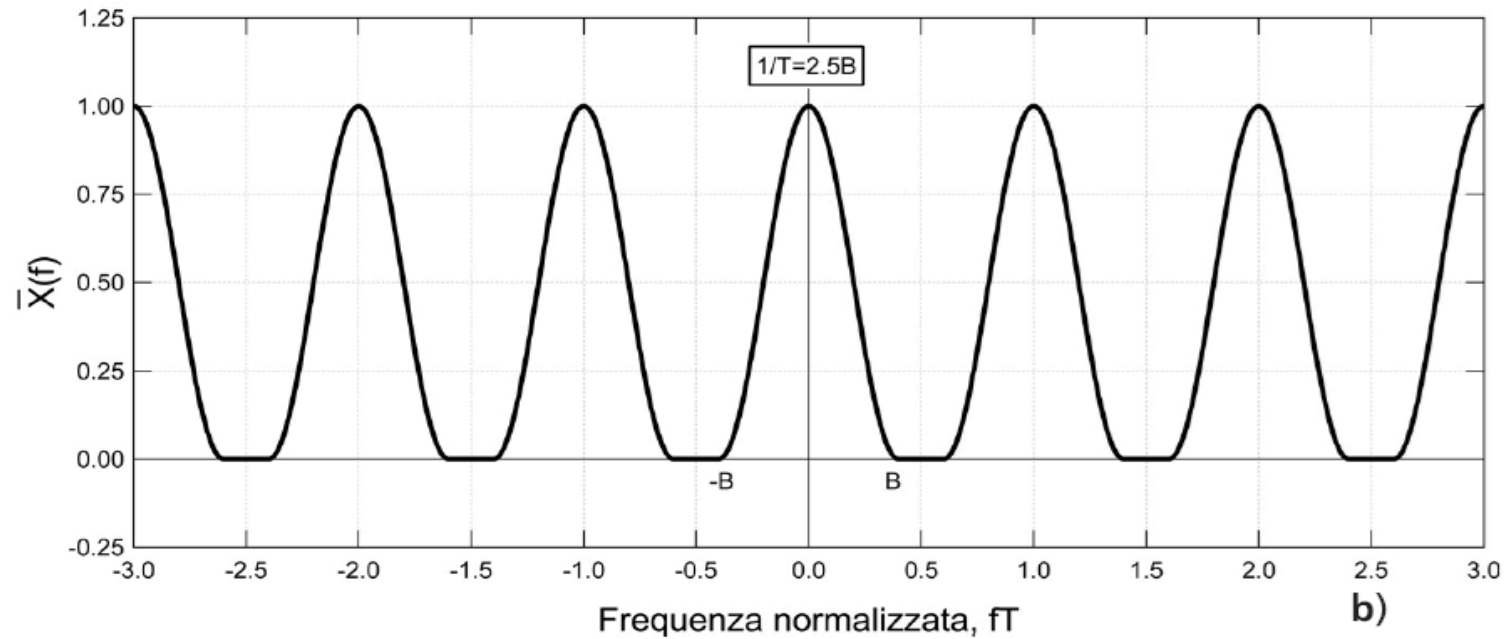
Using a sampling frequency $f_c = 5B/2$

Aliasing Error !



Using a sampling frequency $f_c = 5B/4$

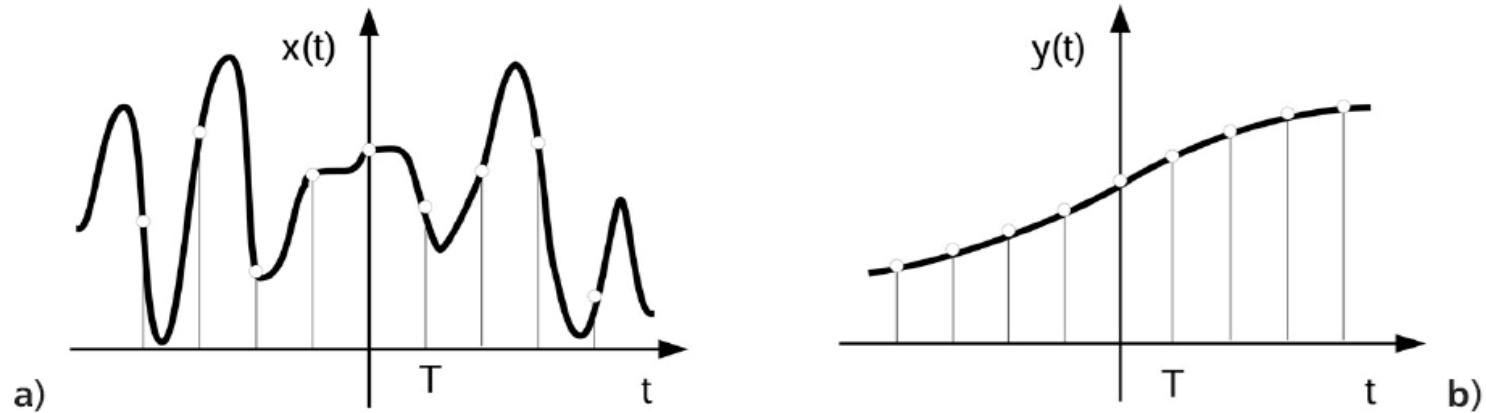
Nyquist rule



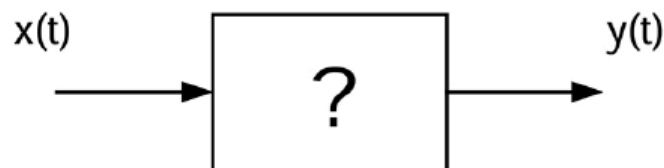
Not to have aliasing, we have to make sure that

$$f_c \geq 2B$$

Nyquist rule in the time domain

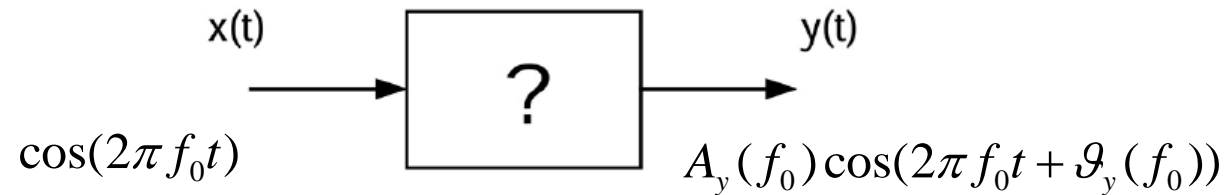


The sampling period is adequate for $y(t)$, but it is clearly too large for $x(t)$. In this sense, the sampling frequency must be commensurate with the signal bandwidth (with the rate of change of the signal), as the Nyquist rule suggests



- A **filter** is a «black box» (HW, SW in an ARM DSP processor) with a linear behavior.
- If $x(t)$ is sinusoidal, then $y(t)$ is sinusoidal at the same frequency: frequencies are not changed
- BUT what is changed is the amplitude/phase of the oscillation:

$$x(t) = \cos(2\pi f_0 t) \quad \Rightarrow \quad y(t) = A_y \cos(2\pi f_0 t + \mathcal{G}_y)$$



- Not all frequencies are the same: according to the properties of the filter systems, some frequencies are more altered than others (for instance, enhanced/attenuated)
- The filter *responds* differently according to the particular frequency of the input: the *frequency response* of the filter is

$$H(f_0) = A_y(f_0) e^{j\vartheta_y(f_0)}$$

What's the purpose of Filtering ?

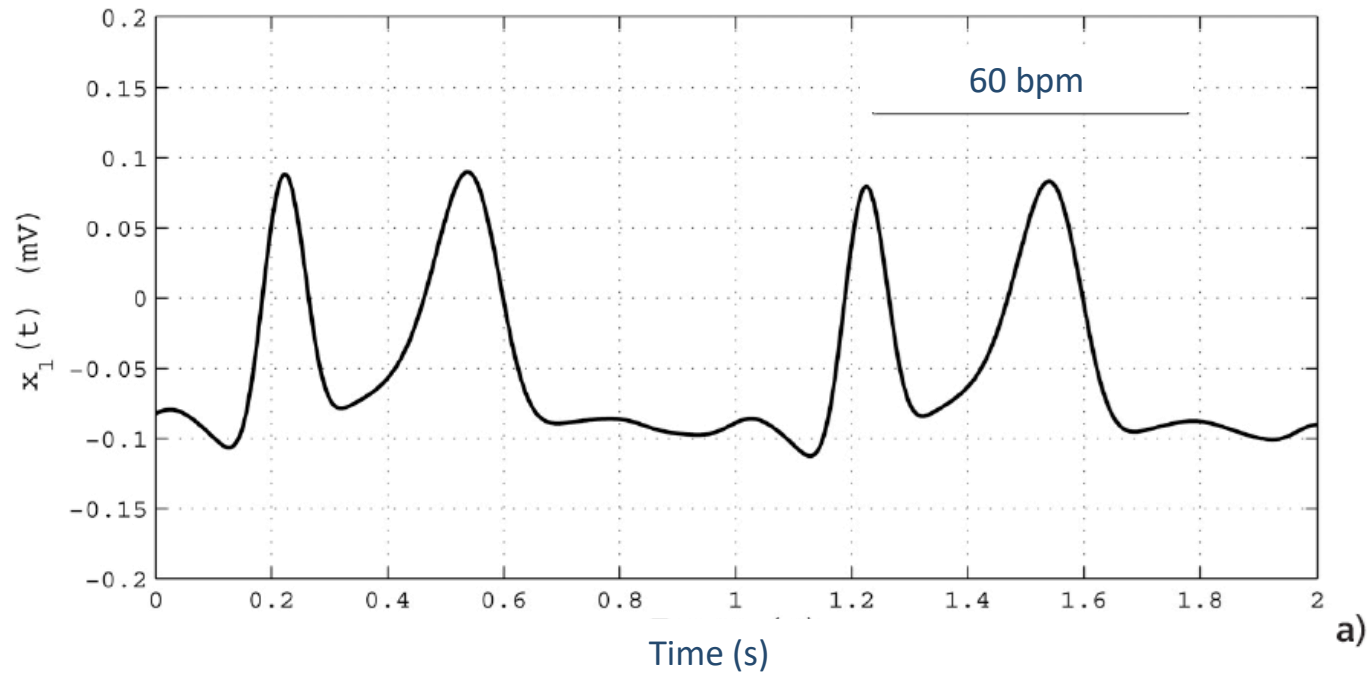
- A certain signal is made of the superposition of two components:

$$x(t) = x_1(t) + x_2(t)$$

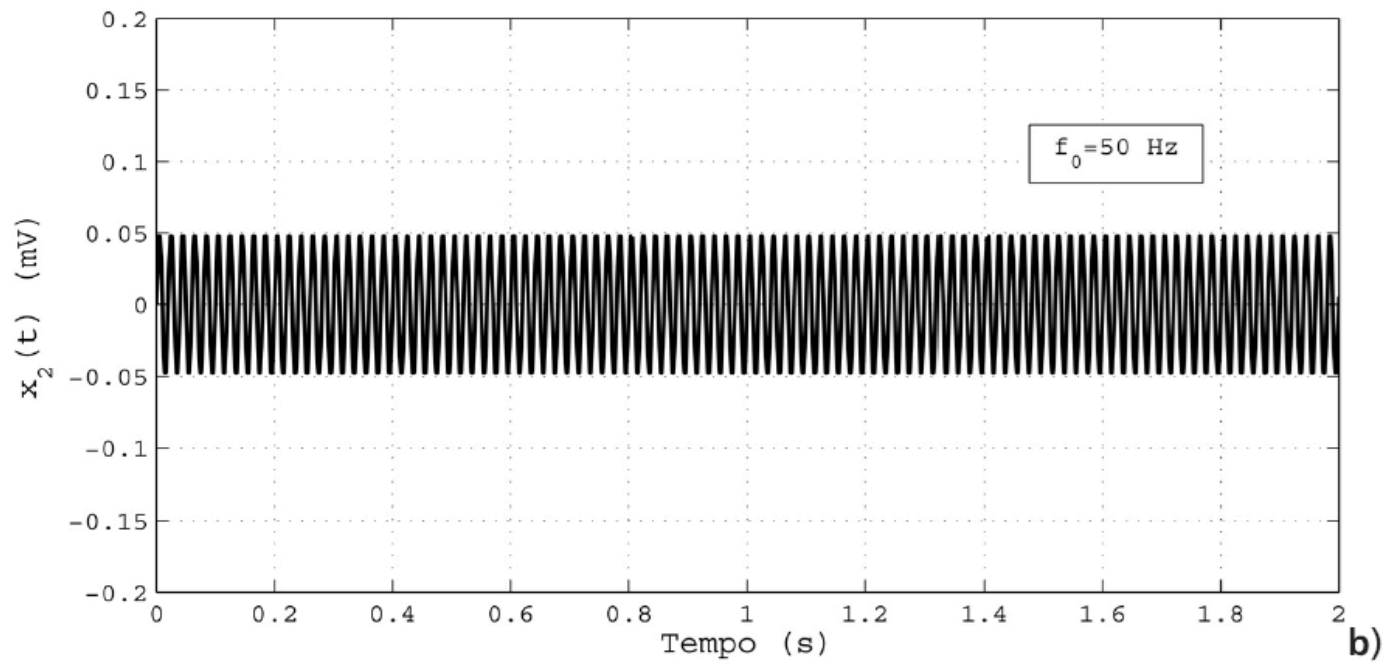
The first component is the useful signal, the second is just *noise* (disturbance), an unwanted component we would like to rid of.

Example: (low-power, low amplitude) ECG signal plus an unwanted component (interference) coming from the 50 Hz AC power supply.

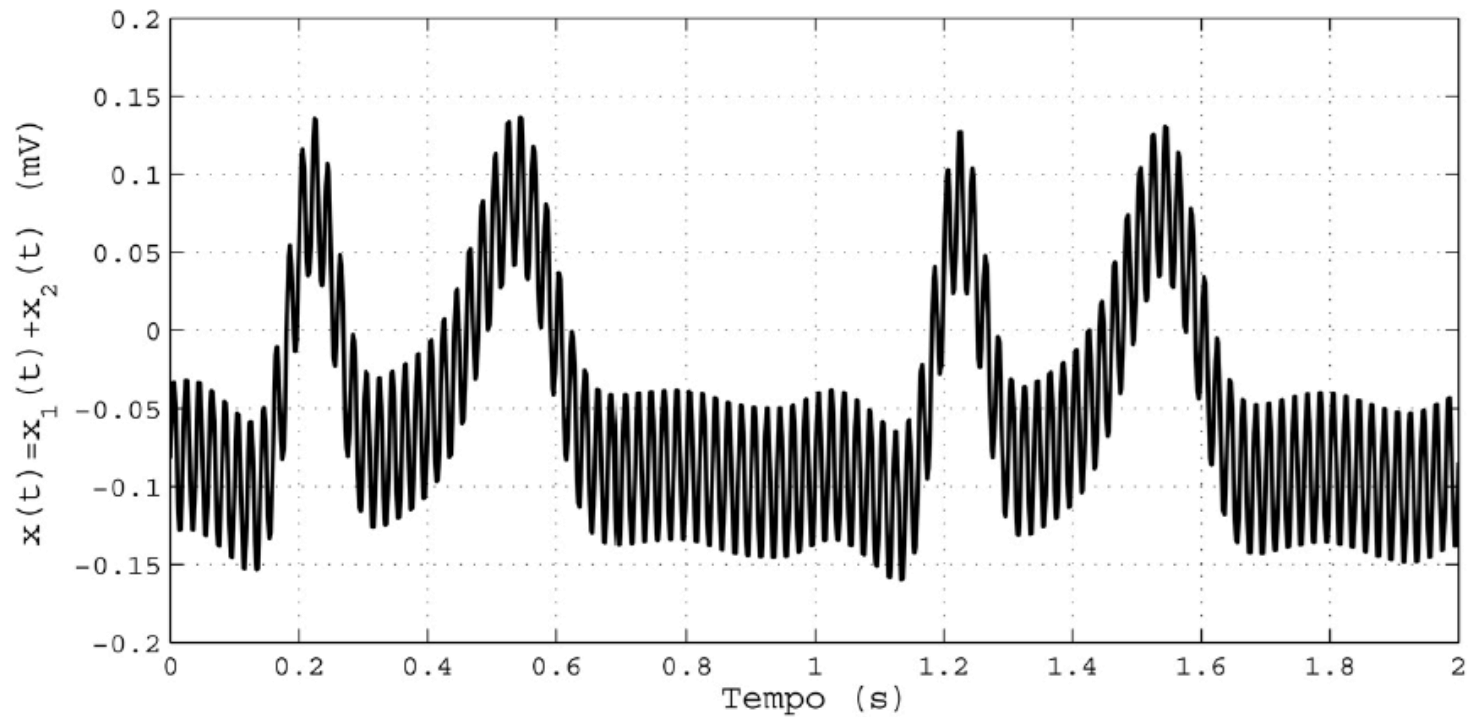
The Notion of FILTER



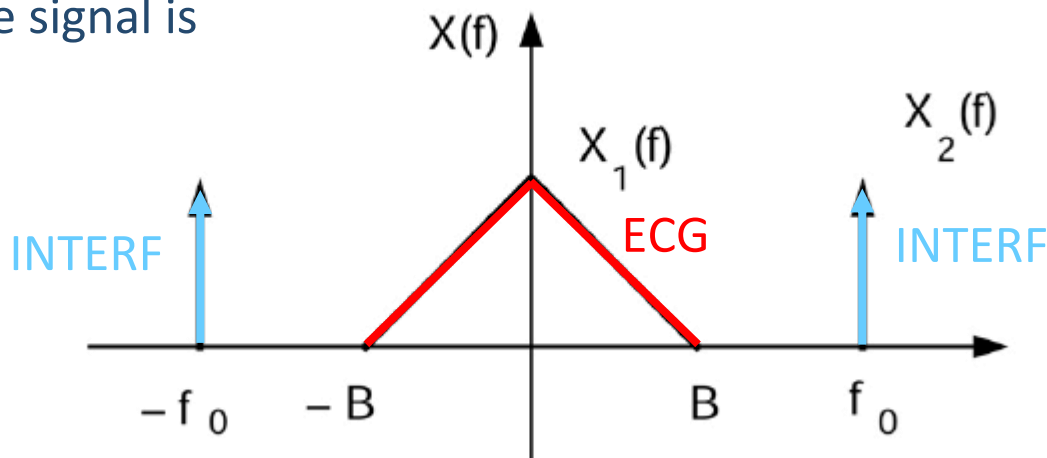
The Notion of FILTER



The Notion of FILTER



Is there any hope to *reject* (eliminate) the unwanted component?
Let us examine the problem in the *frequency domain*. The spectra of the signal is

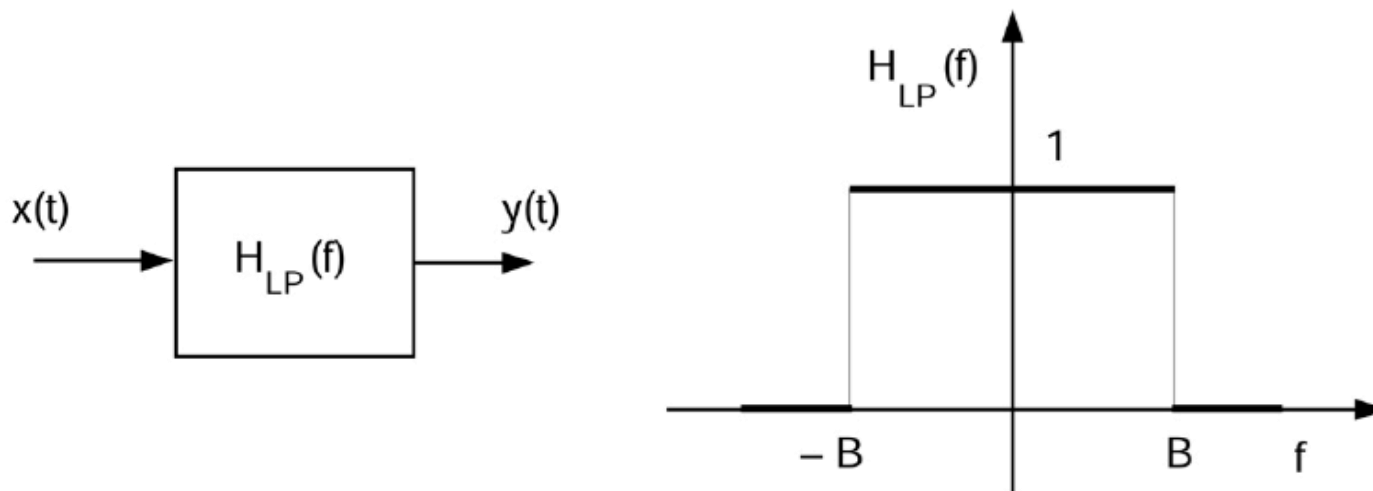


$$X(f) = X_1(f) + X_2(f)$$

Whilst the two signals fully overlap in time, their spectra lie in two separate frequency intervals (*bands*) ! We can separate the two (rejecting the interference) with a filter having an appropriate *frequency response* (with unequal treatment of frequency bands)

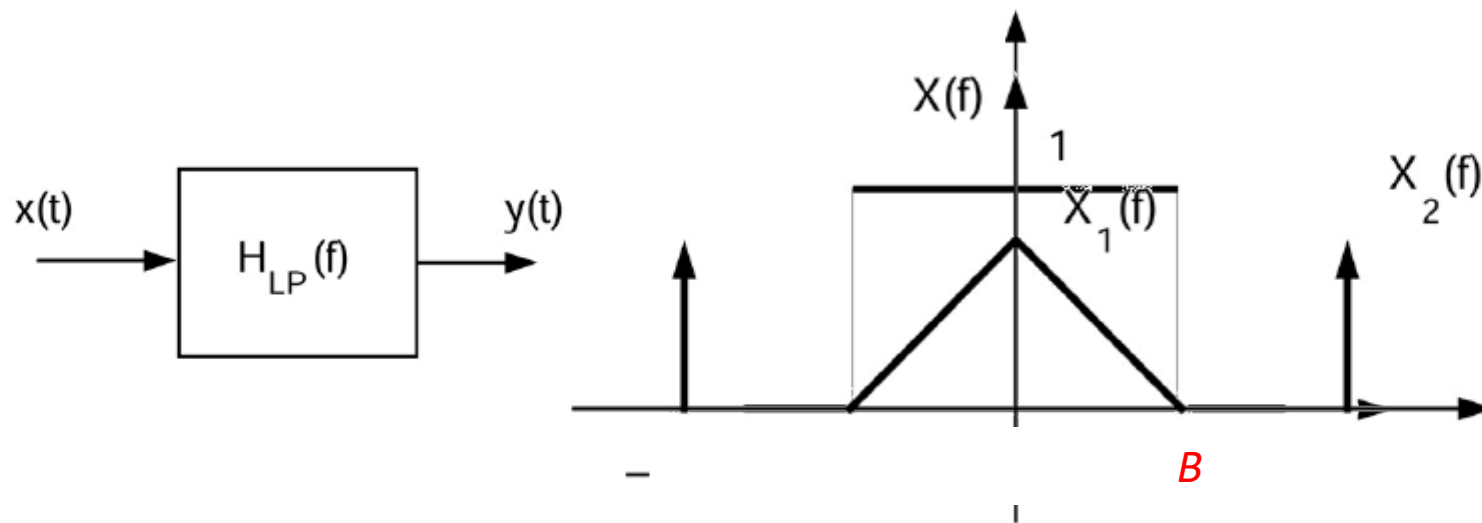
Low-pass Filter

We can *filter out* (reject) the higher-frequency, unwanted component $x_2(t)$ applying to the whole signal $x(t)$ an LTI system that *passes* the low-frequency components and *blocks* the high-frequency ones: a low-pass filter with frequency response



$$Y(f) = X(f) H(f)$$

Low-pass Filter



B is the value of the filter bandwidth (to be considered on positive frequencies only)

$$Y(f) = X(f) H(f)$$

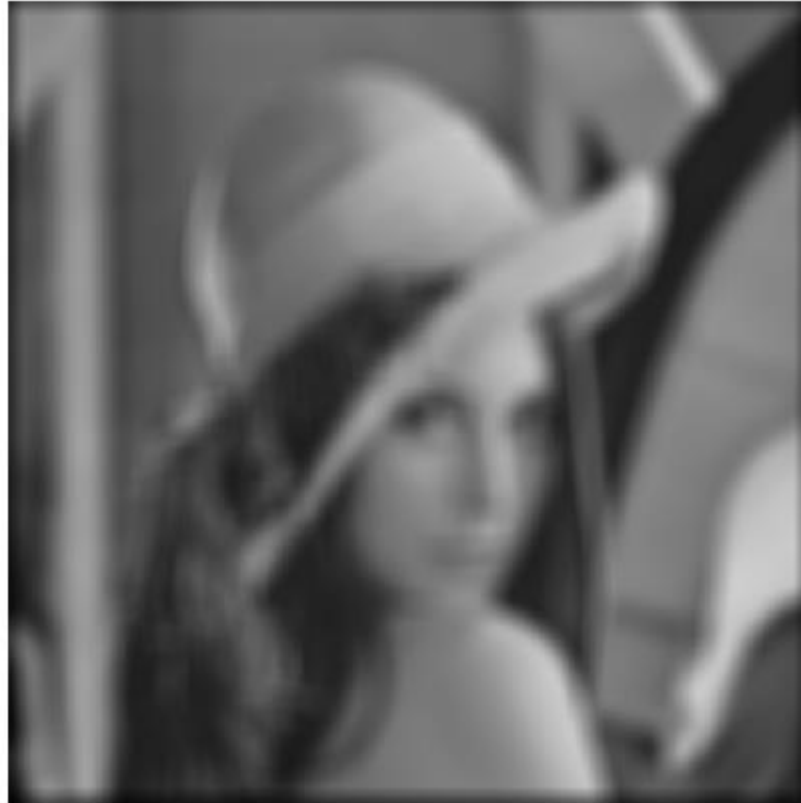
Filtering Example



a)

Input signal $x(t_1, t_2)$

Filtering Example



b)

Low-pass filtered signal $y(t_1, t_2)$

Filtering Example



a)

High-pass filtered signal $y(t_1, t_2)$

Filtering Example

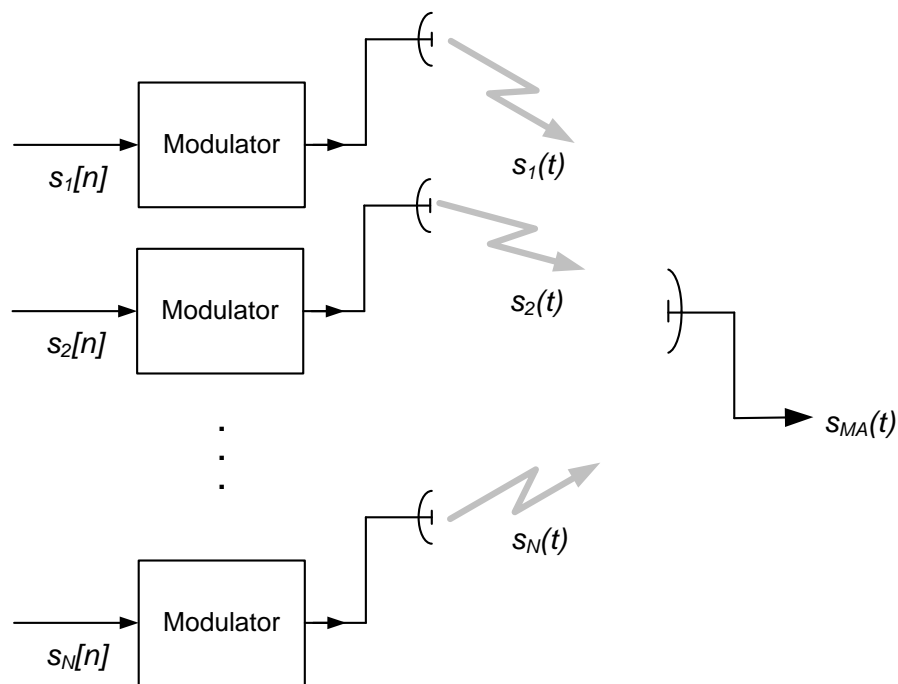


b)

Same, with inverted brightness scale

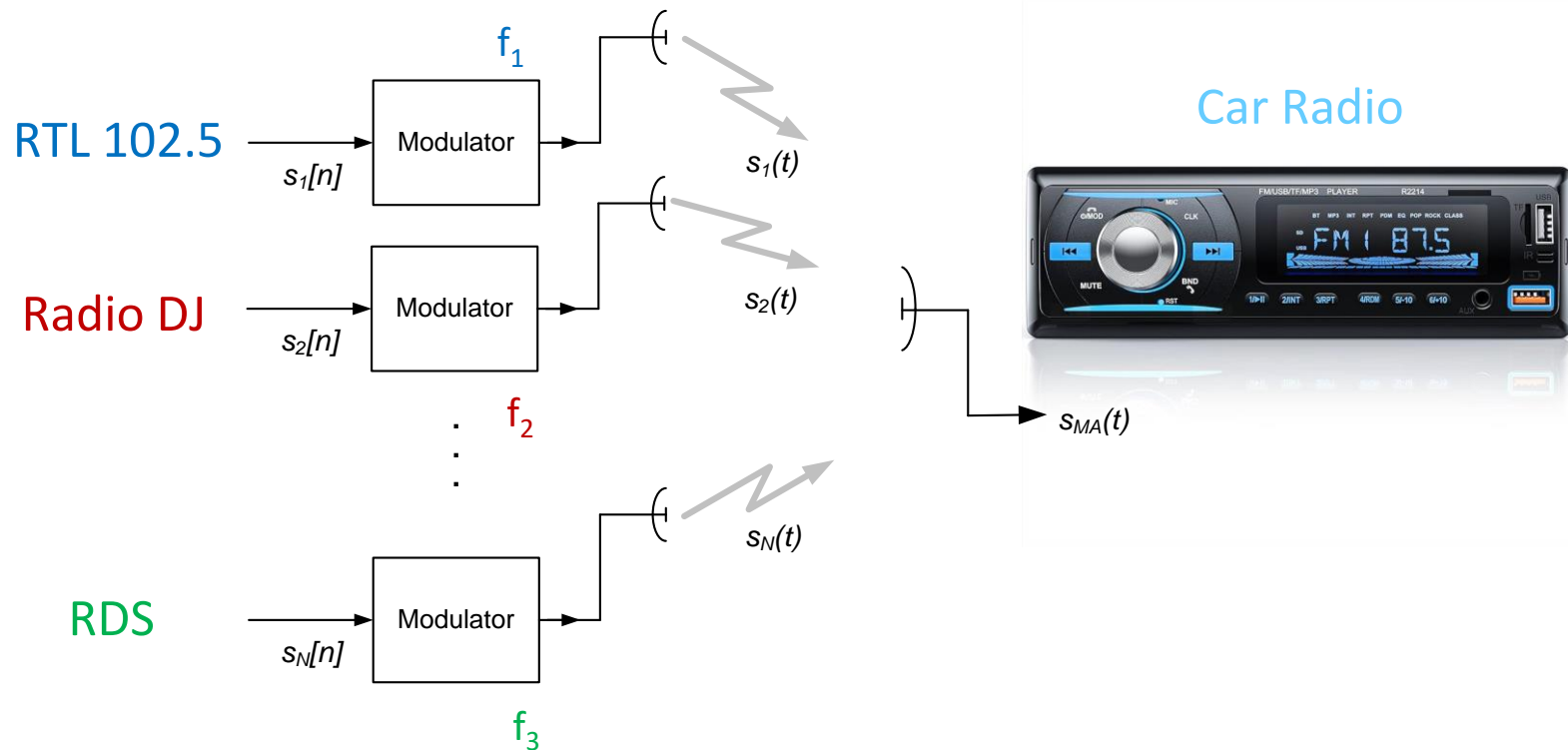
More on Filtering: Frequency-Division Multiple Access

- Many users share the same physical medium (for example, the same band of frequency). How can they co-exist ?

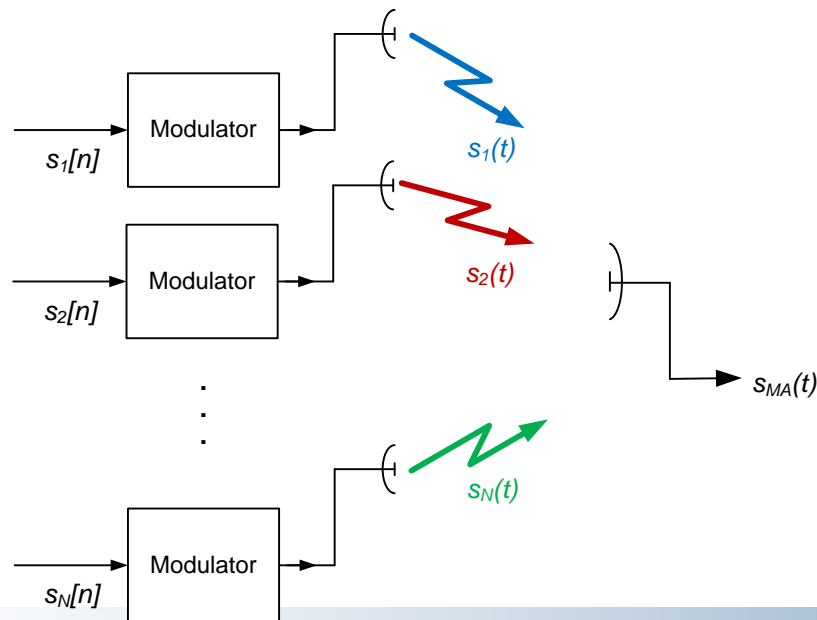
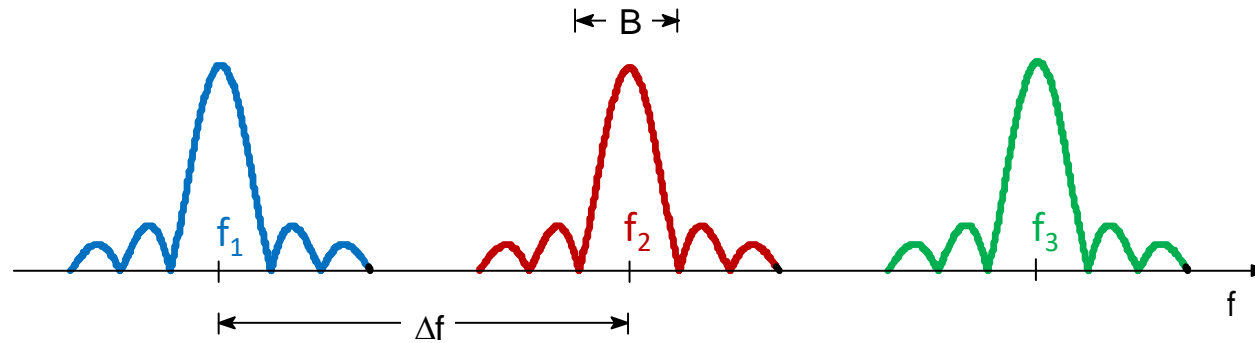


Example: FM Radio

- Many users share the same physical medium (for example, the same band of frequency). How can they co-exist ?



Frequency Division Multiple Access



Different radio stations co-exist on the same FM frequency band 88-108 MHz by using separate bandwidths and different carrier frequencies

$B=150$ kHz

$\Delta f=250$ kHz

Demultiplexing and.. listening

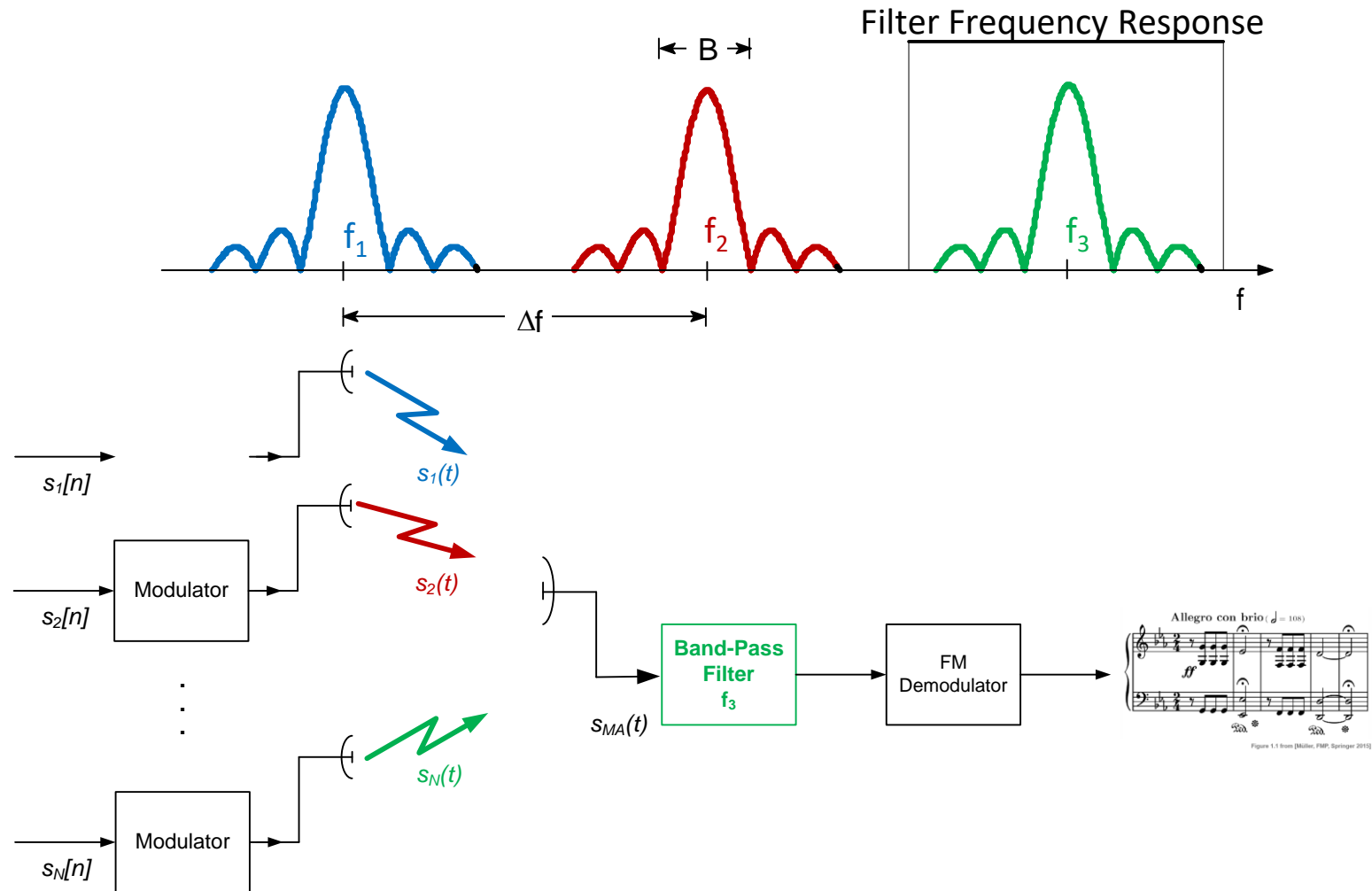
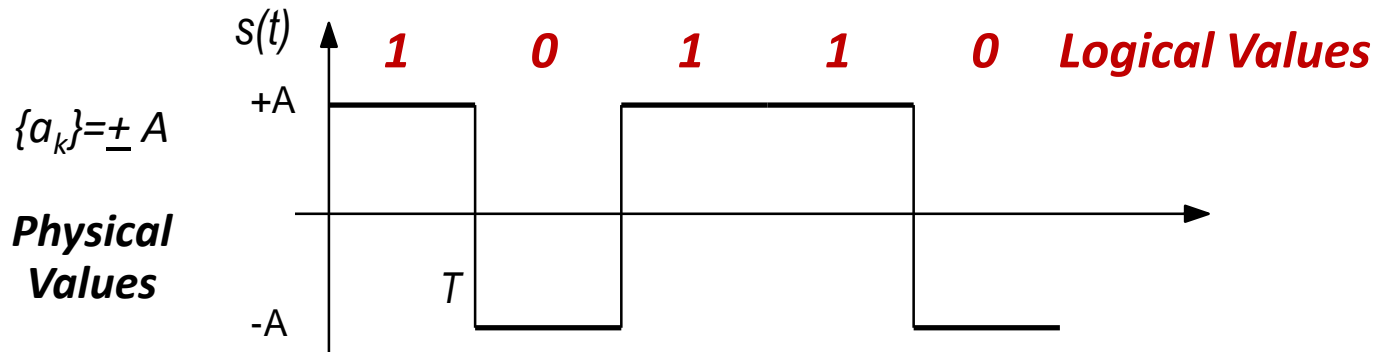


Figure 1.1 from [Müller, FMP, Springer 2015]

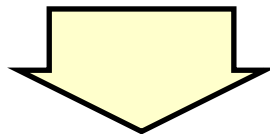
Baseband Physical Data Signal



Binary Baseband Digital Signal

Pulse/Symbol Interval: $T = \text{Bit Interval: } T_b$

Pulse/Symbol Rate: $R=1/T = \text{Bit Rate: } R_b=1/T_b$

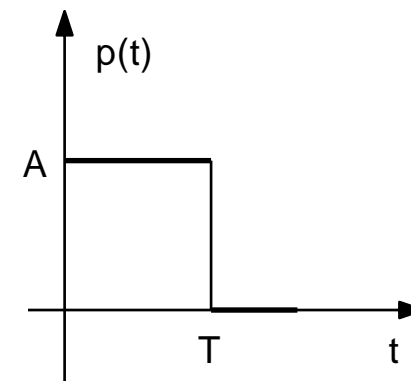


**Bipolar
Format**

$$s(t) = \sum_{k=-\infty}^{\infty} a_k \cdot p(t - kT)$$

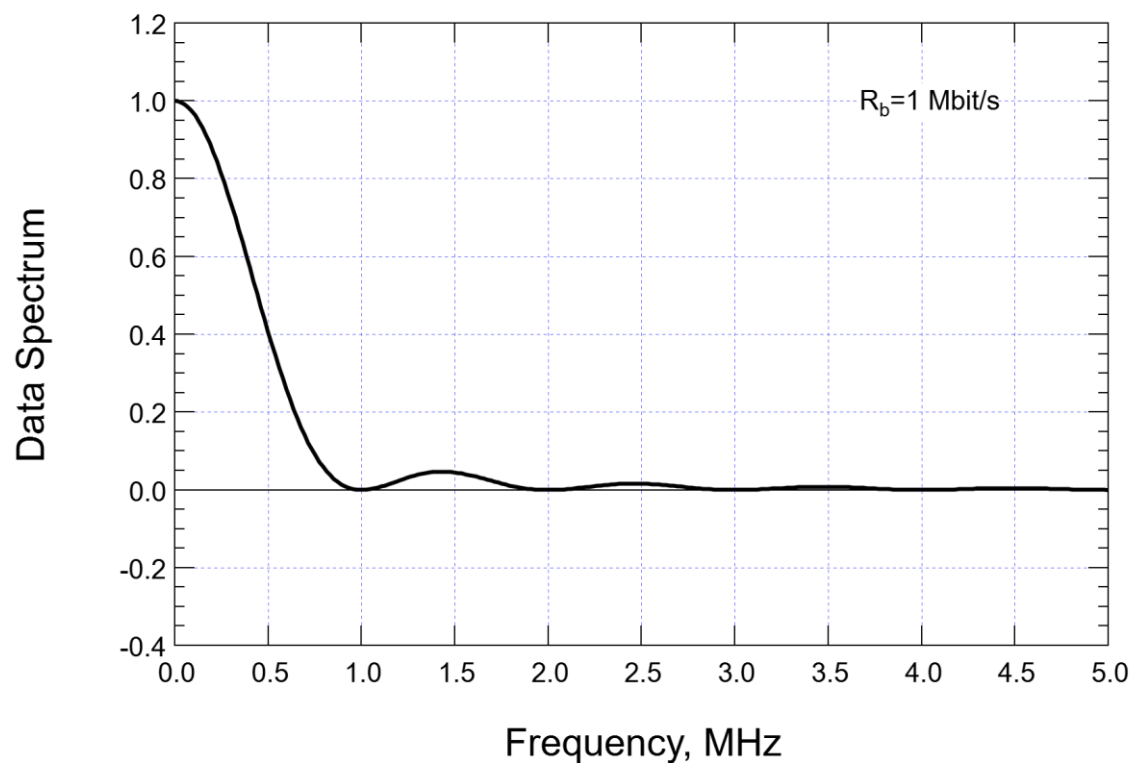
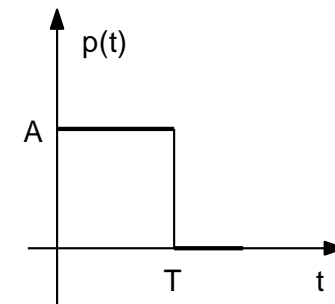


Basic Pulse $p(t)$:



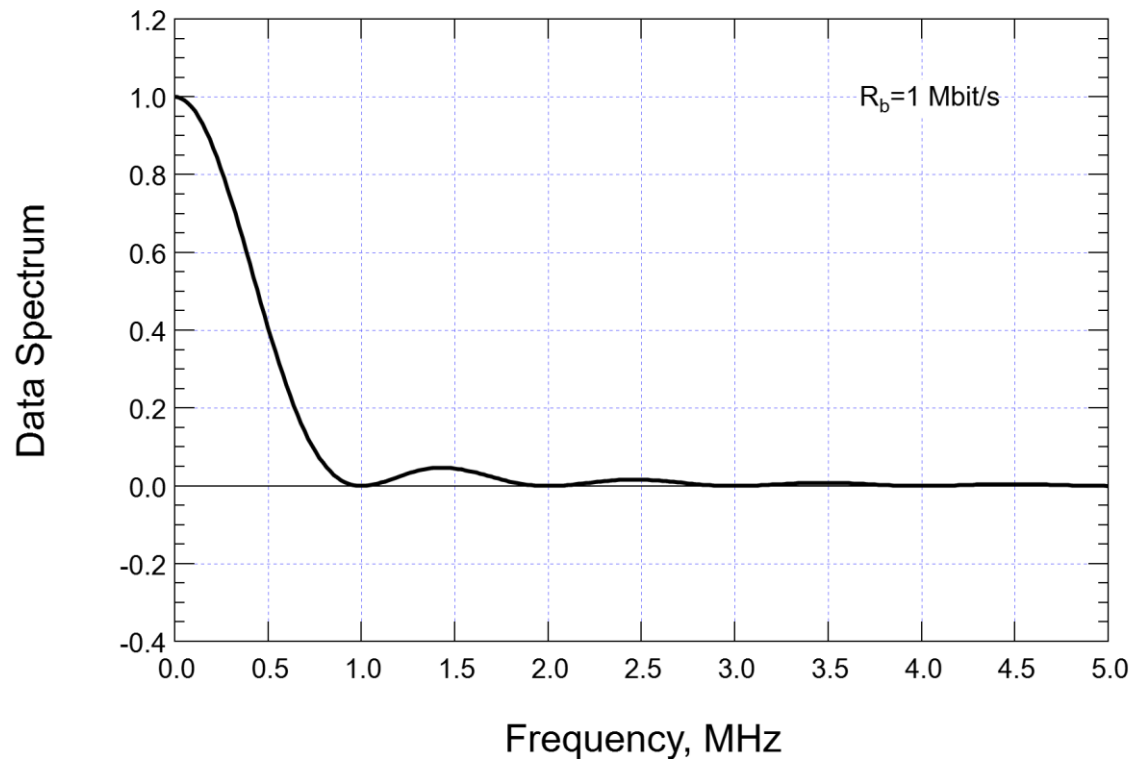
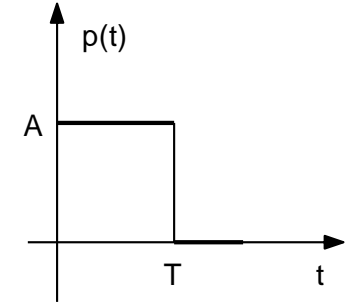
Power Spectral Density (PSD) of the Baseband Data Signal

$$S(f) = A^2 T^2 \left(\frac{\sin(\pi f T)}{\pi f T} \right)^2$$



Power Spectral Density (PSD) of the Baseband Data Signal

$$\text{Bandwidth} = 1/T = 1/T_b = R_b$$



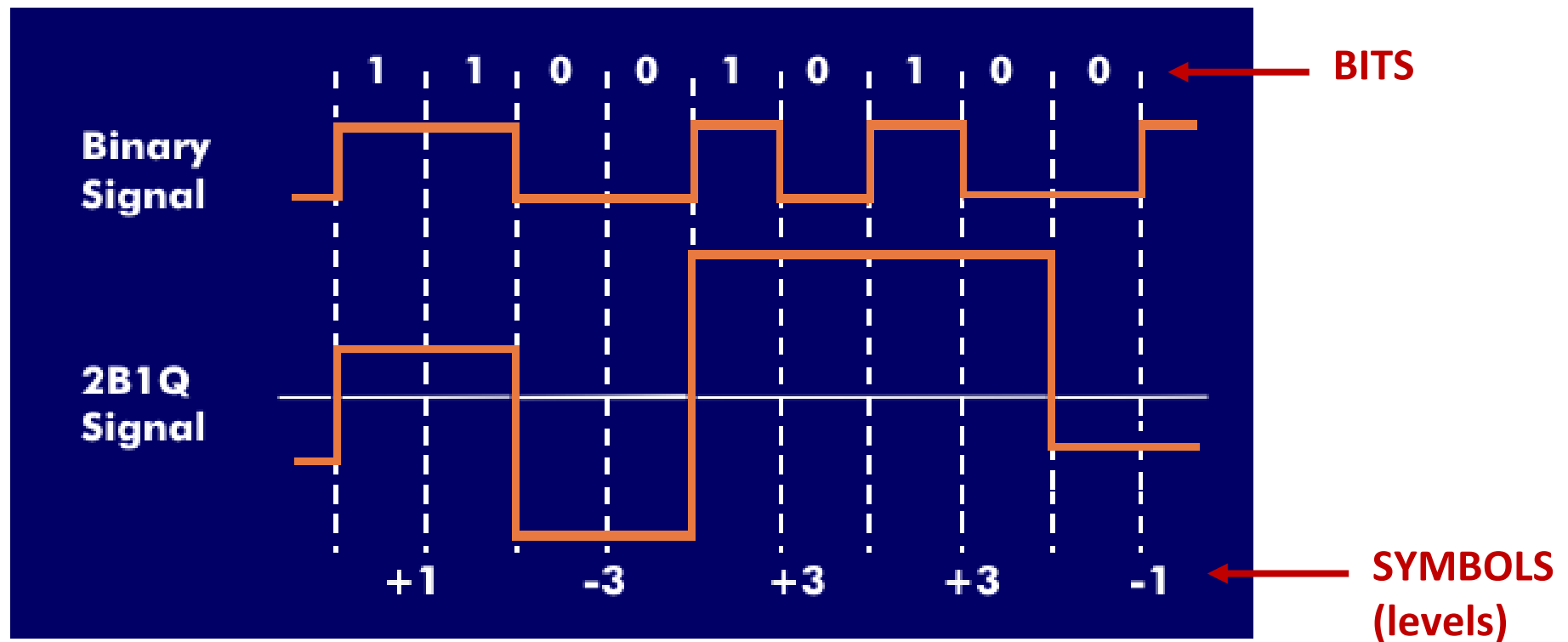
Is it true that the bandwidth (Hz) is the same thing as the bit rate (bit/s) of the link?

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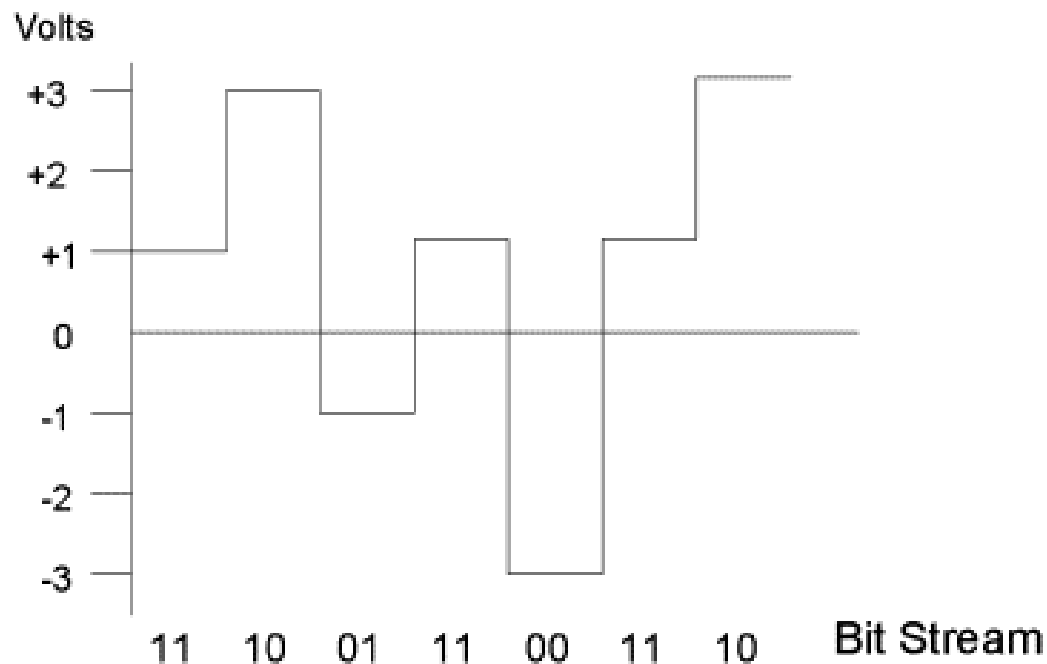
Pre-Internet Technology: ISDN

- 2B+D digital link on telephone lines @ $R_b = 2 \cdot 64 + 16 = 144$ kbit/s

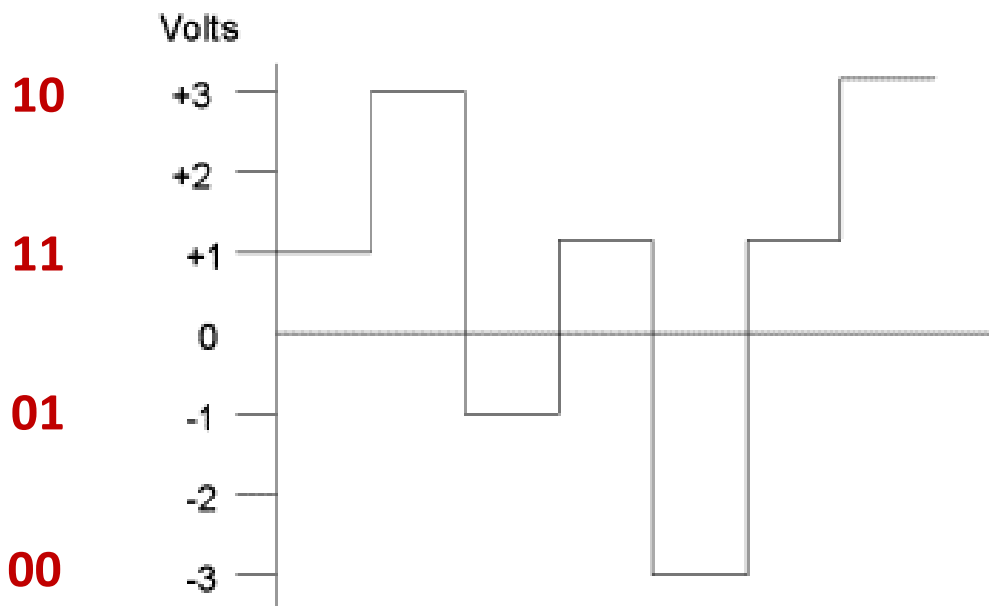


Multilevel data signal...

- There is a clear difference between *bits* and *symbols*
 - They are both represented in the physical world by rectangular NRZ pulses
 - BUT a symbol pulse is wider than a bit pulse
 - BECAUSE a symbol carries more than 1 bit...



- Every symbol level is *mapped* to a (small) packet of bits
 - The number of bit in a packet is ust equal to the factor by which the symbol is wider in time than a bit
 - The receiver uses the reverse mapping to extract logical bits form physical symbol levels



Bit Rate and Symbol Rate

In this case,

$$T_s = 2T_b$$

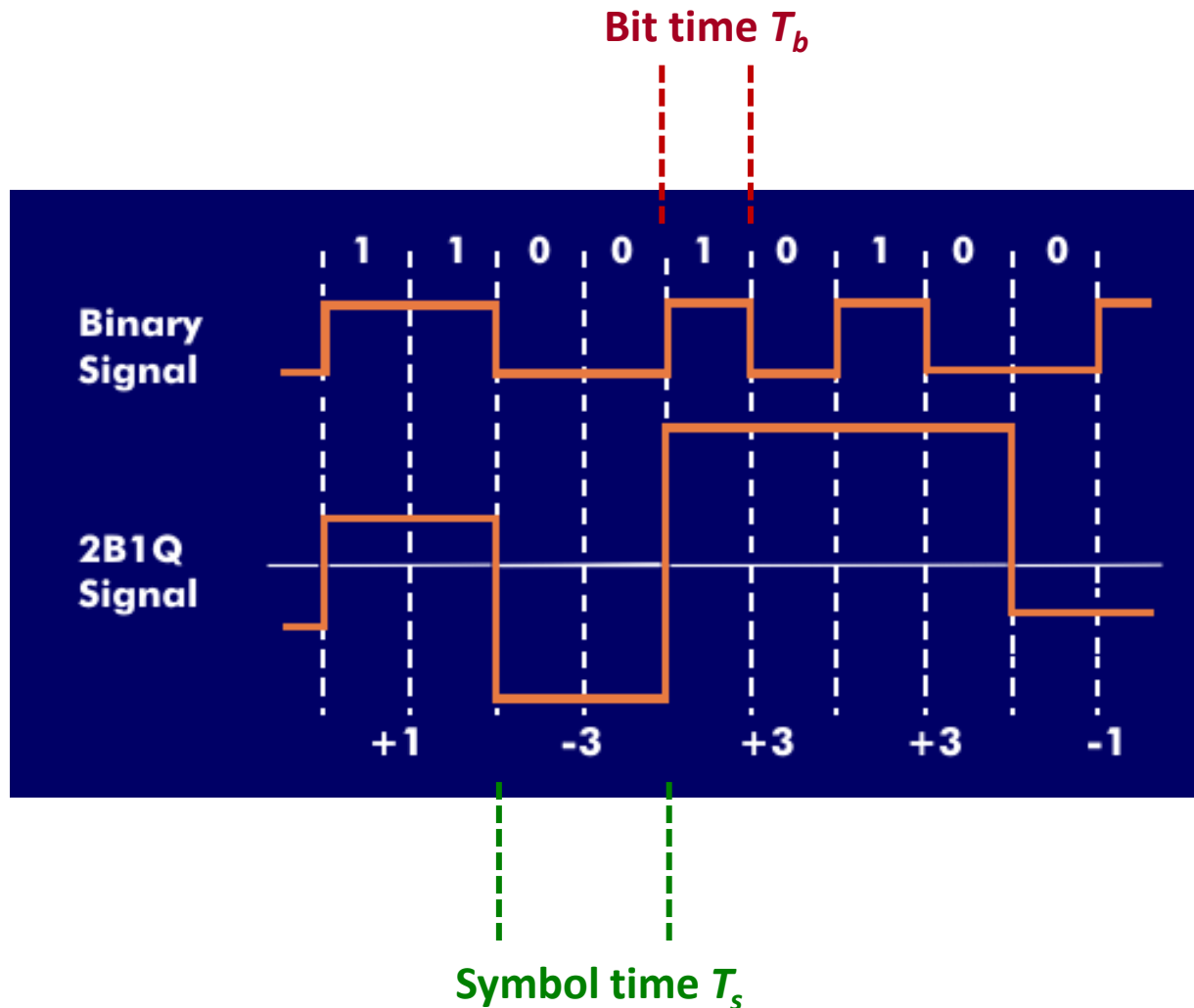
The symbol rate is

$$R_s = 1/T_s$$

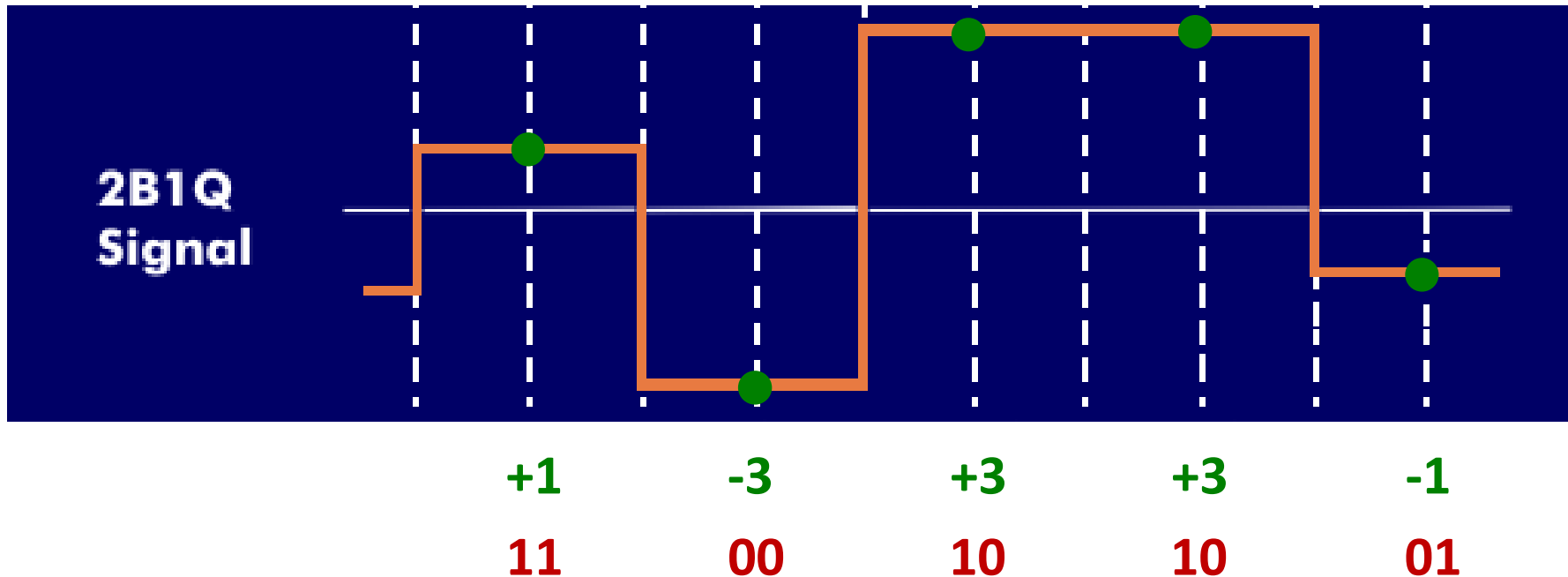
and the bit rate is

$$R_b = 2R_s$$

The bit rate is always *larger* than the symbol rate because one symbol “carries” more than one bit



Alice & Bob (encoding and decoding)



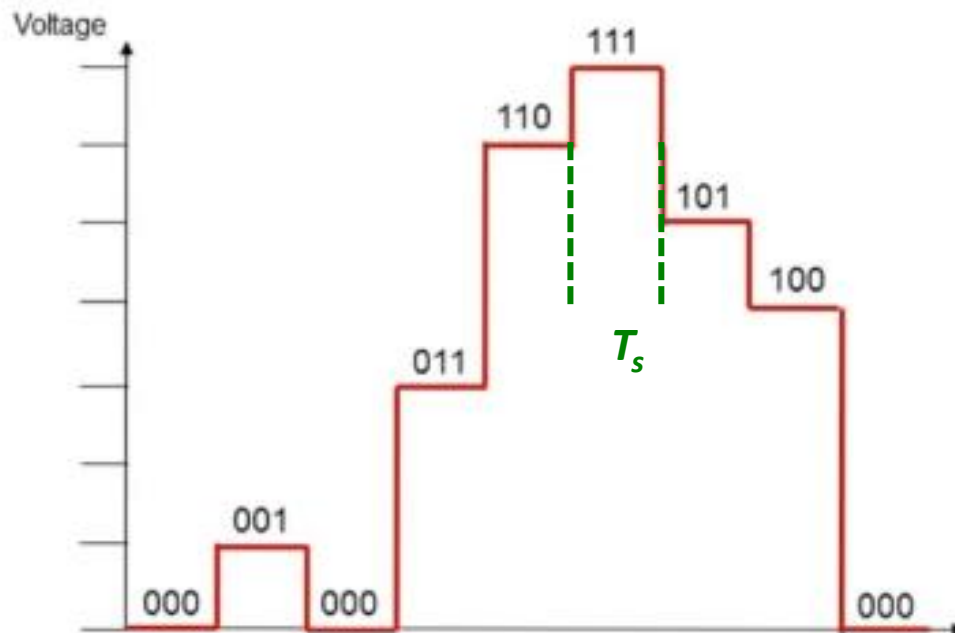
At the receiver, Bob re-clocks the signal at the center of each pulse (symbol) and from the observed values re-construct the stream of **symbols** and **bits**

Bit Rate, Symbol Rate, and Bandwidth 1/2

- The time width of any pulse carrying a symbol is T_s
- Therefore, the *bandwidth* of the multilevel data signal is $B=1/T_s$
- The bandwidth is equal to SYMBOL RATE, not to the bit rate !!!



M=8 symbol levels → 3 bits/symbol

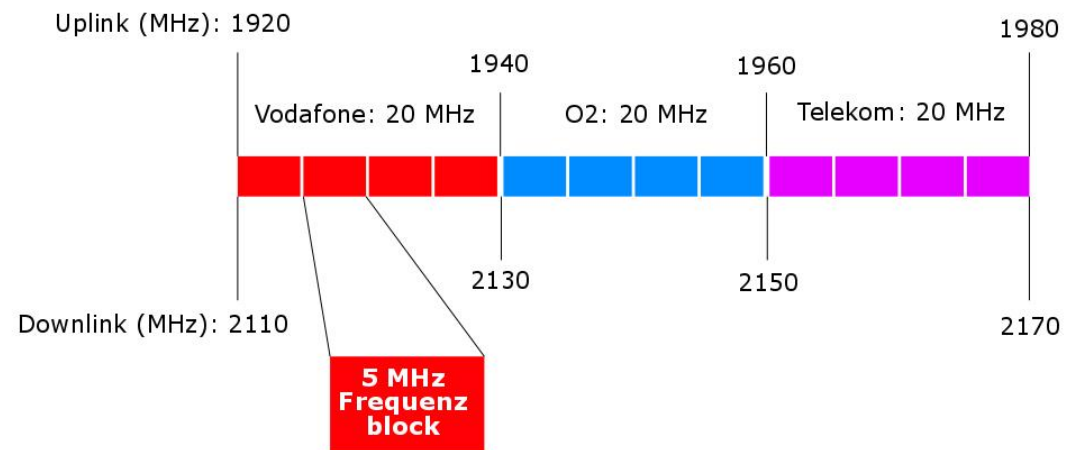


WHAT IS THE PURPOSE OF ALL THIS ?

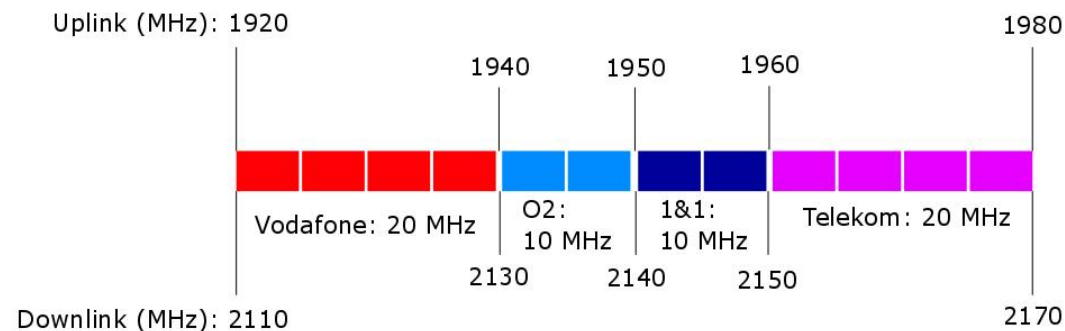
5G Bandwidth Allocation: a Sample from Germany

- A certain mobile operator is assigned very well defined chunks (blocks) of frequency under the regulation of public bodies (Ministry, Agency etc.)
 - Operators compete for frequencies in an AUCTION
- Total revenue to the Ministry from 5G frequencies auction in Italy (same in Germany): 6 G€
- You strive to squeeze as much bitrate as possible into such a bandwidth to serve as much users as possible with high-bit rate services

01.01.2021 - 31.12.2025

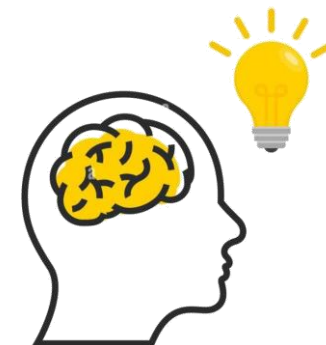


01.01.2025 - 31.12.2040



The idea

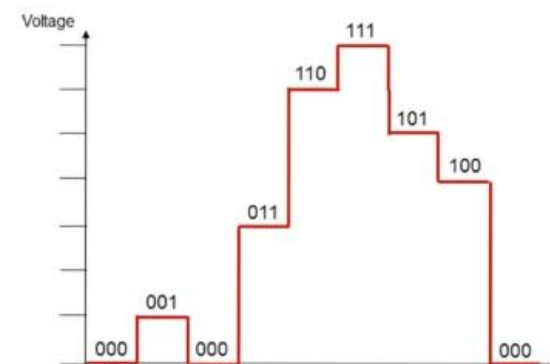
- The bandwidth B is assigned
- $R_s=B$ therefore, the (maximum) symbol rate R_s is assigned as well
- If I want to squeeze as much bitrate as possible into my bandwidth, I have to increase M



$$R_b = \log_2(M) \cdot R_s = \log_2(M) \cdot B$$

Number of bits carried
by one symbol

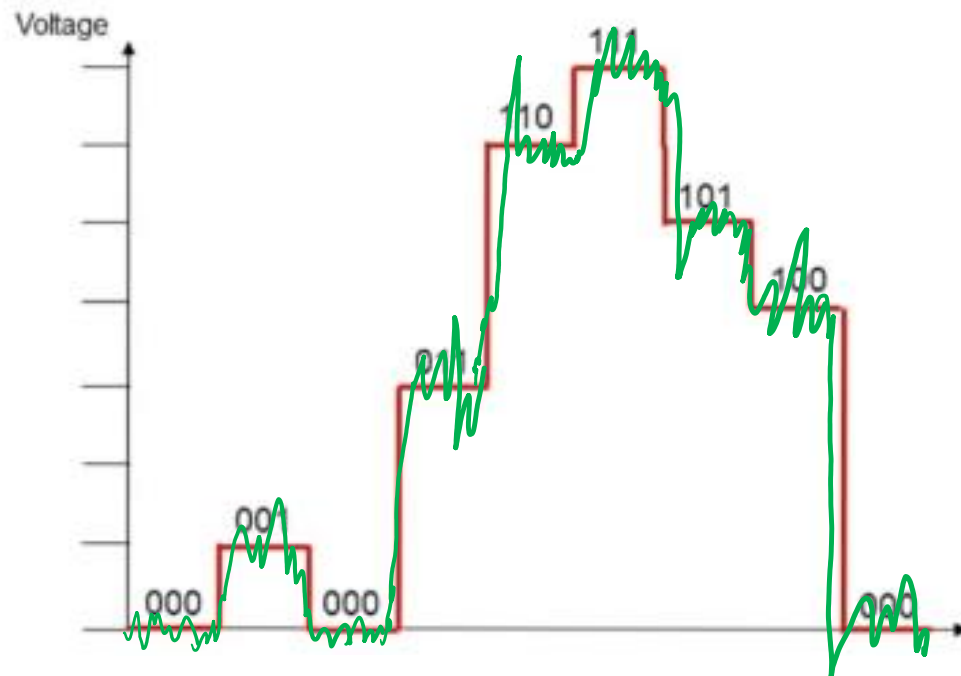
- The bit rate is indeed proportional to the bandwidth, but receives a “boost” from multilevel constellations –
up to $M=256$ levels in 5G !



Why not increasing M as much as possible?

The actual physical signal received by Bob is corrupted by noise, distortion, interference especially when the distance between Alice & Bob is large and the received signals is weak – we have the **green** waveform instead of the ideal, theoretical **red**

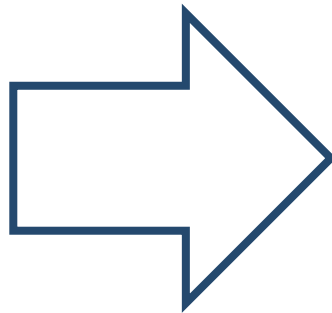
$M=8$ symbol levels \rightarrow 3 bits/symbol



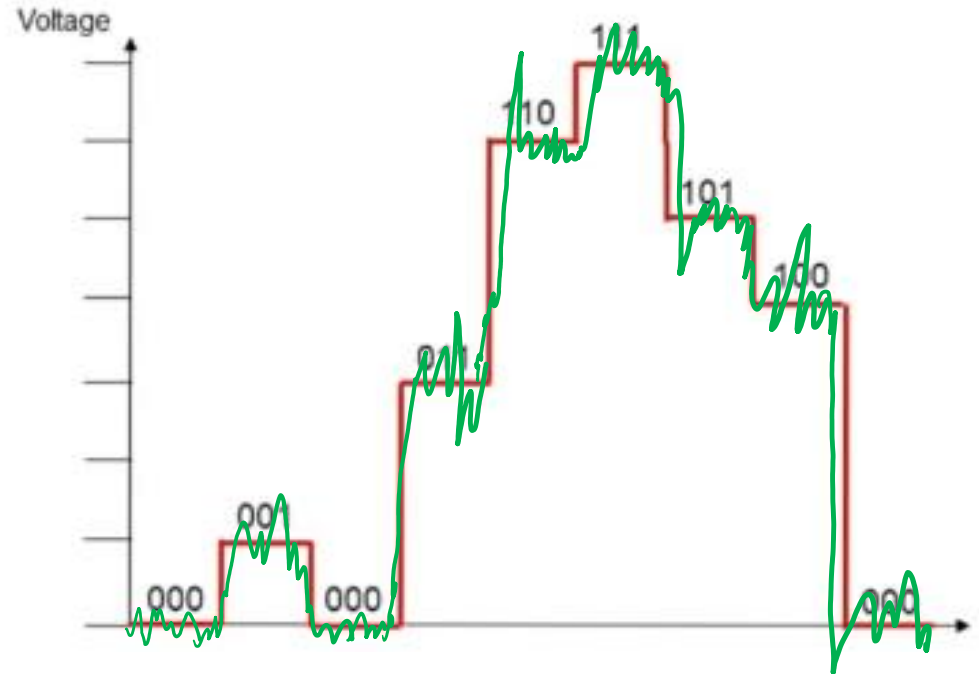
Then what?

Why not increasing M as much as possible?

Look at the skyline from this
“side” observation point

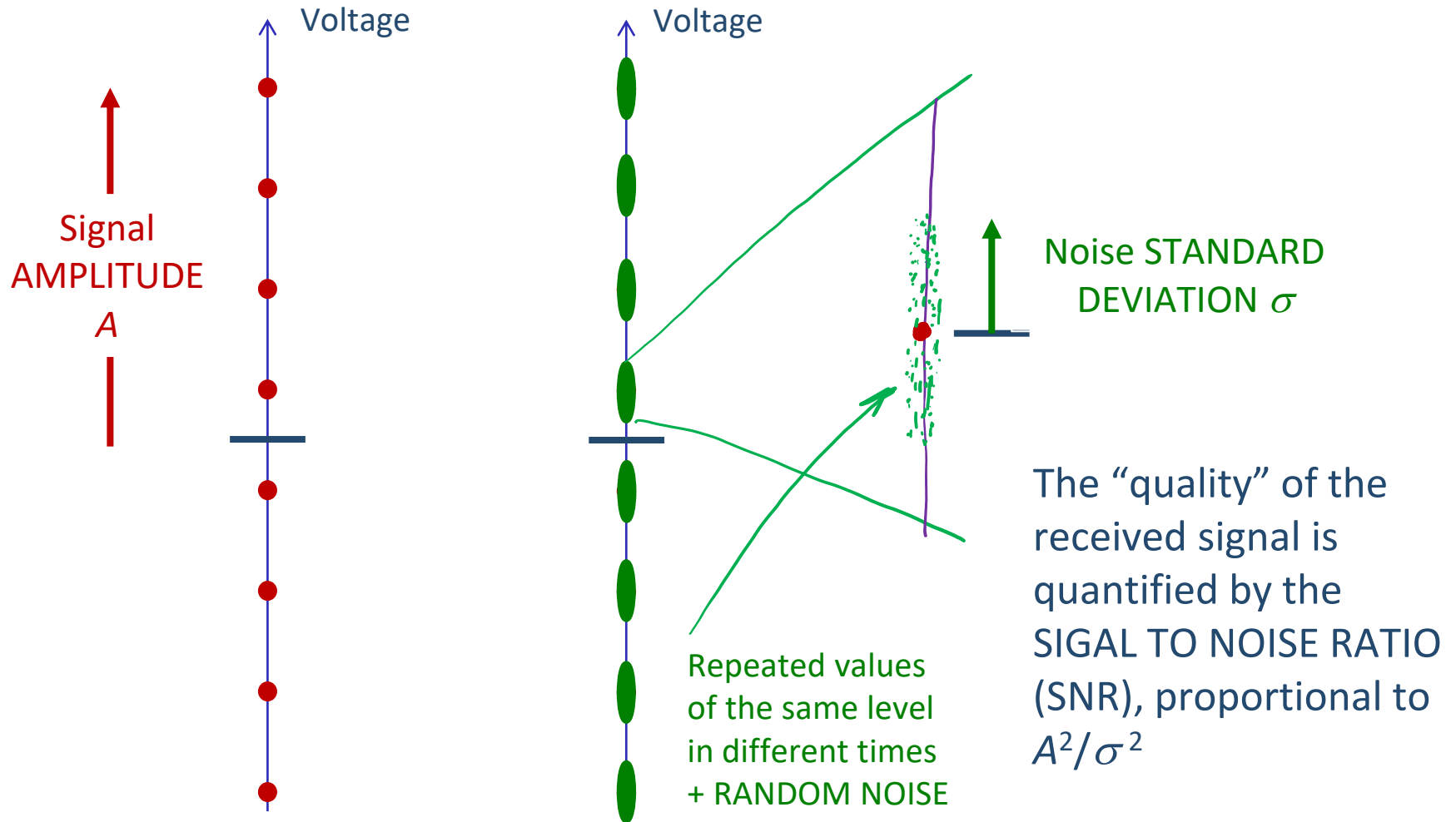


$M=8$ symbol levels \rightarrow 3 bits/symbol

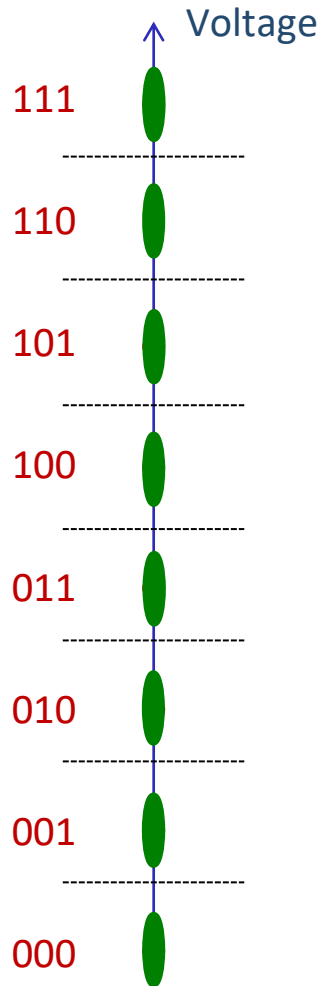


Can we compare the “theoretical” (red) view
with the “actual” (green) one?

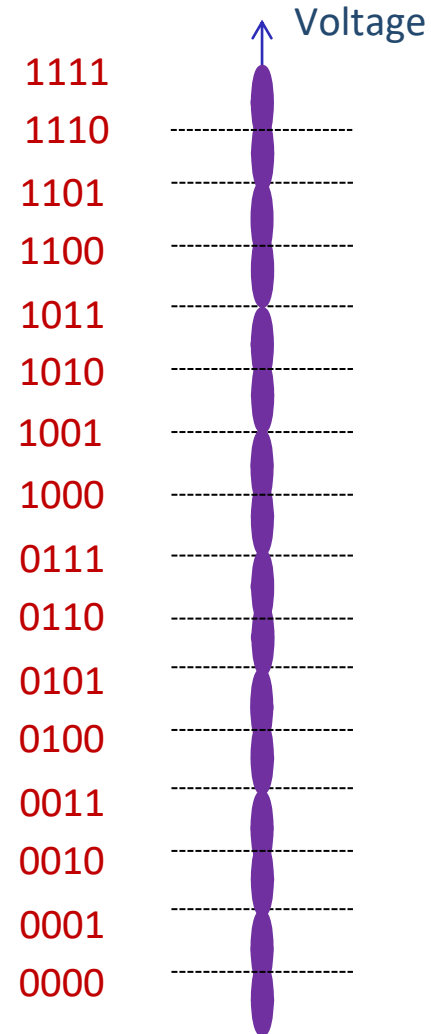
SIGNAL and NOISE (The Skyline)



Doubling M



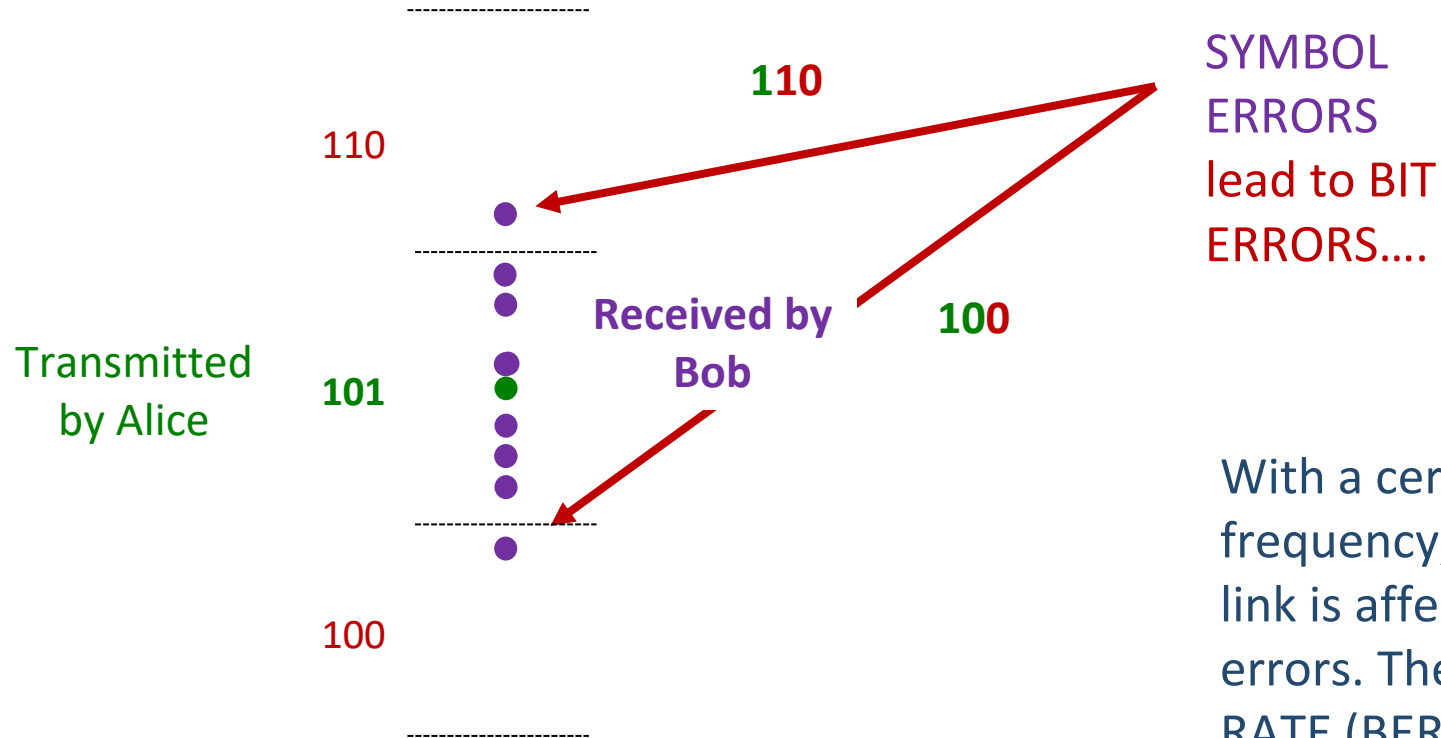
With this SNR, the “clouds” are still inside their own respective “decision regions” separated by the dashed boundaries



For the *same* SNR, the “clouds” invade the neighboring decision regions (the levels are *too many*).

Then what ?

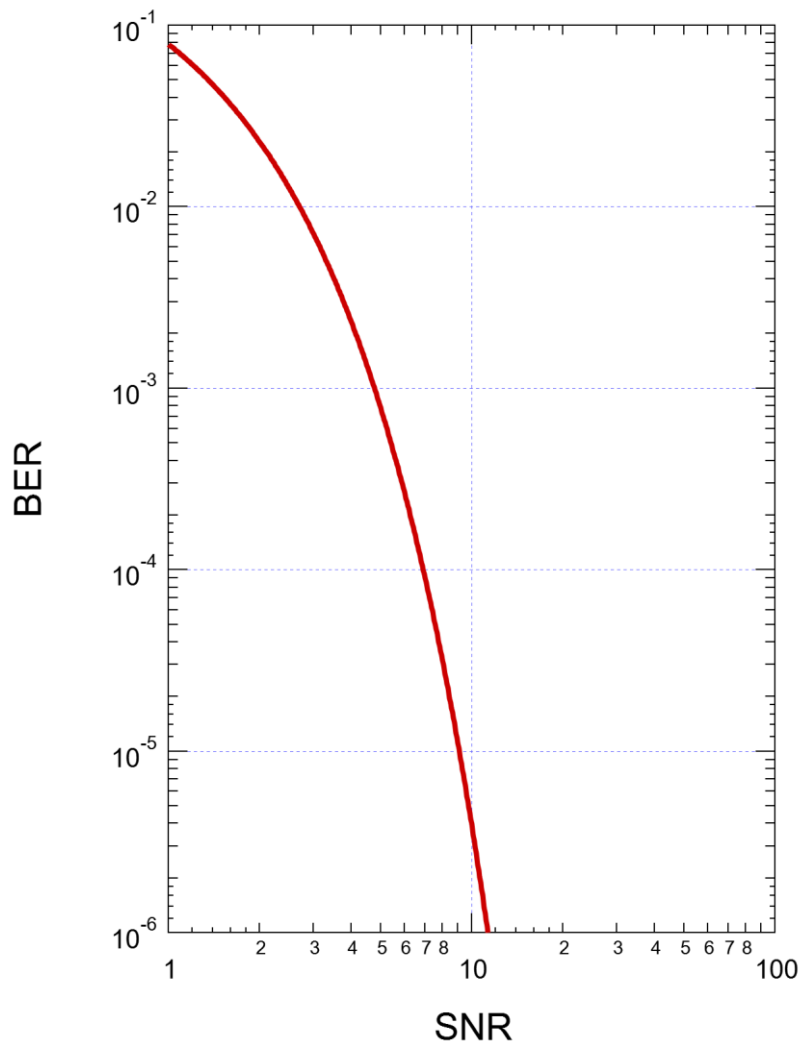
The SER / BER



SYMBOL
ERRORS
lead to BIT
ERRORS....

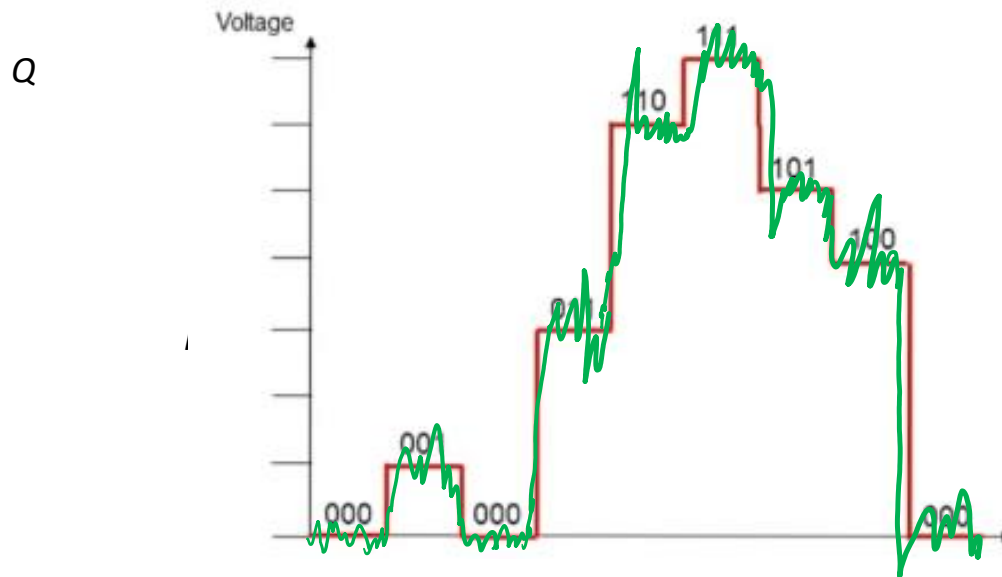
With a certain frequency, the digital link is affected by bit errors. The BIT ERROR RATE (BER) depends of the SNR on the link

BER vs SNR



- The dependence of the BER on the SNR is exponential
- The SNR depends on the application: 100 ÷ 1000 for cellular, 1 ÷ 10 for satellite, up to 10⁴ ÷ 10⁵ for ADSL
- The effect of the BER strongly depends on the application:
 - 1 error in 1000 bits (10⁻³) is good for voice, terrible for image/video
 - Commercial TV needs 10⁻¹⁰
 - Bank transfers <10⁻¹²

BER Sample – 5 wrong pixels in 10^4 



The higher the number of levels M , the less robust is the signal versus noise. The upper limit of the bit-rate (i.e., of the constellation size) for a certain SNR to attain RELIABLE COMMUNICATIONS is given by Shannon's capacity formula

$$R_b \leq B \log_2(1 + \text{SNR}) \quad (\text{bit/s})$$