Game Theory and Optimization in Communications and Networking

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Special classes of static games
Among noncooperative static games, there are special classes of games which are particularly relevant to address problems in wireless and communication networks:

- Supermodular games
- Potential games
- Generalized Nash games
A *supermodular* game is a static noncooperative game, which has:

1. a set of players \( \mathcal{K} = \{1, \ldots, K\} \)
2. \( A_k \) is the set of strategies (actions) available to player \( k \)
3. \( u_k(a) \) is the utility (payoff) for player \( k \)

where, for any \( a'_k \geq a_k \) and for any \( a'_k \geq a'_k \) (component-wise),

\[
 u_k([a'_k, a'_k]) - u_k([a_k, a'_k]) \geq u_k([a'_k, a_k]) - u_k([a_k, a_k])
\]

The utility function has *increasing differences*: if a player takes a higher action, then the other players are better off when taking higher actions as well.
Supermodular games (2/2)

Checking that a game is supermodular is attractive because:

The set of pure-strategy Nash equilibria is not empty

The best-response dynamics converges to the smallest element-wise vector in the set of Nash equilibria
Let's consider again the near-far power control game with continuous power sets $\mathcal{A}_k = [0, p]$: 

The inefficiency of the Nash equilibrium occurs because each terminal ignores the interference it generates to the other.
Pricing (a.k.a. taxation) is an effective tool used in microeconomics to deal with this problem.

We can in fact take advantage of a modified utility:

\[
\tilde{u}_k(a) = u_k(a) - \alpha \cdot a_k = R \cdot \frac{f(\gamma_k)}{a_k} - \alpha \cdot a_k
\]

where the pricing factor \( \alpha \) is used to introduce some form of externality, that charges players proportionally to the powers they radiate (that causes multiple-access interference)
Supermodular games: An example (3/6)

Unlike the game without pricing ($\alpha = 0$), the utility function is not quasi-concave in $a_k$.

However, $\tilde{u}_k(a)$ has increasing differences, and hence the game is supermodular. To check it, it is sufficient to prove that $\tilde{u}_k(a)$ is twice differentiable and such that

$$\frac{\partial^2 \tilde{u}_k(a)}{\partial a_k \partial a_j} \geq 0 \quad \forall k, j \in \mathcal{K}, k \neq j$$

Exercise 1: Compute $\frac{\partial^2 \tilde{u}_k(a)}{\partial a_k \partial a_j}$ in the case $u_k(a) = R f(\gamma_k)/a_k$

solution: $\frac{\partial^2 \tilde{u}_k(a)}{\partial a_k \partial a_j} = -R \cdot \frac{h_j a_j}{M h_k a_k^3} \cdot \frac{\partial^2 f(\gamma_k)}{\partial \gamma_k^2}$
Supermodular games: An example (4/6)

In the case \( f(\gamma_k) = \left[1 - e^{-\gamma_k/2}\right]^L \),

\[
\frac{\partial^2 f(\gamma_k)}{\partial \gamma_k^2} = \frac{L}{4} e^{-\gamma_k/2} \cdot \left[1 - e^{-\gamma_k/2}\right]^{L-2} \cdot (L e^{-\gamma_k/2} - 1)
\]

and thus

\[
\frac{\partial \tilde{u}_k(a)}{\partial a_k \partial a_j} \geq 0 \quad \Leftrightarrow \quad \gamma_k \geq 2 \log L \quad \forall k \in K
\]
Supermodular games: An example (5/6)

To include pricing in our power-control game, we can modify the definitions of the game ingredients as follows:

1. \( \mathcal{K} = [1, 2] \)
2. \( A_k = \left\{ a_k \in [0, \bar{p}] : \gamma_k(a) = \frac{M h_k a_k}{\sigma^2 + h_k a_k} \geq 2 \log L \right\} \)
3. \( \tilde{u}_k(a) = u_k(a) - \alpha \cdot a_k = R \cdot \frac{f(\gamma_k)}{a_k} - \alpha \cdot a_k \)
Supermodular games: An example (6/6)

This modified formulation improves the **efficiency** of the Nash equilibrium (NE):

\[ \alpha^*: \text{NE of the original game} \]
\[ \tilde{\alpha}^*: \text{NE of the pricing game} \]

Intuitively, this form of externality encourages players to use the resources **more efficiently**.
A **potential** game is a static noncooperative game, which has:

1. A set of players $\mathcal{K} = [1, \ldots, K]$.
2. $A_k$ is the set of strategies $a$ (actions) available to player $k$.
3. $u_k(a)$ is the utility (payoff) for player $k$.

where, for all players and all other players’ profile $a \setminus k$,

$$u_k([a_k, a \setminus k]) - u_k([a'_k, a \setminus k]) = \Phi([a_k, a \setminus k]) - \Phi([a'_k, a \setminus k]) \quad \forall a_k, a'_k \in A_k$$

(exact potential game)
A potential game is a static noncooperative game, which has:

1. A set of players $\mathcal{K} = [1, \ldots, K]$
2. $A_k$ is the set of strategies (actions) available to player $k$
3. $u_k(a)$ is the utility (payoff) for player $k$

where, for all players and all other players’ profile $a_{\setminus k}$,

$$\text{sgn}[u_k([a_k, a_{\setminus k}]) - u_k([a'_k, a_{\setminus k}])] = \text{sgn}[\Phi([a_k, a_{\setminus k}]) - \Phi([a'_k, a_{\setminus k}])] \quad \forall a_k, a'_k \in A_k$$

(ordinal potential game)
Potential games: An example [26] (1/5)

Uplink power allocation for an OFDMA cellular network, with N subcarriers and K terminals

Each terminal $k$’s objective: allocate the power so as to maximize its own achievable rate given a constraint on the maximum total radiated power $\bar{p}_k$ to exploit the channel frequency diversity.
The strategic-form representation of the game is the following:

(1) **players:** $\mathcal{K} = [1, \ldots, K]$ terminals in the network

(2) **strategies:** $\mathcal{A}_k = \left\{ p_k \in \mathbb{R}^N : p_k(n) \geq 0 \forall n, \sum_{n=1}^{N} p_k(n) \leq \bar{p}_k \right\}$

(3) **utilities:** $u_k([p_k, p_{\setminus k}]) = \sum_{n=1}^{N} \log(1 + \gamma_k(n))$

where

$$\gamma_k(n) = \frac{h_k(n)p_k(n)}{\sigma^2 + \sum_{j \neq k} h_j(n)p_j(n)}$$

and $h_k(n)$ are terminal $k$’s SINR and channel power gain over subcarrier $n$, resp.
Potential games: An example (3/5)

This is an **exact** potential game, with potential function

\[
\Phi([p_k, p_{\neq k}]) = \sum_{n=1}^{N} \log \left( \sigma^2 + \sum_{j=1}^{K} h_j(n)p_j(n) \right)
\]

**Exercise 2:** Check that

\[
u_k([p_k, p_{\neq k}]) - u_k([p'_k, p_{\neq k}]) = \Phi([p_k, p_{\neq k}]) - \Phi([p'_k, p_{\neq k}])
\]
In an infinite potential game (as occurs in this case), a pure-strategy Nash equilibrium exists if:

- the strategy sets $A_k$ are compact
- the potential function $\Phi$ is upper semi-continuous on $A = A_1 \times \ldots \times A_K$

The interest in potential games stems from the guarantee of existence of pure-strategy Nash equilibria, and from the study of a single-variable function (the potential one).

If $A$ is a compact and convex set, and $\Phi(a)$ is a continuously differentiable function and strictly concave, then the Nash equilibrium of the potential game is unique.
Potential games: An example (5/5)

The convergence to one of the Nash equilibria of the game can be achieved using an iterative algorithm based on the best-response criterion:

\[
p_k^{(t+1)} = \arg \max_{p_k \in \mathbb{R}^N} \Phi \left( \left[ p_k, p_{\setminus k}^{(t)} \right] \right)
\]

\[
\text{s.t. } \sum_{n=1}^{N} p_k(n) \leq \bar{p}_k \text{ and } p_k(n) \geq 0, \forall n
\]

The solution is given by the well-known iterative water-filling algorithm:

\[
p_k^{(t+1)}(n) = \max \left( 0, \frac{1}{\lambda_k} - \frac{p_k^{(t)}(n)}{\gamma_k^{(t)}(n)} \right)
\]

where \( \lambda_k \) is such that

\[
\sum_{n=1}^{N} \max \left( 0, 1/\lambda_k - p_k^{(t)}(n)/\gamma_k^{(t)}(n) \right) = \bar{p}_k
\]

fed back by the base station
A generalised Nash game is a static noncooperative game, with:

1. A set of players $\mathcal{K} = [1, \ldots, K]$
2. Player $k$’s set of strategies $\mathcal{A}_k$ depends on all others’ actions $a_{\setminus k}$
3. $u_k(a)$ is the utility (payoff) for player $k$

The interplay between strategy sets typically occurs when placing constraints on the game formulation.
Let’s focus again on the uplink of an OFDMA cellular network, this time during the (contention-based) network association phase, in which the subcarriers are shared by the terminals in a multicarrier CDMA fashion.

Each terminal $k$’s objective: allocate the power so as to minimize the energy spent to get a successful association given quality of service (QoS) constraints in terms of false code locks.
Generalized Nash games: An example (2/2)

This situation can be modeled by the following strategic-form representation:

1. **Players**: \( \mathcal{K} = \{1, \ldots, K\} \) terminals in the network

2. **Strategies**: power strategy sets \( \mathcal{A}_k = [0, p_k] \)

3. **QoS constraints**: max. probability of false alarm
   max. mean square error on timing estimation

4. **Utilities**: 
   \[
   u_k(a) = \frac{\Pi_k(a)}{I \cdot \alpha_k} 
   \]

The solution of generalized Nash games is the **generalized Nash equilibrium**: in this case, it exists and is unique if and only if the number of terminals \( K \) is below a certain threshold.
Beyond Nash equilibrium: other equilibrium concepts

In addition to the concept of mixed-strategy and pure-strategy Nash equilibria in noncooperative static games, it is worth mentioning:

- **the correlated equilibrium**: a generalization of the Nash equilibrium, where an arbitrator (not necessarily an intelligent entity) helps the players to correlate their actions, so as to favor a decision process in between non-cooperation and cooperation.
  
  **Example**: it may select one of the two pure-strategy Nash equilibria in the multiple-access game.

- **the Wardrop equilibrium**: a limiting case of the Nash equilibrium when the population of users becomes infinite.

  **Example**: it can be used as a decision concept to select routing strategies in ad-hoc networks.
The **correlated equilibrium** is a generalization of the Nash equilibrium, in which an arbitrator can generate some random signals (according to a mechanism known by the players) that can help them to **coordinate** their actions, so as to **improve** the performance in a social sense.

**Basics of noncooperative game theory: special classes of static games**

Robert J. Aumann, 1930-

**(DEF:)** A probability distribution \( \pi(a) = \{ \pi(a) \}_{a \in A} \) is a **correlated equilibrium** of game \( G \) if, for all \( k \in K, a_k, a'_k \in A_k, \) and \( a_{\backslash k} \in A_{\backslash k}, \)

\[
\sum_{a_{\backslash k} \in A_{\backslash k}} \pi(a_k, a_{\backslash k}) \cdot \left[ u_k(a_k, a_{\backslash k}) - u_k(a'_k, a_{\backslash k}) \right] \geq 0
\]
Let's consider the multiple access game again:

- \( K=2 \), \( \mathcal{A}_k = \{w, t\} \)
- \( \mathcal{A} = \{(w,w), (w,t), (t,w), (t, t)\} \)

### Correlated equilibrium (2/3)

The correlation function is given by:

\[
\pi(w,w) \cdot [0 - (t - c)] + \pi(w,t) \cdot [0 - (-c)] \geq 0
\]

\[
\pi(t,w) \cdot [(t - c) - 0] + \pi(t,t) \cdot [(-c) - 0] \geq 0
\]

\[
\pi(w,w) \cdot [0 - (t - c)] + \pi(t,w) \cdot [0 - (-c)] \geq 0
\]

\[
\pi(w,t) \cdot [(t - c) - 0] + \pi(t,t) \cdot [(-c) - 0] \geq 0
\]

\[
\pi(w,w) + \pi(w,t) + \pi(t,w) + \pi(t,t) = 1
\]

\[
\pi(w,w) \geq 0, \pi(w,t) \geq 0, \pi(t,w) \geq 0, \pi(t,t) \geq 0
\]
Correlated equilibrium (3/3)

This system of inequalities admits an infinite number of solutions (i.e., correlated equilibria). Notable solutions are:

- \( \pi_{(w,w)} = 0, \pi_{(w,t)} = 1, \pi_{(t,w)} = 0, \pi_{(t,t)} = 0 \)

- \( \pi_{(w,w)} = 0, \pi_{(w,t)} = 0, \pi_{(t,w)} = 1, \pi_{(t,t)} = 0 \)

- \( \pi_{(w,w)} = \frac{c^2}{t^2}, \pi_{(w,t)} = \frac{c}{t} \left(1 - \frac{c}{t}\right), \pi_{(t,w)} = \frac{c}{t} \left(1 - \frac{c}{t}\right), \pi_{(t,t)} = \left(1 - \frac{c}{t}\right)^2 \)

- \( \pi_{(w,w)} = 0, \pi_{(w,t)} = \frac{1}{2}, \pi_{(t,w)} = \frac{1}{2}, \pi_{(t,t)} = 0 \)

- \( \pi_{(w,w)} = \frac{c}{t - 2c}, \pi_{(w,t)} = \pi_{(t,w)} = \frac{t-c}{t-2c}, \pi_{(t,t)} = 0 \)

Exercise 2: Check that the fourth solution maximizes both the individual expected payoffs and the expected sum-utility.
Basics of noncooperative game theory

Taxonomy revisited

- Games
  - Cooperative games
  - Noncooperative games
    - Complete information
      - Bayesian games
    - Incomplete information
      - Dynamic games
      - Static games

Classes: zero-sum, potential, etc.
Tools: iterated dominance, mixed strategies, etc.
Solution: Nash equilibria, correlated equilibria, etc.
Dynamic games
Dynamic games

There could be situations in which the players are allowed to have a sequential interaction, meaning that the move of one player is conditioned by the previous moves in the game: This is the class of dynamic games.

Consider a sequential multiple-access game, where the packet transmission is successful when there is no collision, in which:

1. player 1 can either transmit (with a transmission cost c) or wait;
2. then, after observing player 1’s action, player 2 chooses whether to transmit or not.
A convenient way to describe a sequential game such as the sequential multiple-access game is the **extensive-form representation**, which consists of:

1. a set of **players**;
2. the order of moves – i.e., who moves when;
3. what the players’ **choices** are when they move;
4. the **information** each player has when he/she makes his/her choices;
5. the **payoff** received by each player for each combination of strategies that could be chosen by the players.
The extensive-form representation of the sequential multiple-access game [29] is

\begin{itemize}
  \item this is a game with \textit{perfect information}, because the player with the move knows the full history of the play of the game thus far;
  \item this is a game with \textit{complete information}, because the $u_k(a)$’s are common knowledge;
  \item this game is a \textit{finite-horizon game}, because the number of stages is finite.
\end{itemize}
Any game (both static and dynamic) can **indifferently** be described by its strategic-form representation and its extensive-form representation. The options (the **strategy**) we show for player 2 are his/her response conditioned on what player 1 does (wait/transmit), i.e., a plan for every history of the game.

<table>
<thead>
<tr>
<th>Player 1 (leader)</th>
<th>Player 2 (follower)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>W</strong></td>
<td><strong>W</strong></td>
</tr>
<tr>
<td>0, 0</td>
<td>0, 0</td>
</tr>
<tr>
<td>(t - c, 0)</td>
<td>(-c, -c)</td>
</tr>
</tbody>
</table>

There are three **Nash equilibria**: \((T, (W,W))\), \((T,(T,W))\), and \((W,(T,T))\)
Let’s go back to the extensive-form representation:

Are the three Nash equilibria, \((T, (W,W))\), \((T, (T,W))\), and \((W, (T,T))\), all credible?

To answer this question, let’s resort to the backward induction of the game.

The strategy \((T, (T,W))\) is the only subgame-perfect Nash equilibrium.
Subgame-perfect Nash equilibrium: A summary

A subgame-perfect Nash equilibrium is a strategy profile which prescribes actions that are optimal at each game stage for every history of the game, i.e., for any possible unfolding of the game.

Otherwise stated, a subgame-perfect Nash equilibrium is a Nash equilibrium of any proper subgame of the original game (i.e., it is a credible Nash equilibrium that has survived backward induction).

Some implications:

- any finite game with complete information has a subgame-perfect Nash equilibrium, perhaps in mixed strategies;
- any finite game with complete and perfect information has (at least) one pure-strategy subgame-perfect Nash equilibrium;
Example 1

Let’s consider the following game, called the entry game:

Player 1: challenger
- enter
- stay out

Player 2: incumbent
- don’t fight
- fight

Utility matrix:
- 2 M€, 1 M€ for Player 1
- 0, 0 for Player 2
- 1 M€, 2 M€ for Player 1

Let’s build the strategic-form representation of this game.

Then, let’s identify the Nash equilibria of the game.

Finally, using backward induction, let’s find the (only) subgame-perfect NE.
Assume you play tic-tac-toe:

In theory, there are:

- $3^9 = 19,683$ possible board layouts
- $9! = 362,880$ different sequences of X’s and O’s on the board

When O is making the first move, we have:

- 131,184 games won by O
- 77,904 games won by X
- 46,080 tied games
Despite its apparent simplicity, it requires some complex mathematics to study its outcomes. However, tic-tac-toe can be easily recognized as a finite extensive game with perfect information.

Thus, there exists at least one subgame-perfect Nash equilibrium. In this particular game, each player has a subgame perfect equilibrium that guarantees a tie, unless the other player deviates from its own: in such case, the equilibrium leads to a win.

What about chess? Chess is not a finite game! However, if you assume to declare a tie once a position is repeated a finite number (e.g., 3) of times, then you can model it as a finite extensive game with perfect information (with a huge complexity!)
Games such as the sequential multiple-access game are a particular class of dynamic games: **Stackelberg games**, that have a **leader** and a **follower**.

Stackelberg games are particularly suitable to analyze wireless networks: e.g., they can investigate power allocation schemes for **femtocell networks**.

*This multi-leader/multi-follower formulation leads to multiple Stackelberg equilibria.*
Dynamic games with imperfect information

Imperfect information occurs when, at some stage(s) of the game, some players do not know the full history of the play thus far.

Example: a situation in which the two terminals play the sequential multiple-access game first, and then the (simultaneous) multiple-access game in case of a collision during the first stage, is a game with imperfect information.

Static games are a special case of dynamic games with imperfect information.

Complete information ≠ Perfect information

(knowledge of the structure of the game: players, strategies, utilities, etc.) ≠ (knowledge of the full history of the game)
Repeated games are a subclass of dynamic games, in which the players face the same single-stage (static) game every period.

Suppose the forwarder’s dilemma is played $N$ times [33], with $N < \infty$.

Both players move simultaneously after knowing all previous actions.

<table>
<thead>
<tr>
<th></th>
<th>player 1</th>
<th>player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>drop</td>
<td>$c, c$</td>
<td>$t+c, 0$</td>
</tr>
<tr>
<td>forward</td>
<td>$0, t+c$</td>
<td>$t, t$</td>
</tr>
</tbody>
</table>

$u_1(a_1, a_2), u_2(a_1, a_2)$
Repeated games (2/4)

Suppose that the game is played twice \( (N=2) \). Its extensive-form representation is given by the following tree:
To find the Nash equilibria of this two-stage game, we can build its equivalent strategic-form representation:

The unique Nash equilibrium of the game is \((d, d), (d, d)\)

Exercise 3: Check that it is also subgame-perfect
Using backward induction, both players will end up dropping the packets at every stage, and no cooperation is enforced: since the stage game has a unique Nash equilibrium, it is played at every stage.

This is true in general: if the stage game $G$ has a unique Nash equilibrium, then the finitely-repeated game $G_N$ (i.e., with $N < \infty$) has a unique subgame-perfect Nash equilibrium, given by playing the Nash equilibrium of $G$ at every stage.
What if the stage game is repeated an infinite number of times \((N = \infty)\)? In this case (*infinite-horizon* game), a *free rider* behavior may be punished in subsequent rounds.

Consider the following strategy (known as *grim-trigger strategy*):

- Keep playing the *social-optimal* solution unless the other player has defected: in this case, switch to the noncooperative (selfish) solution.
Implicitly, we have derived the outcomes of the game $G_N$ by summing up each stage’s payoffs.

We can introduce a discount factor $\delta$ ($0 \leq \delta \leq 1$) that measures the patience of the players:

$$u_k^\delta(a) = (1 - \delta) \sum_{n=0}^{\infty} \delta^n u_k(a(n))$$

- the smaller $\delta$, the less important future payoffs are (players are impatient): myopic (short-sighted) optimization
- when $\delta = 1$, €1 earned today is equivalent to €1 earned in 2020: long-sighted optimization
Suppose $\delta$ is given and common to both players, and player 2 uses the grim-trigger strategy $a_2^{GT} = \{a_2(n)\}_{n=0}^{\infty}$, with

$$a_2(n) = \begin{cases} f, & \text{if } a_1(n-1) = a_2(n-1) = f \\ d, & \text{otherwise} \end{cases}$$

If player 1 plays 'f' every time, i.e., $a_1 = a_1^f = \{a_1(n)\}_{n=0}^{\infty}$, $a_1(n) = f \ \forall n$:

$$u_1^\delta(a_1^f, a_2^{GT}) = (1 - \delta) \sum_{n=0}^{\infty} \delta^n \cdot u_1(f, f) = t$$

If player 1 plays 'd' every time, i.e., $a_1 = a_1^d = \{a_1(n)\}_{n=0}^{\infty}$, $a_1(n) = d \ \forall n$:

$$u_1^\delta(a_1^d, a_2^{GT}) = (1 - \delta) \sum_{n=0}^{\infty} \delta^n \cdot u_1(a_1(n), a_2(n))$$

$$= (1 - \delta) \cdot ((t + c) + c\delta + c\delta^2 + \ldots) = (1 - \delta)t + c$$

**Exercise 4:** Check that $u_1^\delta(a_1, a_2^{GT}) < u_1^\delta(a_1^d, a_2^{GT})$ for $a_1 \neq a_1^d, a_1^f$.
Infinite-horizon repeated games (4/5)

What is the best strategy for player 1? To answer this question, we must compare the two expected payoffs:

\[ t \geq (1 - \delta)t + c \]

- If \( \delta \geq c/t \), i.e., if player 1 is patient enough, \( a_1^{GT} \) is the best strategy for player 1 too.
- If \( \delta < c/t \), player 1’s best strategy is to defect at every stage.

Due to the symmetry of the problem, \( a^* = (a_1^{GT}, a_2^{GT}) \) is a subgame-perfect Nash equilibrium of the game \( G_\infty \) when \( \delta \geq c/t \).

In the repeated forwarder’s dilemma, cooperation is enforced by letting the players interact an infinite (i.e., unknown) number of times: this is due to threatening future punishments for those who defect.
Infinite-horizon repeated games (5/5)

Is the grim-trigger strategy the only one that induces cooperation?

**Exercise 5:** Consider the strategy known as tit-for-tat, where

\[ a_k(n) = a_k(n - 1) \]

\[ a_k(n - 1) = f \quad a_k(n - 1) = d \quad a_k(n - 1) = d \]

\[ a_k(n) = f \quad a_k(n) = d \quad a_k(n - 1) = f \]
The set of **feasible** discounted payoffs of $G_\infty$ is any **convex** combination of the pure-strategy payoffs of the constituent game $G$:

$$a(2n) = (f, f), a(2n + 1) = (f, d), \delta = 1$$

$$a(2n) = (f, f), a(2n + 1) = (d, f), \delta = 1$$

$$a(3n) = (f, f), a(3n + 1) = (d, f), a(3n + 2) = (d, f), \delta = 1$$

$$a(3n) = (f, f), a(3n + 1) = (d, f), a(3n + 2) = (f, d), \delta = 1$$
(Friedman, 1971) Let $G$ be a finite static game with complete information, with a unique pure-strategy Nash equilibrium $a^\ast$. Let $\{u_k(a^\ast)\}_{k=1}^{K}$ be the payoffs at the Nash equilibrium. For any feasible set $\{\hat{u}_k\}_{k=1}^{K}$ such that $\hat{u}_k > u_k(a^\ast)$ for all $k$, there exists a discount factor $\hat{\delta} < 1$ such that, for all $\delta \in (\hat{\delta}, 1)$, there exists a subgame-perfect Nash equilibrium of the infinitely repeated version of the stage game $G$ that achieves $\{\hat{u}_k\}_{k=1}^{K}$ as the average payoffs.
Let’s suppose to play the continuous-power near-far power control game an infinite number of times:

There exists a critical $\delta$ such that, for all $\delta > \delta$ (i.e., for delay-tolerant users), the grim-trigger strategy achieves the maximum social welfare.
Textbooks


Non-technical readings

Bibliography/Webography (2/10)

Tutorial-style readings on wireless communications


Bibliography/Webography (3/10)

Tutorial-style readings on wireless communications (cont’d)


Noncooperative game theory

Static finite games


Static infinite games


Noncooperative game theory (cont’d)

Supermodular games


Potential games


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Generalized Nash games


Dynamic games


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Noncooperative game theory (cont’d)

Stackelberg games


Repeated games


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Bayesian games


Games with strict incomplete information

Cooperative game theory

Bargaining problems


Coalitional game theory


Cooperative game theory (cont’d)

Canonical coalitional games


Coalition formation games


Coalition graph games