8/01/2008

4) 10 palline Rosse
5 palline nere

a) I)[1] $P = \frac{C_{5,2}}{C_{15,2}} = \frac{5!}{3!\,2!}\,\frac{13!\,2!}{15!} = \frac{5\cdot 4}{2}\cdot\frac{2}{15\cdot 14} = \frac{5\cdot 4}{15\cdot 14} = 0.095$

II) $p_1 = \frac{5}{15}$ $\quad p_2 = \frac{4}{14}$ $\quad$ p_1 = prob. estrarre 1 pallina nera (1ª estrazione)
p_2 = prob. estrarre 1 pallina nera, dopo averne estratta 1 nera

$p = p_1 \cdot p_2$

b) I) $p = \frac{C_{10,5}}{C_{15,5}} = \frac{10!}{5!\,5!}\cdot\frac{5!\,10!}{15!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{15\cdot 14\cdot 13\cdot 12\cdot 11} = 0,0839$

II) $p_1 = \frac{10}{15}$ $\quad p_2 = \frac{9}{14}$ $\quad p_3 = \frac{8}{13}$ $\quad p_4 = \frac{7}{12}$ $\quad p_5 = \frac{6}{11}$

$p = p_1 p_2 p_3 p_4 p_5$

c) $P(\text{almeno 1 rosse}) = 1 - P(\text{tutte palline nere}) = 1 - \frac{C_{5,5}}{C_{15,5}} = 1 - \frac{10!\,5!}{15!} =$

$= 1 - \frac{5\cdot 4\cdot 3\cdot 2}{15\cdot 14\cdot 13\cdot 12\cdot 11} = 0,9997$

d) $p = \frac{5}{15} = \frac{1}{3}$ $\quad q = \frac{2}{3}$ $\quad$ $n = 5$ num. estrazioni $\quad$ 3 successi

$P_5(3) = \binom{5}{3} p^3 q^2 = \frac{5!}{3!\,2!}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 = \frac{5\cdot 4}{2}\cdot\frac{1}{27}\cdot\frac{4}{9} = 0,165$

II° modo c

casi favorevoli $F = C_{10,5} + C_{10,4}\cdot C_{5,1} + C_{10,3}\cdot C_{5,2} + C_{10,2}\cdot C_{5,3} + C_{10,1}\cdot C_{5,4}$

casi possibili $\quad F + C_{5,5}$

$p = \frac{F}{F + C_{5,5}}$

[1] con I e II si intendono due possibili modi per risolvere l'esercizio

8/01/2008

5) $S_n = \frac{2^{-|n|} e^{j\frac{\pi n}{2}}}{n^2}$ $n \neq 0$, $S_0 = 0$

a) $S_{-n} = \frac{2^{-|-n|} e^{-j\frac{\pi n}{2}}}{(-n)^2} = \frac{2^{-|n|} e^{-j\frac{\pi n}{2}}}{n^2} = S_n^*$ $\Rightarrow$ s(t) reale

b) $S_{-n} \neq -S_n$ s(t) no dispari

$S_{-n} \neq S_n$ s(t) no pari

c) $S_1 = \frac{1}{2} e^{j\frac{\pi}{2}} = \frac{j}{2}$ $\quad S_{-1} = -\frac{j}{2}$ $\quad |S_1| = |S_{-1}| = 1/2$ $\quad \angle S_1 = \pi/2$

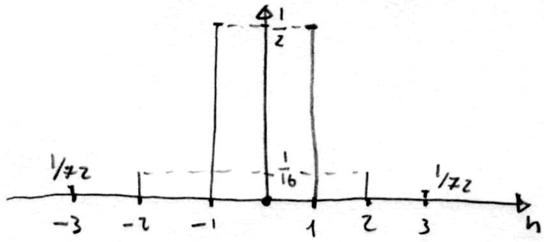
$\angle S_{-1} = -\pi/2$

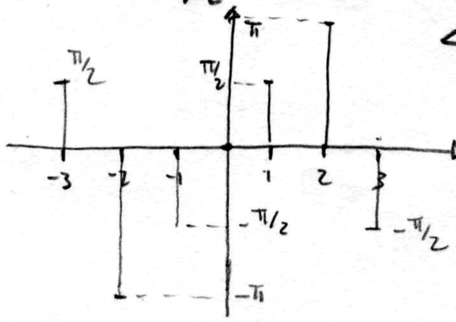
$S_2 = \frac{1}{4} \frac{e^{j\pi}}{4} = -\frac{1}{16}$ $\quad S_{-2} = \frac{1}{4} \cdot \frac{e^{-j\pi}}{4} = -\frac{1}{16}$ $\quad |S_2| = |S_{-2}| = \frac{1}{16}$ $\quad \angle S_2 = \pi$

$\angle S_{-2} = -\pi$

$S_3 = \frac{1}{8} \frac{e^{j\frac{3}{2}\pi}}{9} = \frac{1}{72} e^{-j\frac{\pi}{2}} = -\frac{j}{72}$ $\quad S_{-3} = \frac{1}{72} e^{j\frac{\pi}{2}} = \frac{j}{72}$ $\quad |S_3| = |S_{-3}| = \frac{1}{72}$

$\angle S_3 = -\pi/2$ $\quad \angle S_{-3} = \frac{\pi}{2}$

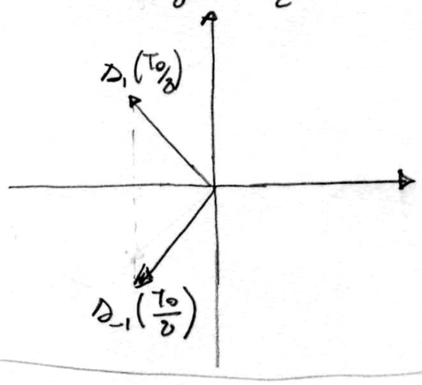




d) $s_1(t) = S_1 e^{j2\pi \frac{t}{T_0}} = \frac{j}{2} e^{j2\pi\frac{t}{T_0}}$

$s_{-1}(t) = S_{-1} e^{-j2\pi\frac{t}{T_0}} = -\frac{j}{2} e^{-j2\pi\frac{t}{T_0}}$

$s_1\left(\frac{T_0}{8}\right) = \frac{j}{2} e^{j\frac{\pi}{4}} = \frac{1}{2} e^{j\frac{3}{4}\pi}$ $\quad s_{-1}\left(\frac{T_0}{8}\right) = -\frac{j}{2} e^{-j\frac{\pi}{4}} = \frac{1}{2} e^{-j\frac{3}{4}\pi}$



6) $s(t) \overset{\mathcal{F}}{\rightleftharpoons} S(f)$

a) $s(t-t_0) \overset{\mathcal{F}}{\rightleftharpoons} S(f) e^{-j2\pi f t_0} \Rightarrow S(f) e^{j2\pi f T/2} \overset{\mathcal{F}}{\rightleftharpoons} s(t+T/2)$. sol. 6

b) $s(\alpha t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{|\alpha|} S\left(\frac{f}{\alpha}\right) \Rightarrow \alpha = \frac{1}{2}$ $\quad s\left(\frac{t}{2}\right) \overset{\mathcal{F}}{\rightleftharpoons} 2S(2f)$

$\Downarrow$

$S(2f) = \frac{1}{2} s\left(\frac{t}{2}\right)$ sol. 5

7) $f_{min} = 4KHz$ $f_{max} = 6KHz$ $A \in [0, 10V]$

f_c minima ?

Passa basso: $f_c \geq 2 f_{max} = 12KHz$

Passa banda: $B = f_{max} - f_{min} = 2KHz$ $\frac{f_{max}}{B} = \frac{6KHz}{2KHz} = 3 \Rightarrow m=3$

$$f_c = \frac{2 f_{max}}{m} = 4KHz$$

$$f_c \text{ minima} = 4KHz$$

$$\varepsilon = \frac{10V}{2^{12}} = \frac{10V}{4096}$$

8) $n=25$

σ popolazione ignota $\Rightarrow$ statistica t

$$t = \frac{11,9 - 9,8}{3.5/\sqrt{25}} = \frac{2,1}{0,7} = 3$$

ipotesi alternativa bilaterale $c = \pm 3,745$

$$-3,745 < t < 3,745 \Rightarrow$$

l'ipotesi nulla di uguaglianza tra la freq. di micronuclei osservata nei campioni irradiati e quella osservata nella popolazione di controllo, non può essere rifiutata