Es. statistica

Distrib. $f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$f_x(x) = 0,1629\, e^{-\frac{x^2}{12}}$

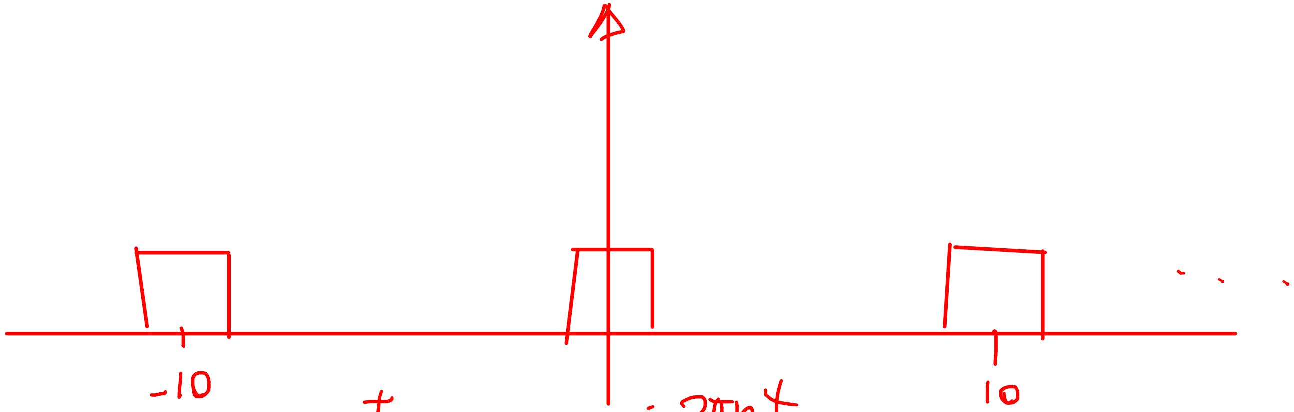
Valore atteso $\rightarrow \int_{-\infty}^{+\infty} x\, f_x(x)\, dx = E[x] = 0$

$\rightarrow \int_{-18}^{18} f_x(x)\, dx$ $\qquad \int_{0}^{+\infty} f_x(x)\, dx$ $\qquad \int_{0}^{36} f_x(x)\, dx$

$0,5$ $\qquad < 0,5$

visto che è simmetrica
e gli estremi sono distanti
dal valore medio ($>5\sigma$)
questa è la stat. maggiore

$$s(t) = \sum_{k=-\infty}^{\infty} \mathrm{rect}\left(\frac{t-kT_0}{T}\right) \qquad T_0 = 10 \quad T = 2$$



$$S_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} s(t)\, e^{-j 2\pi n \frac{t}{T_0}}\, dt$$

sfruttando le simmetrie $\quad S_n = \frac{2}{T_0} \int_{0}^{\frac{T_0}{2}} s(t) \cos 2\pi n \frac{t}{T_0}\, dt$

$$s(t) = rep_{T_0}(s_1(t))$$

Relazione tra TCF e S_n

$$S_n = \frac{1}{T_0} S_1\left(\frac{n}{T_0}\right) \quad \text{dove} \quad S_1(f) \overset{\mathcal{F}}{\Longleftrightarrow} s(t)$$

$$s_1(t) = \mathrm{rect}\left(\frac{t}{T}\right) \qquad S_1(f) = T\,\mathrm{sinc}(fT)$$

$$S_n = \frac{T}{T_0}\,\mathrm{sinc}\left(n\frac{T}{T_0}\right)$$

$$S_0 = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} s(t)\, e^{-j 2\pi 0 \frac{t}{T_0}}\, dt = \frac{1}{T_0}\int_{-1}^{1} 1\, dt = \frac{2}{T_0}$$

$$S_n = \frac{1}{T_0}\int_{-1}^{1} e^{-j 2\pi n \frac{t}{T_0}}\, dt = \frac{1}{T_0}\,\frac{1}{-j\frac{2\pi n}{T_0}}\, e^{-j 2\pi n \frac{t}{T_0}}\Big|_{-1}^{+1} =$$

$$= \frac{1}{T_0}\,\frac{1}{-j\frac{2\pi n}{T_0}}\left(e^{-j\frac{2\pi n}{T_0}} - e^{+j\frac{2\pi n}{T_0}}\right) = \frac{1}{T_0}\,\frac{1}{-j\frac{2\pi n}{T_0}}\,(-2j)\sin\frac{2\pi n}{T_0}$$

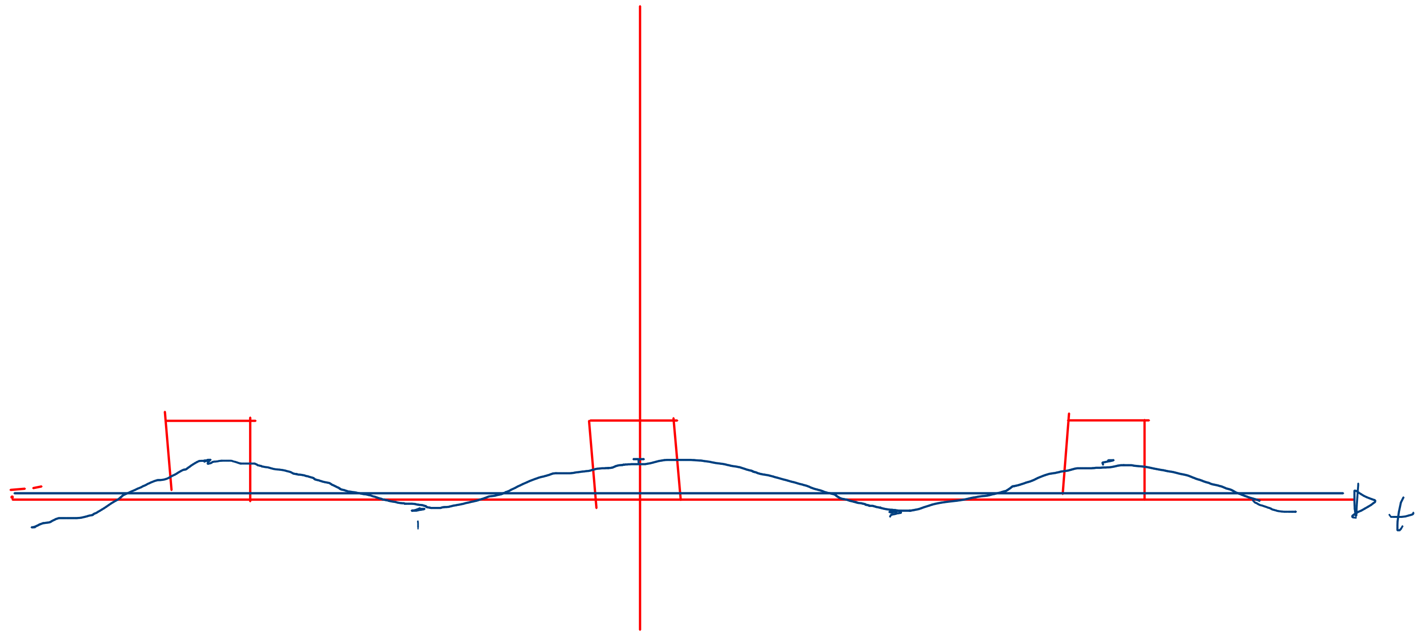
$$= \frac{2}{T_0}\,\mathrm{sinc}\left(\frac{2n}{T_0}\right)$$

$$S_1 = \frac{1}{5}\,\mathrm{sinc}\left(\frac{1}{5}\right) = 0{,}187 \qquad S_{-1} = S_1^* = 0{,}187$$

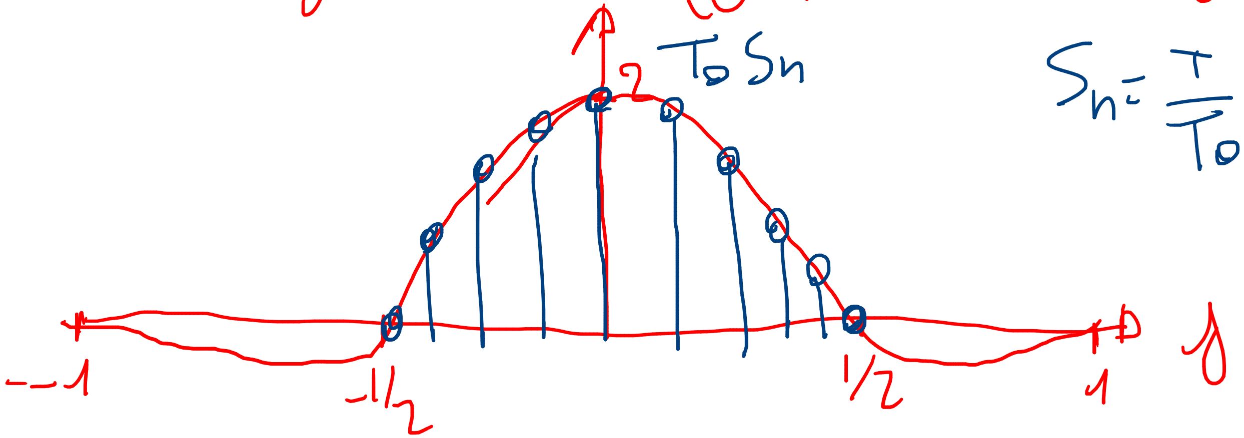
$$S_5 = \frac{1}{5}\,\mathrm{sinc}(1) = 0 \qquad S_{-5} = 0$$

$$s(t) = S_0 + S_1 e^{j2\pi\frac{t}{T_0}} + S_{-1} e^{-j2\pi\frac{t}{T_0}} =$$

$$= \frac{1}{5} + 0{,}37 \cos 2\pi\frac{t}{10}$$


t

$$S(f) = T\,\mathrm{sinc}(fT) = 2\,\mathrm{sinc}(2f)$$



$$S_n = \frac{T}{T_0}\,\mathrm{sinc}\left(n\frac{T}{T_0}\right)$$

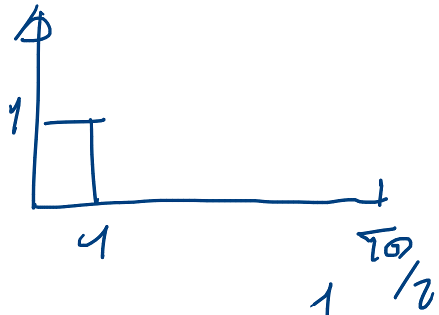
$$\frac{n}{T_0} = \frac{n}{10}$$

Abbiamo sfruttato la relazione tra

S_n del segnale periodicizzato

e $S(f)$ del segnale aperiodico

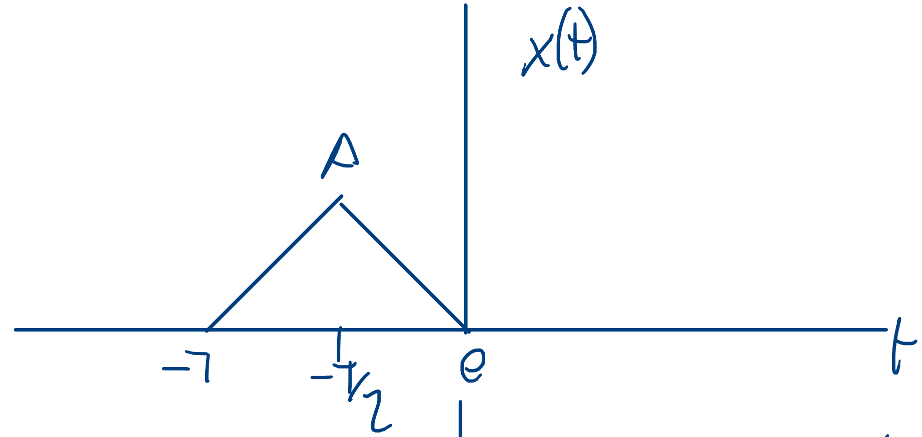
Sfruttando le simmetrie . . . .

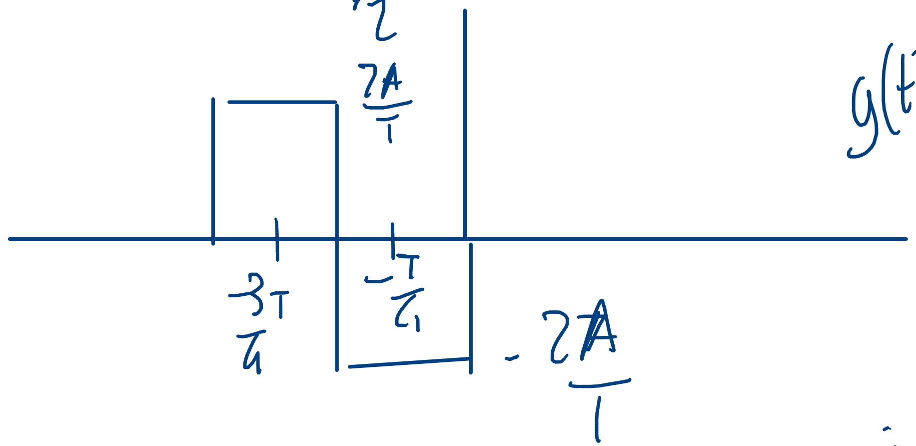


$$S_n = \frac{2}{T_0}\int_0^{\frac{T_0}{2}} s(t)\cos 2\pi n \frac{t}{T_0}\, dt =$$

$$= \frac{2}{T_0}\int_0^{1} \cos 2\pi n \frac{t}{T_0}\, dt = \frac{2}{T_0}\,\frac{1}{\frac{2\pi n}{T_0}}\,\sin 2\pi n \frac{t}{T_0}\Big|_0^1 =$$

$$= \frac{2}{T_0}\,\frac{1}{\frac{2\pi n}{T_0}}\,\sin \frac{2\pi n}{T_0}$$



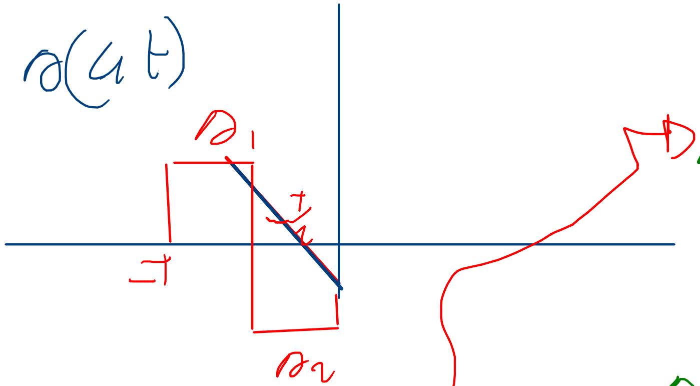


$$y(t) = \frac{2A}{T}\,\mathrm{rect}\left(\frac{t+\frac{3T}{4}}{T/2}\right) - \frac{2A}{T}\,\mathrm{rect}\left(\frac{t+T/4}{T/2}\right)$$

$$Y(f) = \frac{2A}{T}\,\frac{T}{2}\,\mathrm{sinc}\left(f\frac{T}{2}\right)e^{j\frac{3}{4}T2\pi f} - \frac{2A}{T}\,\frac{T}{2}\,\mathrm{sinc}\left(f\frac{T}{2}\right)e^{j\frac{1}{4}2\pi f}$$

$$\rightarrow \quad y(t) = \int_{-\infty}^{t} x(\alpha)\, d\alpha \qquad Y(f) = \frac{X(f)}{j2\pi f}$$

$\rightarrow \quad s(4t)$



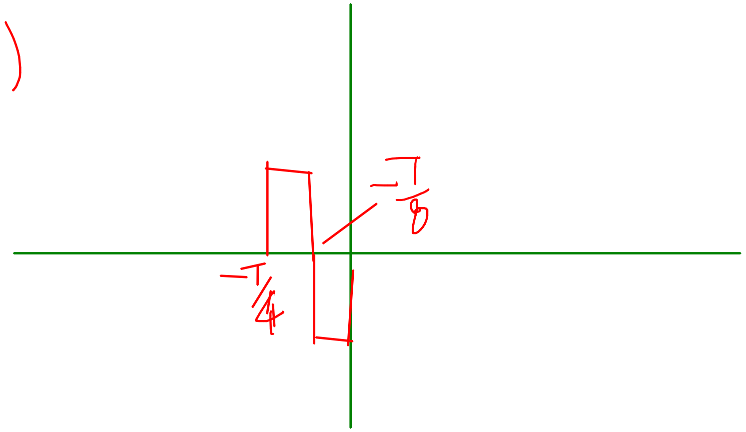
Cosa succede a t se cambio scala?

$$\rightarrow s(t) = \begin{cases} s_1 & -T < t \leq -T/2 \\ s_2 & -T/2 < t \leq 0 \end{cases}$$

$$s(4t) = \begin{cases} s_1 & \text{se } -T < 4t \leq -T/2 \\ s_2 & \text{se } -T/2 < 4t \leq 0 \end{cases}$$

$$= \begin{cases} s_1 & \text{se } -\frac{T}{4} < t \leq -\frac{T}{8} \\ s_2 & \text{se } -\frac{T}{8} < t \leq 0 \end{cases}$$

$s(4t)$



in frequenza . . .

$$Y_1(f) = \frac{1}{|4|} S\left(\frac{f}{4}\right)$$

$f_{min} = 105 \text{ KHz}$ $\quad f_{max} = 150$

$B = 150 - 105 = 45$ $\quad \frac{f_{max}}{B} = 3,\#$

$\Downarrow$

$n = 3$

$\rightarrow f_c = \frac{2 f_{max}}{n} = 100 \text{ KHz}$

$$s(t) = 1 + \sin\left(\frac{\pi}{8} t\right) + \sin\left(\frac{\pi}{6} t\right)$$

$\sin(2\pi f_A t)$ $\quad f_1 = \frac{1}{16}$ $\quad f_2 = \frac{1}{12}$

$$f_c \geq 2 f_{max} = \frac{1}{6} \qquad f_c \geq \frac{1}{6}$$

$$dt \leq 6$$

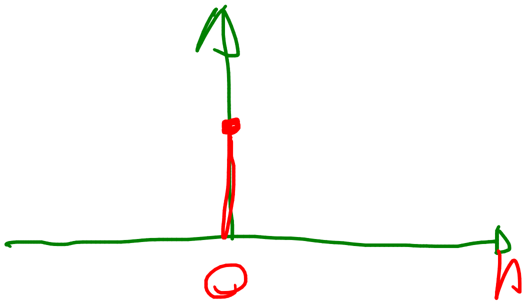
Si consideri l'operatore convoluzione lineare indicato con il simbolo ⊗, dire qua

A. □ $x[n] = \delta[-n] \otimes x[-n]$ B. □ $x[n] = \big[u[n] - u[n-1]\big] \otimes x[n]$

$$x[n] = u[n] \otimes x[n]$$

$\delta[n]$

$$x[n] = \delta[n] \otimes x[n] = x[n]$$



$$x[-n] \otimes \delta[n] = x[-n]$$ ← la A non è corretta

la soluzione è la B visto che $u[n] - u[n-1] = \delta[n]$

Esercizio 6. Data il sistema tempo discreto des

y[n]=x[n-2]-x[n-4]+0.4005y[n-1]-0.81y[n-2], s

$$Y(z) = z^{-2} X(z) - z^{-4} X(z) + 0{,}4005\, z^{-1} Y(z) - 0{,}81\, z^{-2} Y(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-2} - z^{-4}}{1 - 0{,}4005\, z^{-1} + 0{,}81\, z^{-2}} = \frac{z^{2} - 1}{z^{2}\left(z^{2} - 0{,}4002\, z + 0{,}81\right)}$$

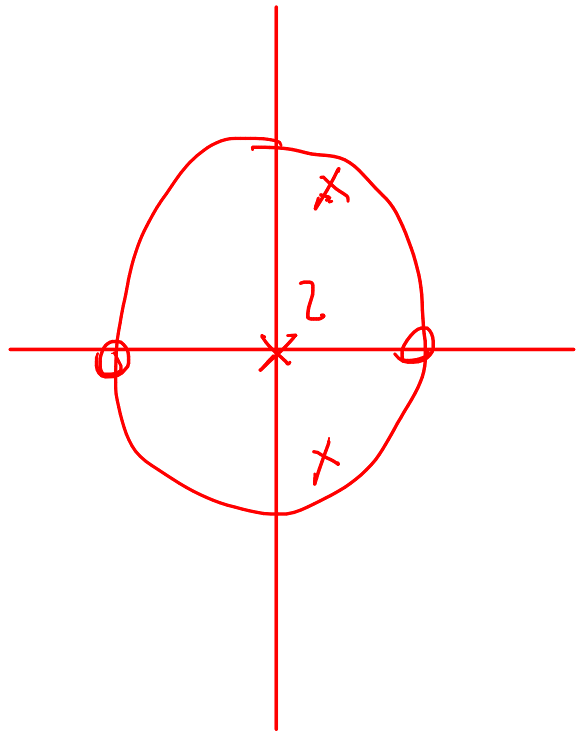
$$z_{o1} = 1 \qquad z_{o2} = -1$$

$$z_{P1} = z_{P2} = 0$$

$$\Delta = \sqrt{0{,}4^{2} - 4 \cdot 0{,}81} = \sqrt{-3{,}08}$$

$$z_{P3} = 0{,}2 - 0{,}87j$$

$$z_{P4} = 0{,}2 + 0{,}87j$$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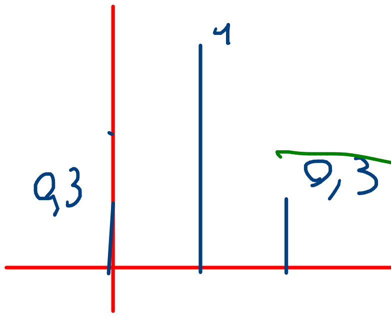
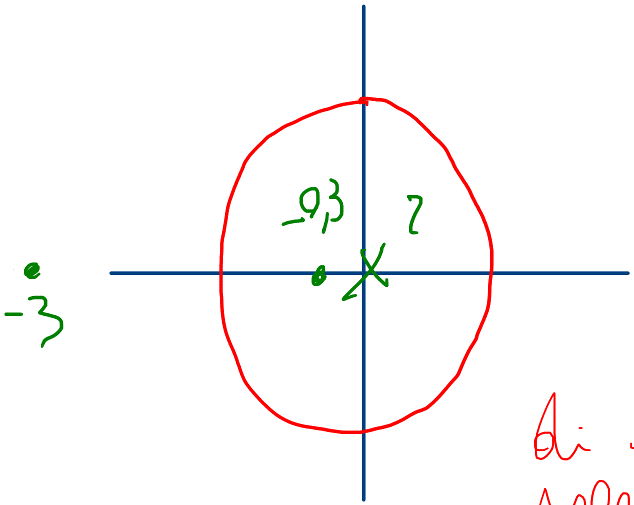


$$h[n] = 0{,}3\,\delta[n] + 1\,\delta[n-1] + 0{,}3\,\delta[n-2]$$

$$H(z) = 0{,}3 + z^{-1} + 0{,}3\,z^{-2} = \frac{0{,}3\,z^2 + z + 0{,}3}{z^2}$$



i coeff. della risposta imp. di un filtro FIR sono i coeff. della $H(z)$