— 14.7.08

4.

probabilità difetto $p=\frac{1}{10}$ $\Rightarrow$ prob. protesi non difettosa $q=\frac{9}{10}$

a) $P_8(4)=\binom{8}{4}p^4q^{8-4}=\frac{8!}{4!4!}\left(\frac{1}{10}\right)^4\left(\frac{9}{10}\right)^4=\frac{\overset{2}{\cancel{8}}\cdot 7\cdot \overset{2}{\cancel{6}}\cdot 5}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}\,10^{-4}\cdot 0{,}6561=0{,}46\%$

b) $p=\sum_{K=3}^{8}P_8(K)=1-\sum_{K=0}^{2}P_8(K)=P_8(3)+P_8(4)+P_8(5)+P_8(6)+P_8(7)+P_8(8)$

$=\binom{8}{3}\left(\frac{1}{10}\right)^3\left(\frac{9}{10}\right)^5+\binom{8}{4}\left(\frac{1}{10}\right)^4\left(\frac{9}{10}\right)^4+\binom{8}{5}\left(\frac{1}{10}\right)^5\left(\frac{9}{10}\right)^3+\binom{8}{6}\left(\frac{1}{10}\right)^6\left(\frac{9}{10}\right)^2+\binom{8}{7}\left(\frac{1}{10}\right)^7\left(\frac{9}{10}\right)^1+\binom{8}{8}\left(\frac{1}{10}\right)^8\left(\frac{9}{10}\right)^0=$

$=1-\binom{8}{2}\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^6-\binom{8}{1}\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^7-\binom{8}{0}\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^8=3{,}8\%$

Altro modo

prob. non difetto $p=\frac{9}{10}$ prob. difetto $q=\frac{1}{10}$

a) $P\{4 \text{ difettose}\}=P\{4 \text{ non difettose}\}=P_8(4)=\binom{8}{4}p^4q^{8-4}=\frac{8!}{4!4!}\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)^4$

b) probabilità $P\{\text{almeno 3 difettose}\}=P\{\text{non difettose}\leq 5\}=$

$=\sum_{K=0}^{5}P_8(K)=P_8(0)+P_8(1)+P_8(2)+P_8(3)+P_8(4)+P_8(5)=$

$=\binom{8}{0}\left(\frac{9}{10}\right)^0\left(\frac{1}{10}\right)^8+\binom{8}{1}\left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^7+\binom{8}{2}\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)^6+\binom{8}{3}\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^5+\binom{8}{4}\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)^4+$

$+\binom{8}{5}\left(\frac{9}{10}\right)^5\left(\frac{1}{10}\right)^4=1-\sum_{K=6}^{8}P_8(K)=1-\binom{8}{6}\left(\frac{9}{10}\right)^6\left(\frac{1}{10}\right)^2-\binom{8}{7}\left(\frac{9}{10}\right)^7\left(\frac{1}{10}\right)^1-\binom{8}{8}\left(\frac{9}{10}\right)^8\left(\frac{1}{10}\right)^0$

N.B.

$\binom{n}{k}=\binom{n}{n-k}$

$\binom{8}{0}=\binom{8}{8}$

$\binom{8}{2}=\binom{8}{6}$

14/07/2008

5.

$$S_n = -j\frac{\sin^2\left(\frac{\pi n}{2}\right)}{n} \quad \text{per } n \neq 0 \qquad S_0 = 1$$

$$S_{-n} = -j\frac{\sin^2\left(\frac{-\pi n}{2}\right)}{-n} = j\frac{\sin^2\left(\frac{\pi n}{2}\right)}{n} = -S_n = S_n^*$$

a) $S_{-n} = S_n^* \Rightarrow s(t)$ reale

b) $S_n = -S_{-n} \Rightarrow s(t)$ dispari

c) per grafico vedi in fondo

$n=1 \quad S_1 = e^{-j\frac{\pi}{2}} \qquad S_{-1} = j = e^{j\frac{\pi}{2}}$

$n=2 \quad S_2 = 0 \qquad S_{-2} = 0$

$n=3 \quad S_3 = -\frac{j}{3} = \frac{1}{3}e^{-j\frac{\pi}{2}} \qquad S_{-3} = \frac{j}{3} = \frac{1}{3}e^{j\frac{\pi}{2}}$

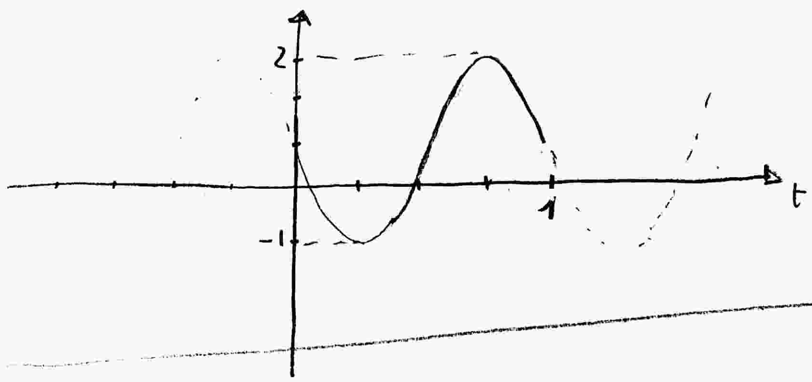
d) $s_{-1}(t) = S_{-1}\, e^{-j2\pi\frac{t}{T_0}} = e^{-j\frac{\pi}{2}}e^{-j\frac{2\pi t}{T_0}} = e^{-j\left(\frac{2\pi t}{T_0}+\frac{\pi}{2}\right)} = e^{-j\left(2\pi t + \frac{\pi}{2}\right)}$ | N.B. | $T_0 = 1\,s$

$s_0(t) = S_0$ N.B. $s_0(t) = S_0\, e^{j\frac{2\pi n t}{T_0}}\Big|_{n=0} = S_0 = 1$

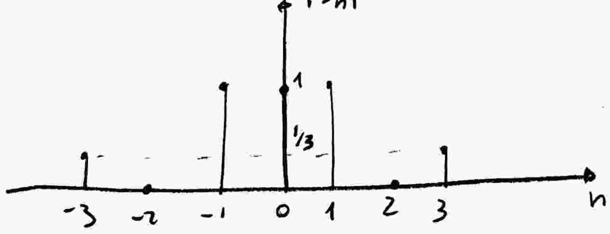
$s_1(t) = S_1\, e^{j\frac{2\pi t}{T_0}} = e^{j\frac{\pi}{2}}e^{j\frac{2\pi t}{T_0}} = e^{j\left(2\pi t + \frac{\pi}{2}\right)}$

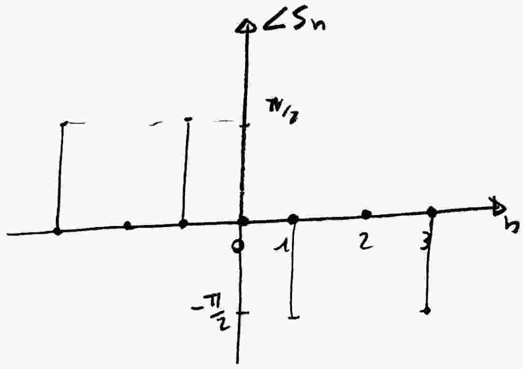
$$s_{-1}(t) + s_1(t) + s_0(t) = e^{-j\left(2\pi t+\frac{\pi}{2}\right)} + e^{j\left(2\pi t+\frac{\pi}{2}\right)} + 1 =$$

$$= 1 + 2\cos\left(2\pi t + \frac{\pi}{2}\right) = 1 - 2\sin(2\pi t)$$



c)





6.

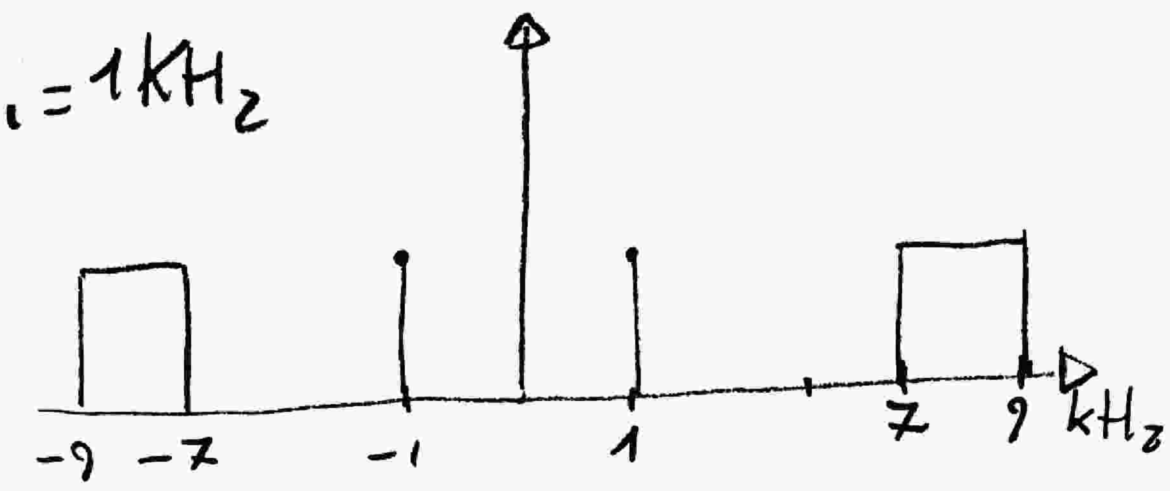
$f_{min} = 7\ KHz \quad f_{max} = 9\ KHz$

PASSA BASSO $\quad f_c \geq 2 f_{max} = 18\ KHz \qquad f_c \geq 18\ KHz$

PASSA BANDA $\quad B = f_{max} - f_{min} = 2\ KHz \qquad \dfrac{f_{max}}{B} = 4.5 \Rightarrow m = 4$

$$f_c = \frac{2 f_{max}}{m} = 4.5\ KHz$$

$s_1(t) = s(t) + \cos 2\pi f_1 t \qquad f_1 = 1\ KHz$



$B_1 = 9 - 1 = 8\ KHz \qquad \dfrac{f_{max}}{B_1} = \dfrac{9}{8} = 1.125 \qquad m = 1$

$f_c \geq 18\ KHz$

7. $\quad T = N dt = 0,5\,s \Rightarrow df = \dfrac{1}{0,5\,s} = 2\ Hz$

$dt = 0.05 \qquad f \in [-10 ; 10]\ Hz$ Frequenze visualizzabili senza ambiguità

$N = 10 \Rightarrow \quad f = df \cdot [-5:4] = [-10:1:8] \Rightarrow$ (b)

con zero padding $N=10 \qquad T = 20 \cdot dt = 1\,s \quad df = 1\ Hz$

$dt = 0.05 \Rightarrow f \in [-10 ; 10)\ Hz$

$f = df \cdot [-10:1:9] = [-10:1:9]\ Hz$

8. $\quad n = 6$

misure : 83, 81, 69, 71, 73, 82 mM

$H_0 : \bar{x} = \eta_0$

$H_a : \bar{x} \neq \eta_0$

$$\bar{x} = \frac{83+81+69+71+73+82}{6} = 76,5$$

$$\hat{s} = \sqrt{\frac{(83-\bar{x})^2+(81-\bar{x})^2+(69-\bar{x})^2+(71-\bar{x})^2+(73-\bar{x})^2+(82-\bar{x})^2}{5}} =$$

$$= \sqrt{\frac{6.5^2+4.5^2+(-7,5)^2+(5,5)^2+(-3,5)^2+(5,5)^2}{5}} =$$

$$= \sqrt{\frac{42.25+20.25+56.25+30.25+17,25+30,25}{5}} =$$

$$= \sqrt{\frac{191.5}{5}} = \sqrt{38.3} \cong 6,19$$

Valore atteso popolazione $\eta_0 = 72\ mM$

Varianza popolazione incognita $\Rightarrow t$

Ipotesi alternativa bilatera $\quad t_c = \pm 2,57$

$$t = \frac{\bar{x} - \eta_0}{\hat{s}/\sqrt{n}} = \frac{76.5 - 72}{6.19/\sqrt{6}} = \frac{4.5 \cdot \sqrt{6}}{6.19} = 1.78$$

$-t_c < 1,78 < +t_c$

l'ipotesi nulla non può essere rifiutata