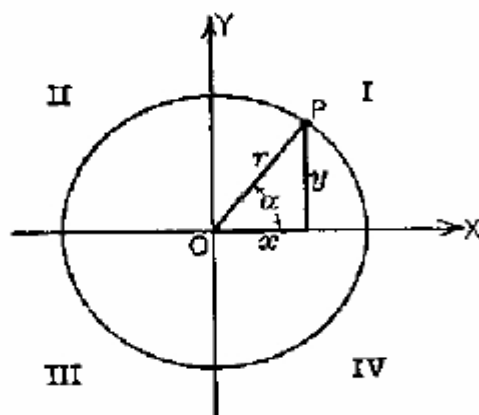


4.1 Trigonometry

Definitions



In quadrant I:

$$\sin \alpha = \frac{y}{r}, \quad \alpha = \arcsin \left(\frac{y}{r} \right) = \sin^{-1} \left(\frac{y}{r} \right)$$

$$\cos \alpha = \frac{x}{r}, \quad \alpha = \arccos \left(\frac{x}{r} \right) = \cos^{-1} \left(\frac{x}{r} \right)$$

$$\tan \alpha = \frac{y}{x}, \quad \alpha = \arctan \left(\frac{y}{x} \right) = \tan^{-1} \left(\frac{y}{x} \right)$$

$$r = \sqrt{x^2 + y^2}$$

Funzioni Matlab

| | |
|-------------------------|---|
| <code>sin(alpha)</code> | Sine of alpha |
| <code>cos(alpha)</code> | Cosine of alpha |
| <code>tan(alpha)</code> | Tangent of alpha |
| <code>asin(z)</code> | Arcsine or inverse sine of z , where z must be between -1 and 1 . Returns an angle between $-\pi/2$ and $\pi/2$ (quadrants I and IV). |
| <code>acos(z)</code> | Arccosine or inverse cosine of z , where z must be between -1 and 1 . Returns an angle between 0 and π (quadrants I and II). |
| <code>atan(z)</code> | Arctangent or inverse tangent of z . Returns an angle between $-\pi/2$ and $\pi/2$ (quadrants I and IV). |
| <code>atan2(y,x)</code> | Four quadrant arctangent or inverse tangent, where x and y are the coordinates in the plane shown in the figure above. Returns an angle between $-\pi$ and π (all quadrants), depending on the signs of x and y . |

Help elfun

N.B. gli angoli sono sempre in radianti

>> $y = \cos(\pi/3)$

>> $z = \sin(2/3*\pi)$

>> $\text{alfa_rad} = \text{atan}(1)$

Come passare da radianti a gradi e vice-versa:

Data la proporzione: **$\alpha_{\text{gradi}} : \alpha_{\text{rad}} = 180 : \pi$**

- Da radianti a gradi: **$\alpha_{\text{gradi}} = \alpha_{\text{rad}} * 180 / \pi$**
- Da gradi a radianti: **$\alpha_{\text{rad}} = \alpha_{\text{gradi}} * \pi / 180$**

Complex numbers

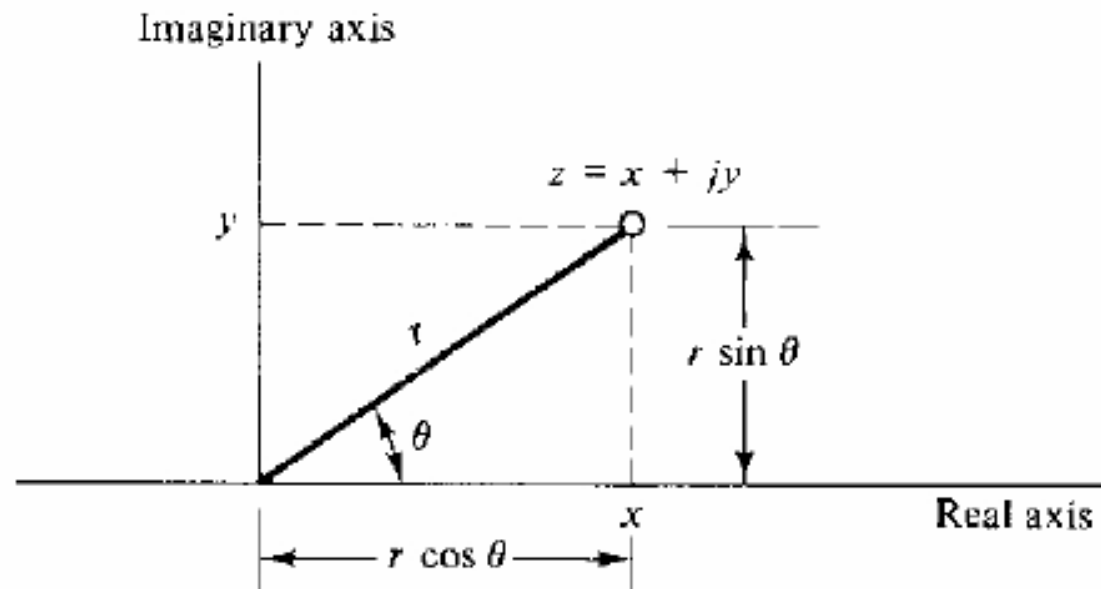


Figure 4.3: The complex number z in the complex plane

Rectangular Representation: A complex number z consists of the “real part” x and the “imaginary part” y and is expressed as

$$z = x + jy$$

Polar Representation: Defining the radius r and the angle θ of the complex number z shown in Figure 4.3, z can be represented in polar form and written as

$$z = r \cos \theta + jr \sin \theta$$

Exponential Representation: The base of the natural logarithms, $e = 2.71828182845904$, is used to develop the exponential representation for complex numbers, through the **Euler** (sounds like oiler) formula $e^{j\theta} = \cos \theta + j \sin \theta$

use them to represent the complex number z in the exponential form

$$\begin{aligned} z = re^{j\theta} &= r \cos \theta + jr \sin \theta \\ &= r(\cos \theta + j \sin \theta) \end{aligned}$$

Rappresentazioni dei numeri complessi:

$$z = x + jy = r\angle\theta = re^{j\theta} = |z|e^{j\angle z}$$

I numeri complessi in Matlab

Imaginary number: The most fundamental new concept in the study of complex numbers is the “imaginary number” j . This imaginary number is defined to be the square root of -1 :

$$j = \sqrt{-1}$$

$$j^2 = -1$$

In MATLAB, `i` and `j` are variable names that default to the imaginary number. You have to be careful with their use, however, as they can be overridden and used as general variables. You can insure that `j` is the imaginary number by explicitly computing it as the square root of -1 :

```
>> j = sqrt(-1)
j =
    0+ 1.0000i
```

Come generare numeri complessi in Matlab:

A general complex number can be formed in three ways:

```
>> z = 1 + j*2
z =
    1.0000+ 2.0000i
>> z = 1 + 2j
z =
    1.0000+ 2.0000i
>> z = complex(1,2)
z =
    1.0000 + 2.0000i
```

- La funzione ***real(z)*** restituisce la parte reale del numero complesso z
- La funzione ***imag(z)*** restituisce la parte immaginaria del numero complesso z

Da rappresentazione rettangolare a polare:

| | Math definition | MATLAB |
|--------------------|--|-----------------------|
| Magnitude: | $ z = r = \sqrt{x^2 + y^2}$ | <code>abs(z)</code> |
| Argument or angle: | $\theta = \tan^{-1}\left(\frac{y}{x}\right) = \sin^{-1}\left(\frac{y}{r}\right) = \cos^{-1}\left(\frac{x}{r}\right)$ | <code>angle(z)</code> |

$$z_{\text{polar}} = \text{abs}(z) * (\cos(\text{angle}(z)) + j * \sin(\text{angle}(z)))$$

Da rappresentazione polare a rettangolare:

$$x = r * \cos(\text{teta})$$

$$y = r * \sin(\text{teta})$$

$$z = x + i * y$$

Da rappresentazione rettangolare a esponenziale:

$$z_{\text{esp}} = \text{abs}(z) * \exp(i * \text{angle}(z))$$

Algebra of Complex Numbers

Addition and Subtraction:

$$\begin{aligned}z_1 + z_2 &= (x_1 + jy_1) + (x_2 + jy_2) \\ &= (x_1 + x_2) + j(y_1 + y_2)\end{aligned}$$

$$\begin{aligned}z_1 - z_2 &= (x_1 + jy_1) - (x_2 + jy_2) \\ &= (x_1 - x_2) + j(y_1 - y_2)\end{aligned}$$

Multiplication:

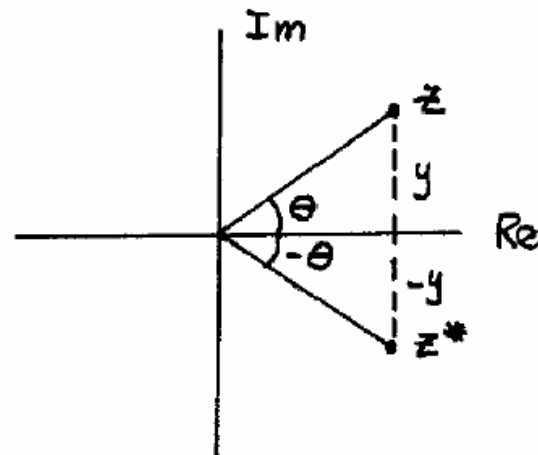
$$\begin{aligned}z_1 z_2 &= r_1 e^{j\theta_1} r_2 e^{j\theta_2} \\ &= r_1 r_2 e^{j(\theta_1 + \theta_2)}\end{aligned}$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$

Complex Conjugate.

$$z^* = x - jy = r e^{-j\theta}$$



To summarize, the following are the MATLAB functions for complex numbers:

| | |
|-----------------------|--|
| <code>abs(z)</code> | Complex magnitude $ z $ (absolute value for real z) |
| <code>angle(z)</code> | Phase angle or argument of z |
| <code>conj(z)</code> | Complex conjugate z^* |
| <code>imag(z)</code> | Complex imaginary part $\text{Im}(z)$ |
| <code>real(z)</code> | Complex real part $\text{Re}(z)$ |