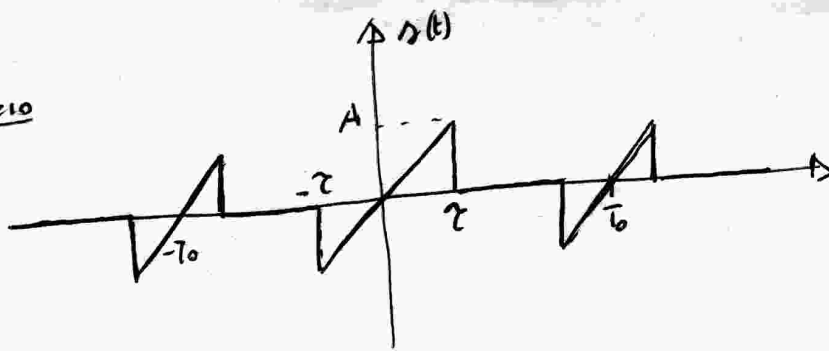


Esercizio

(12) d



$s(t)$  dispari  $\Rightarrow R_n = 0$

$$I_n = -\frac{2}{T_0} \int_0^{T_0/2} s(t) \sin 2\pi n t \frac{dt}{T_0} dt =$$

$$= -\frac{2}{T_0} \int_0^{\tau} \frac{At}{\tau} \sin 2\pi n t \frac{dt}{T_0} = -\frac{2A}{T_0 \tau} \int_0^{\tau} t \sin 2\pi n t \frac{dt}{T_0} = -\frac{2A}{T_0 \tau} \int_0^{\tau} t \frac{1}{2\pi n} \left( -\frac{d \cos 2\pi n t}{dt} \right) dt$$

$$= +\frac{2A}{T_0 \tau} \frac{1}{2\pi n} \left[ t \cos 2\pi n t \frac{dt}{T_0} \right]_0^{\tau} - \int_0^{\tau} \cos 2\pi n t \frac{dt}{T_0} = \frac{A}{\pi n \tau} \left[ \tau \cos 2\pi n \frac{\tau}{T_0} - \right.$$

$$\left. - \frac{1}{2\pi n} \left[ \tau \sin(2\pi n t \frac{dt}{T_0}) \right]_0^{\tau} \right] = \frac{A}{\pi n \tau} \left[ \tau \cos 2\pi n \frac{\tau}{T_0} - \frac{T_0}{2\pi n} \sin 2\pi n \frac{\tau}{T_0} \right]$$

per  $\tau = T_0/4$

$$I_n = \frac{A4}{\pi n T_0} \left[ \frac{T_0}{4} \cos \frac{\pi n}{2} - \frac{T_0}{2\pi n} \sin \frac{\pi n}{2} \right] = \frac{A}{\pi n} \cos \frac{\pi n}{2} - \frac{2A}{(\pi n)^2} \sin \frac{\pi n}{2}$$

$$S_n = R_n + j I_n$$

$$S_0 = 0$$

$$S_1 = j I_1 = j \left( -\frac{2A}{\pi^2} \right) \Rightarrow S_{-1} = j \frac{2A}{\pi^2}$$

$$S_2 = j I_2 = j \cdot \left( -\frac{A}{2\pi} \right) \Rightarrow S_{-2} = j \frac{A}{2\pi}$$

$$S_3 = j I_3 = j \frac{2A}{9\pi^2} \quad S_{-3} = -j \frac{2A}{9\pi^2}$$

$$S_4 = j I_4 = j \frac{A}{4\pi} \quad S_{-4} = -j \frac{A}{4\pi}$$

⋮

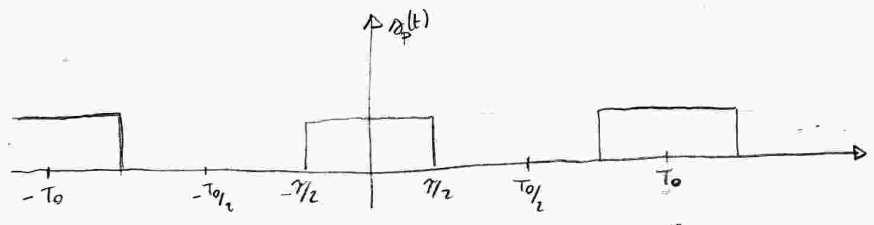
Trasformata continua di Fourier

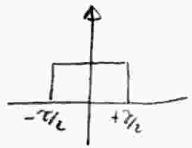
Un segnale aperiodico  $x(t)$ , si può rappresentare come la sovrapposizione di componenti sinusoidali di ampiezza infinitesima e di frequenza variabile con continuità tra  $-\infty$  e  $+\infty$ .

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} df$$

Si può dimostrare in modo intuitivo, andando a vedere come cambia lo spettro di un segnale periodico quando il periodo viene fatto tendere all'infinito, cioè diviene aperiodico.

Se consideriamo un treno di impulsi rettangolari



questo può essere scritto come 
$$s_p(t) = \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-nT_0}{\tau}\right)$$
 dove  $\text{rect}\left(\frac{t}{\tau}\right)$  

Nel limite  $T_0 \rightarrow \infty$  i vari impulsi si allontanano e si ritrova il segnale

$$s(t) = \text{rect}\left(\frac{t}{\tau}\right) = \lim_{T_0 \rightarrow \infty} s_p(t)$$

Vediamo cosa succede ai coefficienti della serie di Fourier al tendere di  $T_0$  all'infinito.

$$S_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} s_p(t) e^{-j2\pi nt/T_0} dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \sum_{q=-\infty}^{+\infty} s(t+qT_0) e^{-j2\pi nt/T_0} dt$$

visto che  $e^{-j2\pi nq} = 1 \Rightarrow e^{-j2\pi nt/T_0} = e^{-j2\pi n(t+q/T_0)T_0}$

allora 
$$S_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \sum_{q=-\infty}^{+\infty} s(t+qT_0) e^{-j2\pi n(t+q/T_0)T_0} dt = \frac{1}{T_0} \sum_{q=-\infty}^{+\infty} \int_{-T_0/2}^{T_0/2} s(t+qT_0) e^{-j2\pi n(t+q/T_0)T_0} dt$$
 (si pone  $t'=t+qT_0$ )

$$= \frac{1}{T_0} \sum_{q=-\infty}^{+\infty} \int_{T_0(q-1/2)}^{T_0(q+1/2)} s(t') e^{-j2\pi nt'/T_0} dt' = \frac{1}{T_0} \int_{-\infty}^{+\infty} s(t') e^{-j2\pi nt'/T_0} dt'$$

se si confrontano

$$S_n = \frac{1}{T_0} \int_{-\infty}^{+\infty} p(t') e^{-j2\pi n t' / T_0} dt' \quad S(f) \hat{=} \int_{-\infty}^{+\infty} p(t) e^{-j2\pi f t} dt$$

si vede che i coefficienti  $S_n$  coincidono con  $S(f)$  per  $f = n f_0$ , moltiplicati  $1/T_0$

$$S_n = f_0 S(n f_0)$$

i coefficienti sono definiti per valori successivi della frequenza

distanti  $\Delta f = (n+1)f_0 - n f_0 = f_0 \Rightarrow S_n = S(n f_0) \Delta f$

Ritroviamo il segnale

$p(t)$

$$p(t) = \sum_{n=-\infty}^{+\infty} S_n e^{j2\pi n f_0 t} = \sum_{n=-\infty}^{+\infty} S(n f_0) \Delta f e^{j2\pi n f_0 t}$$

nel limite  $T_0 \rightarrow \infty$

$p(t) = p(t)$ , e la sommatoria si trasforma in un integrale

$$s(t) = \int_{-\infty}^{+\infty} S(f) e^{j2\pi f t} df$$

$p(t)$  è la somma di infinite funzioni esponenziali del tipo

$$S(f) df e^{j2\pi f t}$$

rappresentabili come vettori di ampiezza infinitesima  $|S(f)| df$

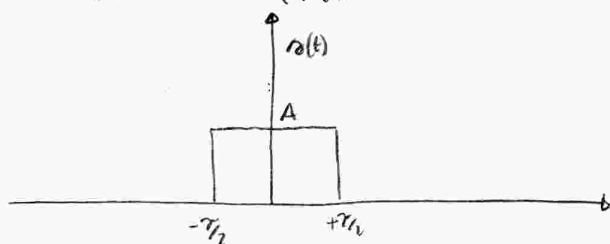
fase iniziale  $\theta(f) = \angle S(f)$  e ruotanti con velocità angolare  $\omega = 2\pi f$

Se si pone  $S(f) = |S(f)| e^{j\theta(f)}$

si ha che  $|S(f)|$  è lo spettro di ampiezza

$\theta(f)$  è lo spettro di fase

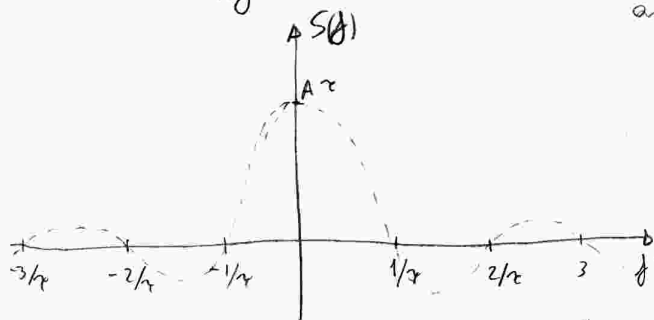
$$s(t) = A \text{rect}(t/\tau)$$



$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt = A \int_{-tau/2}^{tau/2} e^{-j2\pi ft} dt = A \left( \frac{1}{j2\pi f} \right) \left( e^{-j2\pi ft} \right) \Big|_{-tau/2}^{tau/2} =$$

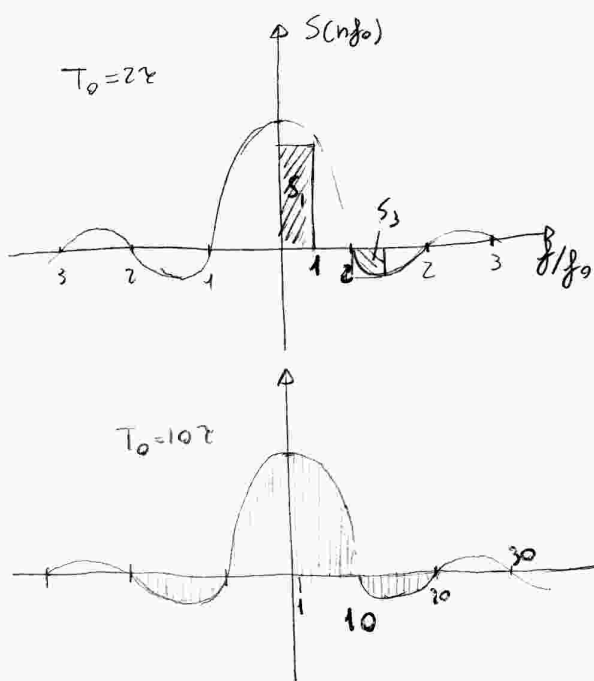
$$= A \frac{\sin \pi f \tau}{\pi f} = A \tau \text{sinc}(f\tau)$$

è reale, si può usare anche un solo grafico



Visto che  $S_n = f_0 S(nf_0)$  è formibile dal grafico ricavare i coefficienti della serie di fourier del segnale periodico, ottenuto ripetendo  $s(t)$

Vediamo il grafico normalizzato e consideriamo  $T_0 = 2\tau$



Per trovare i coefficienti  $S_n$  basta trovare le ordinate in  $\pm nf_0$  e moltiplicarli per  $\Delta f = f_0$  che è la distanza tra due righe successive.

al tendere di  $T_0$  all'infinito le righe si inghiottiscono

Cambiamento di scala

$$\alpha \in \mathbb{R}, \text{ costante} \quad s(t) \leftrightarrow S(f)$$

$$s(\alpha t) \leftrightarrow \frac{1}{|\alpha|} S(f/\alpha)$$

se indichiamo con  $\mathcal{F}_c(s(t))$  la Trasformata continua di  $s(t)$

$$\mathcal{F}_c(s(t)) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt$$

se si pone  $z = \alpha t$ , con  $\alpha > 0 \Rightarrow \mathcal{F}_c(s(z)) = \frac{1}{\alpha} \int_{-\infty}^{\infty} s(z) e^{-j2\pi f z/\alpha} dz = \frac{1}{\alpha} S(f/\alpha)$

per  $\alpha < 0$  si trova  $\mathcal{F}_c(s(\alpha t)) = -\frac{1}{\alpha} S(f/\alpha)$

NB per  $\alpha > 1$   $s(\alpha t)$  rappresenta una "compressione" nella scala temporale

Quindi ad una compressione nella scala dei tempi corrisponde un'espansione nel dominio temporale.

Teorema del ritardo

Se il segnale  $s(t)$  ammette TCF  $S(f)$ , il segnale  $s(t-t_0)$  ha come trasformata  $S(f) e^{-j2\pi f t_0}$

$$s(t-t_0) \leftrightarrow S(f) e^{-j2\pi f t_0}$$

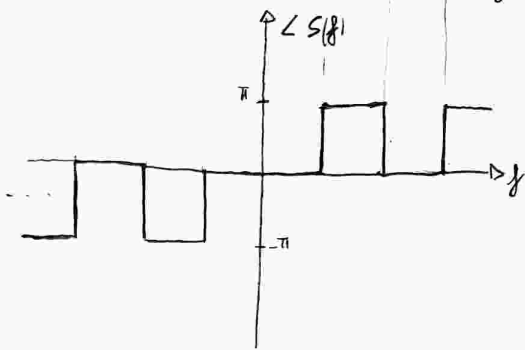
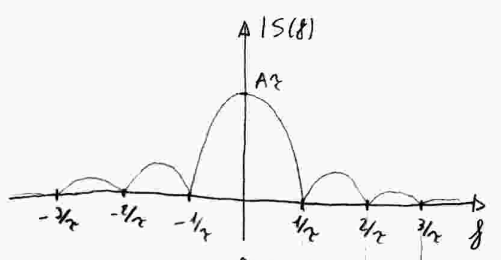
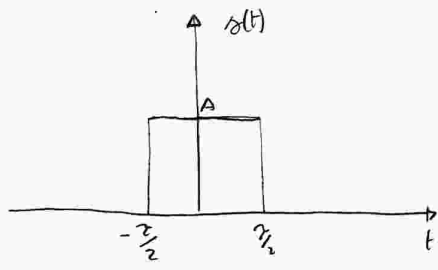
DM

$$\mathcal{F}_c[s(t-t_0)] = \int_{-\infty}^{\infty} s(t-t_0) e^{-j2\pi f t} dt = \text{(si pone } t-t_0=x) = \int_{-\infty}^{\infty} s(x) e^{-j2\pi f (x+t_0)} dx = S(f) e^{-j2\pi f t_0}$$

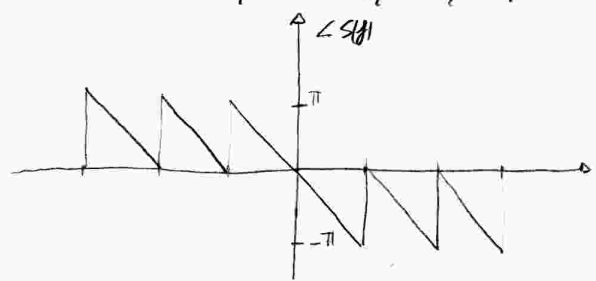
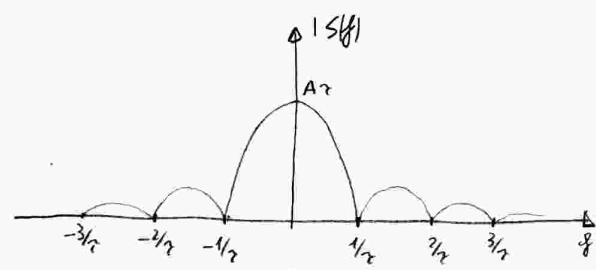
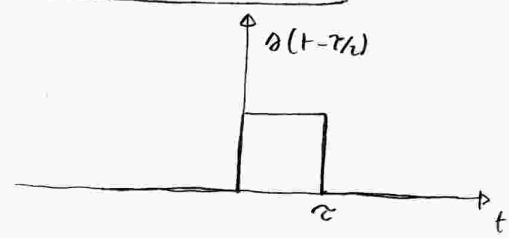
La traslazione temporale non altera il contenuto frequenziale del segnale ma ogni componente viene traslata di  $t_0$  corrispondente ad uno sfasamento di  $-2\pi f t_0$

$$\varphi(t) = A \operatorname{rect}\left(\frac{t}{\tau}\right)$$

$$S(f) = A\tau \operatorname{sinc}(f\tau)$$

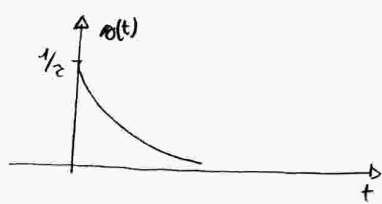


$$\varphi(t - \tau/2) \Leftrightarrow S(f) e^{-j2\pi f \tau/2}$$



esempio

$$\varphi(t) = \begin{cases} 1/2 e^{-t/\tau} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

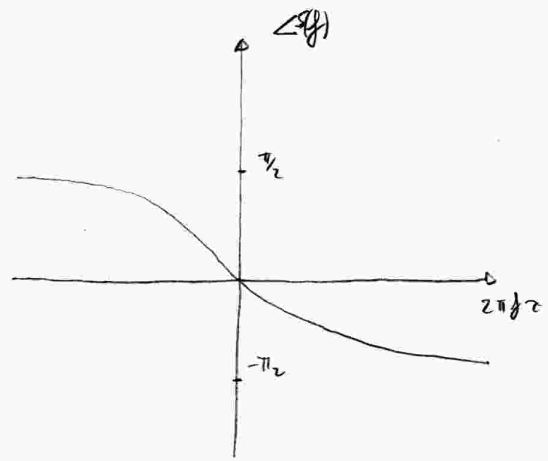
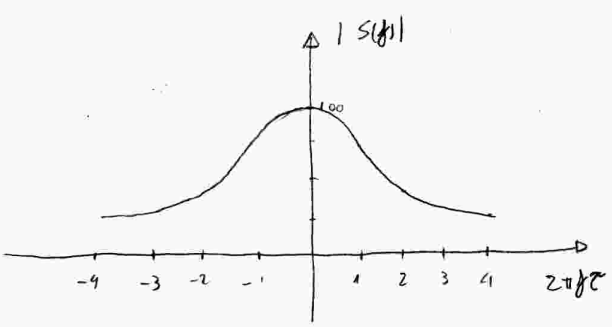


$$S(f) = \int_{-\infty}^{\infty} \varphi(t) e^{-j2\pi f t} dt = \int_0^{\infty} 1/2 e^{-t/\tau} e^{-j2\pi f t} dt = \frac{1}{\tau} \int_0^{\infty} e^{-t(1/\tau + j2\pi f)} dt = \frac{1}{\tau} \left( \frac{1}{1/\tau + j2\pi f} \right) e^{-t(1/\tau + j2\pi f)} \Big|_{t=0}^{t=\infty} =$$

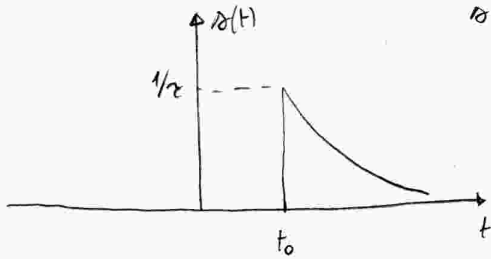
$$= \frac{1}{1 + j2\pi f \tau}$$

$$|S(f)| = \frac{1}{[1 + (2\pi f \tau)^2]^{1/2}}$$

$$\angle S(f) = -\arctan(2\pi f \tau)$$

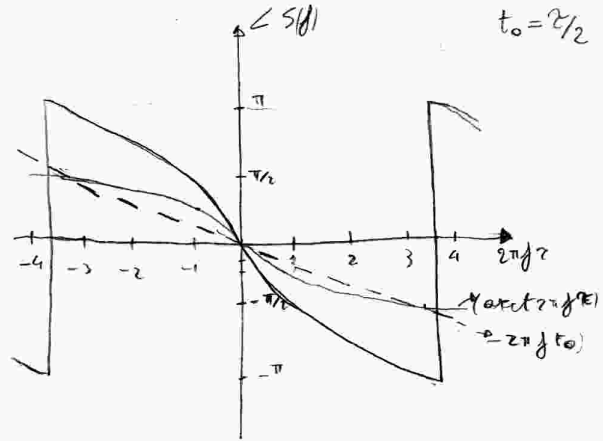
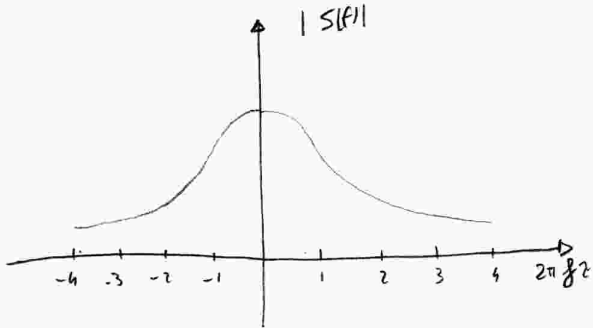


$$s(t) = \frac{1}{2} e^{-\frac{t-t_0}{2}} u(t-t_0)$$



$$\rightarrow \mathcal{F}_c[s(t)] = \frac{1}{1 + j2\pi f} \cdot e^{-j2\pi f t_0}$$

$$\angle S(f) = -\arctan(2\pi f) - 2\pi f t_0$$



- Derivazione (nel tempo)

$$s(t) \leftrightarrow S(f) \quad \Rightarrow \quad \frac{ds(t)}{dt} \leftrightarrow j2\pi f S(f)$$

D.M.  $\frac{ds(t)}{dt} = \frac{d}{dt} \left[ \int_{-\infty}^{\infty} S(f) e^{j2\pi f t} df \right] = \int_{-\infty}^{\infty} j2\pi f S(f) e^{j2\pi f t} df \neq$  antitrasformata di  $j2\pi f S(f)$

- Integrazione (nel tempo)

$$s(t) \leftrightarrow S(f) \quad \text{e} \quad S(0) = 0$$

$$\int_{-\infty}^t s(\alpha) d\alpha \leftrightarrow \frac{S(f)}{j2\pi f}$$

D.M.  $\varphi(t) = \int_{-\infty}^t s(\alpha) d\alpha \quad \text{e} \quad \mathcal{F}_c[\varphi(t)] = \Phi(f)$

$$s(t) = \frac{d\varphi(t)}{dt} \leftrightarrow j2\pi f \Phi(f) \quad \Rightarrow \quad S(f) = j2\pi f \Phi(f)$$

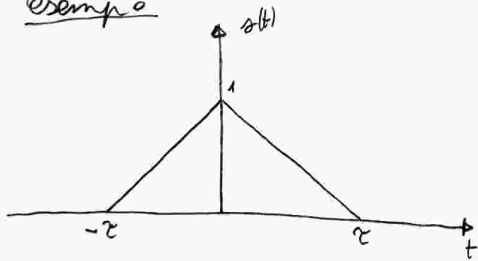
$$\Phi(f) = \frac{1}{j2\pi f} S(f)$$

visto che  $\Phi(f) \leftrightarrow \int_{-\infty}^t s(\alpha) d\alpha$

P.B. la condizione  $S(0) = \int_{-\infty}^{\infty} s(t) dt$  assicura che  $\lim_{t \rightarrow \infty} \varphi(t) = 0$ ; ciò è necessario per l'esistenza di  $\Phi(f)$ . Con l'integrazione della  $S(f)$  la condizione non è più necessaria.

esempio

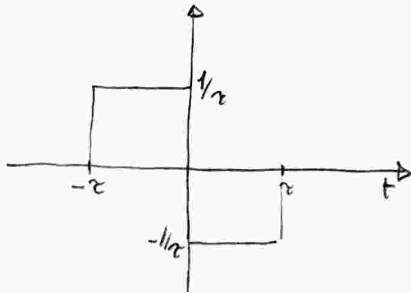
(19)



$$S(f) = ?$$

noi possiamo facilmente calcolare la derivata di  $s(t)$ , la cui trasformata è nota e poi applicare la regola di integrazione

$\dot{s}(t)$



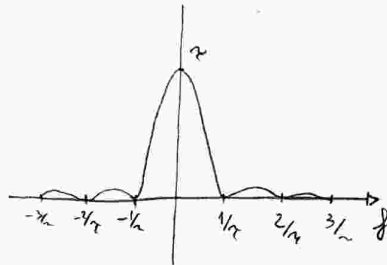
$$\dot{s}(t) = \frac{1}{2} \left[ \text{rect} \left( \frac{t+\tau}{2\tau} \right) - \text{rect} \left( \frac{t-\tau}{2} \right) \right]$$

$\Downarrow$

$$\begin{aligned} \mathcal{F}_c[\dot{s}(t)] &= \frac{1}{2} \left[ \tau \frac{\sin(\pi f \tau)}{\pi f \tau} e^{j2\pi f \frac{\tau}{2}} - \tau \frac{\sin \pi f \tau}{\pi f \tau} e^{-j2\pi f \tau} \right] \\ &= \text{sinc}(f\tau) (e^{j\pi f \tau} - e^{-j\pi f \tau}) = \end{aligned}$$

$$= \text{sinc}(f\tau) 2j \sin(\pi f \tau) = j 2\pi f \tau \text{sinc}^2(f\tau)$$

visto che  $s(t) = \int_{-\infty}^t \dot{s}(a) da \Rightarrow \mathcal{F}_c[s(t)] = \frac{1}{j2\pi f} \mathcal{F}_c[\dot{s}(t)] = \tau \text{sinc}^2(f\tau)$



- Traslazione in frequenza

$$s(t) \iff S(f)$$

$$s(t) e^{j2\pi f_0 t} \iff ?$$

$$\mathcal{F}_c [s(t) e^{j2\pi f_0 t}] = \int_{-\infty}^{\infty} s(t) e^{j2\pi f_0 t} e^{-j2\pi f t} dt = \int_{-\infty}^{\infty} s(t) e^{-j2\pi (f-f_0) t} dt = S(f-f_0)$$

$$s(t) e^{j2\pi f_0 t} \iff S(f-f_0)$$



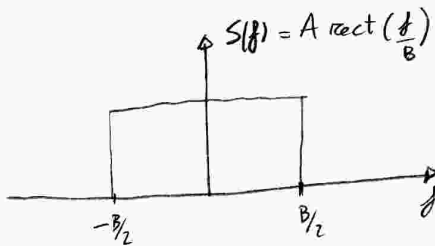
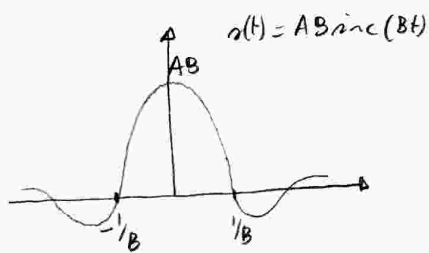
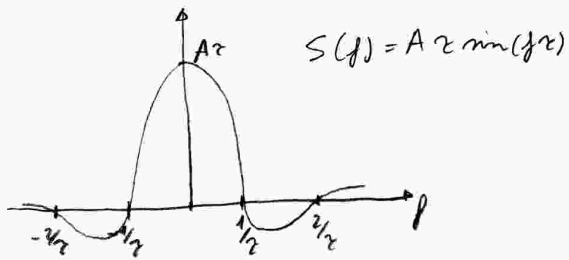
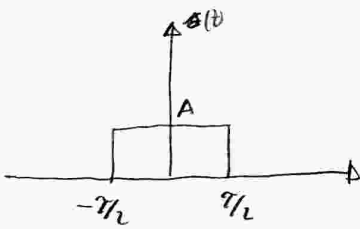
- Dualità

Se  $s(t)$  la come TCF  $S(f)$ , allora se si considera quest'ultima come funzione del tempo

$$S(t) \leftrightarrow s(-f)$$

$$s(t) = \int_{-\infty}^{\infty} S(f) e^{-j2\pi ft} df \quad \text{rango } f \equiv x$$

$$s(-t) = \int_{-\infty}^{\infty} S(x) e^{-j2\pi tx} dx \quad \text{e quindi } s(-f) = \int_{-\infty}^{\infty} S(x) e^{-j2\pi fx} dx$$



- convoluzione temporale

$x(t)$   $y(t)$  continui e di energia finita e tali che  $x(t) \iff X(f)$   
 $y(t) \iff Y(f)$

il segnale continuo  $z(t) = \int_{-\infty}^{\infty} x(\alpha) y(t-\alpha) d\alpha$

si indica con  $z(t) = x(t) \otimes y(t)$  ed è detto prodotto di convoluzione

allora

$$x(t) \otimes y(t) \iff X(f) Y(f)$$

DIM  $\mathcal{F}_c [x(t) \otimes y(t)] = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} x(\alpha) y(t-\alpha) d\alpha \right] e^{-j2\pi ft} dt = \int_{\alpha=-\infty}^{\infty} x(\alpha) \left[ \int_{t=-\infty}^{\infty} y(t-\alpha) e^{-j2\pi ft} dt \right] d\alpha =$

applicando il teorema del ritardo  $= \int_{\alpha=-\infty}^{\infty} x(\alpha) Y(f) e^{-j2\pi f\alpha} d\alpha = Y(f) \int_{\alpha=-\infty}^{\infty} x(\alpha) e^{-j2\pi f\alpha} d\alpha = Y(f) X(f)$

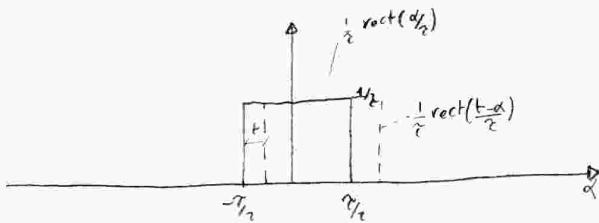
es antitrasformata di  $S(f) = \text{sinc}^2(f\tau)$   $\mathcal{F}_c^{-1}[S(f)] = ?$

$S(f)$  può essere scritta come  $S(f) = \text{sinc}(f\tau) \cdot \text{sinc}(f\tau)$  con  $X(f) = Y(f) = \text{sinc}(f\tau)$

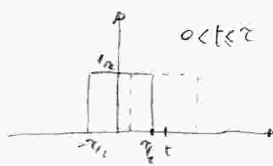
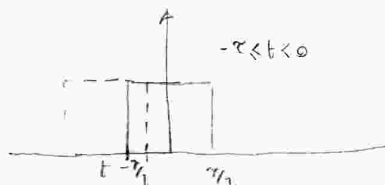
da cui  $z(t) = \int_{-\infty}^{\infty} x(\alpha) y(t-\alpha) d\alpha$

ma  $X(f) \iff \frac{1}{\tau} \text{rect}\left(\frac{f}{\tau}\right) \Rightarrow x(t) = \mathcal{F}_c^{-1}[X(f)] = \frac{1}{\tau} \text{rect}\left(\frac{t}{\tau}\right)$

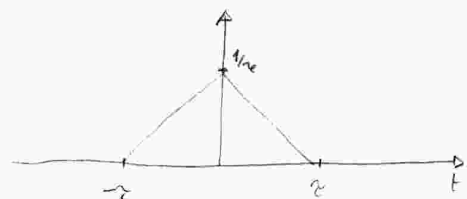
$$z(t) = \int_{-\infty}^{\infty} \frac{1}{\tau} \cdot \text{rect}\left(\frac{\alpha}{\tau}\right) \cdot \frac{1}{\tau} \text{rect}\left(\frac{t-\alpha}{\tau}\right) d\alpha$$



$$z(t) = \begin{cases} 0 & \text{per } t < -\tau \\ \frac{1}{\tau^2} \cdot \left[ t + \frac{\tau}{2} - \left(-\frac{\tau}{2}\right) \right] = \frac{1}{\tau} \left[ 1 + \frac{t}{\tau} \right] & -\tau \leq t < 0 \\ \frac{1}{\tau^2} \cdot \left[ \tau - \left(t - \frac{\tau}{2}\right) \right] = \frac{1}{\tau} \left[ 1 - \frac{t}{\tau} \right] & 0 < t \leq \tau \\ 0 & \text{per } t > \tau \end{cases}$$



$$z(t) = \begin{cases} \frac{1}{\tau} \left[ 1 - \frac{|t|}{\tau} \right] & |t| \leq \tau \\ 0 & \text{per } |t| > \tau \end{cases}$$



### Funzione generalizzata (o impulsiva di Dirac) $\delta(t)$

$$\int_{-\infty}^{\infty} f(t) \delta(t-t_0) dt = \int_{-\infty}^{\infty} f(t) \delta(t_0-t) dt = f(t_0)$$

N.B.

$$\int_{-\infty}^{\infty} \delta(t-t_0) dt = 1$$

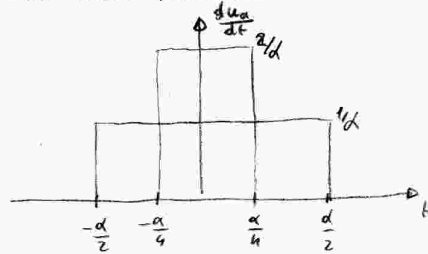
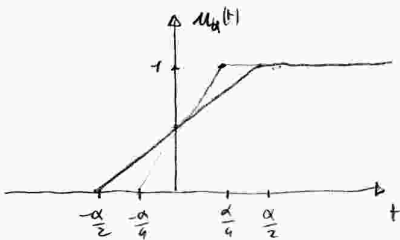
- Se  $f(t)$  è un impulso rettangolare centrato in  $t_0$ , di ampiezza  $A$  e durata  $2\epsilon$

$$\int_{-\infty}^{\infty} A \text{rect}\left(\frac{t-t_0}{2\epsilon}\right) \delta(t-t_0) dt = A \int_{t_0-\epsilon}^{t_0+\epsilon} \delta(t-t_0) dt = A$$

Quindi si ha

$$\int_{-\infty}^t \delta(\tau) d\tau = u(t) \quad \Rightarrow \quad \frac{d}{dt} u(t) = \delta(t)$$

- La relazione precedente va intesa nel limite



nel limite  $\alpha > 0$  la  $\frac{du_\alpha}{dt}$  tende alla  $\delta(t)$

- Se si pone  $f(t) = e^{-j2\pi ft}$

$$\int_{-\infty}^{\infty} e^{-j2\pi ft} \delta(t) dt = 1 \quad \Rightarrow \quad \Delta(f) = 1 \quad \text{risultato che la relazione precedente equivale alla } \mathcal{F}_c[\delta(t)]$$

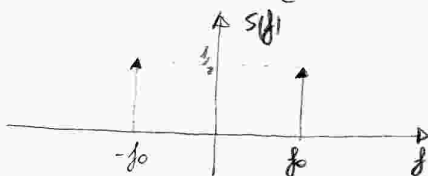
Una volta definita la funzione generalizzata  $\delta(t)$  si può esprimere la TCF di alcuni segnali a potenza media finita.

a)  $A \Leftrightarrow A \delta(f)$  deriva dalla proprietà di simmetria

b)  $e^{j2\pi f_0 t} \Leftrightarrow \delta(f-f_0)$  deriva dalla traslazione in frequenza

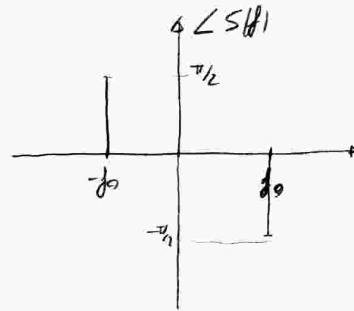
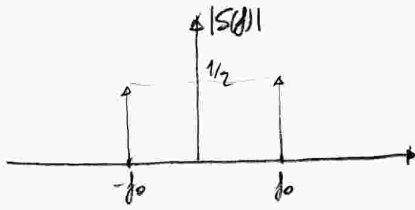
c)  $x(t) = \cos 2\pi f_0 t$

$$\mathcal{F}_c[x(t)] = \mathcal{F}_c\left[\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right] = \frac{1}{2} [\delta(f-f_0) + \delta(f+f_0)]$$



$$d) \quad s(t) = \sin(2\pi f_0 t)$$

$$\mathcal{F}_c[\sin(2\pi f_0 t)] = \mathcal{F}_c\left[\frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j}\right] = \frac{1}{2j} [\delta(f - f_0) - \delta(f + f_0)]$$

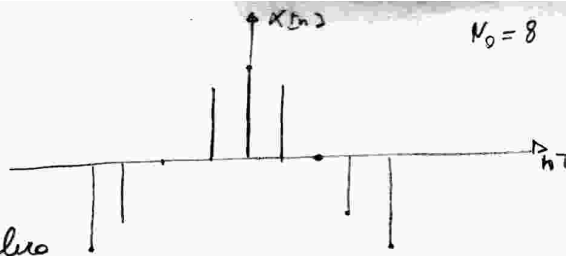


# Esercizi sulla TDF

$N_0 = 8$

(2.9)

$$x[n] = \cos\left(\frac{2\pi n T}{N_0 T}\right)$$

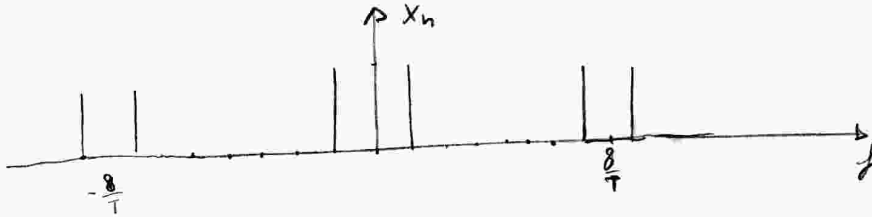


utilizzando le formule di Eulero

$$x[n] = \frac{1}{2} (e^{j2\pi n/N_0} + e^{-j2\pi n/N_0}) =$$

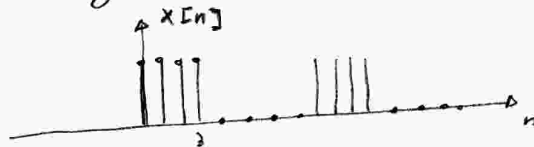
confrontando con 
$$x[n] = \sum_{k=0}^{N_0-1} \bar{X}_k e^{j\frac{2\pi k n}{N_0}}$$

si trova  $\bar{X}_1 = \frac{1}{2}$     $\bar{X}_{-1} = \frac{1}{2}$     $\bar{X}_n = 0$     $n = 0, \pm 2, \dots, \pm \frac{N_0}{2}$



-  $x[n]$  periodica di periodo  $N_0 = 8$  definita sul periodo come

$$x[n] = \begin{cases} 1 & 0 \leq n \leq 3 \\ 0 & 4 \leq n \leq 7 \end{cases}$$



$$\bar{X}_k = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n] e^{-j\frac{2\pi k n}{N_0}} = \frac{1}{8} \sum_{n=0}^3 e^{-j\frac{2\pi k n}{8}} = \frac{1}{8} \sum_{n=0}^3 (e^{-j\frac{\pi k}{4}})^n$$

si usa la formula

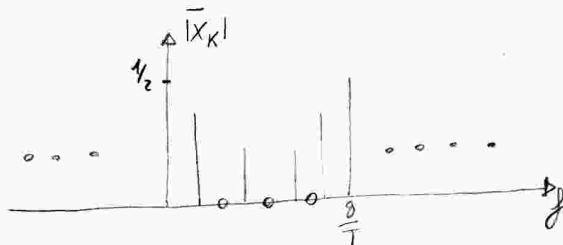
$$\sum_{n=0}^{N_0-1} a^n = \begin{cases} \frac{1-a^{N_0}}{1-a} & \text{per } a \neq 1 \\ N_0 & \text{per } a = 1 \end{cases}$$

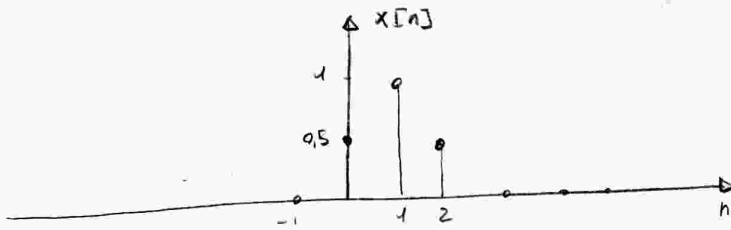
$$\bar{X}_k = \begin{cases} 1/2 & \text{per } k=0 \\ \frac{1}{8} \frac{1-e^{-j\pi k}}{1-e^{-j\pi k/4}} & k \neq 0 \end{cases}$$

$$\bar{X}_2 = \bar{X}_4 = \bar{X}_6 = 0$$

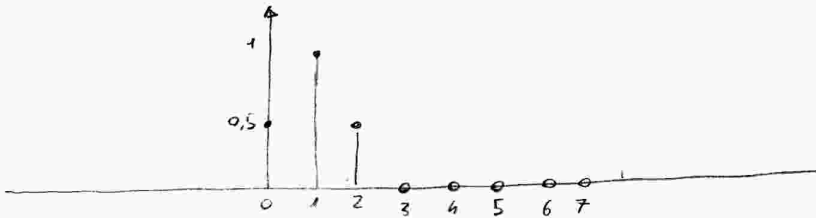
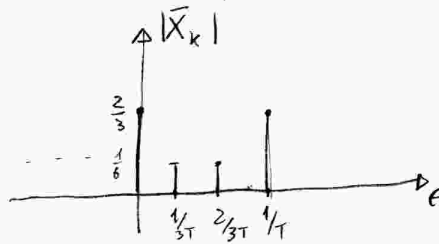
$$|\bar{X}_1| = |\bar{X}_7| = 1/8 \sin \pi/8 = 0,327$$

$$|\bar{X}_3| = |\bar{X}_5| = 1/8 \sin 3\pi/8 = 0,135$$





$$\begin{aligned} \bar{X}_K &= \frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n] e^{-j2\pi kn/N_0} = \frac{1}{3} [x[0] + x[1] e^{-j\frac{2\pi K}{3}} + x[2] e^{-j\frac{4\pi K}{3}}] = \\ &= \frac{1}{3} e^{-j\frac{2\pi K}{3}} \left[ \frac{1}{2} e^{j\frac{2\pi K}{3}} + 1 + \frac{1}{2} e^{-j\frac{2\pi K}{3}} \right] = \frac{1}{3} e^{-j\frac{2\pi K}{3}} [1 + \cos \frac{2\pi K}{3}] \quad K=0,1,\dots,16 \\ &\qquad\qquad\qquad \text{cos } \bar{\omega} \\ &\qquad\qquad\qquad K=0,1,2 \end{aligned}$$



$$\bar{X}_K = \frac{1}{8} e^{-j\frac{2\pi K}{8}} [1 + \cos \frac{2\pi K}{8}] \quad K=0, \dots, 7$$

