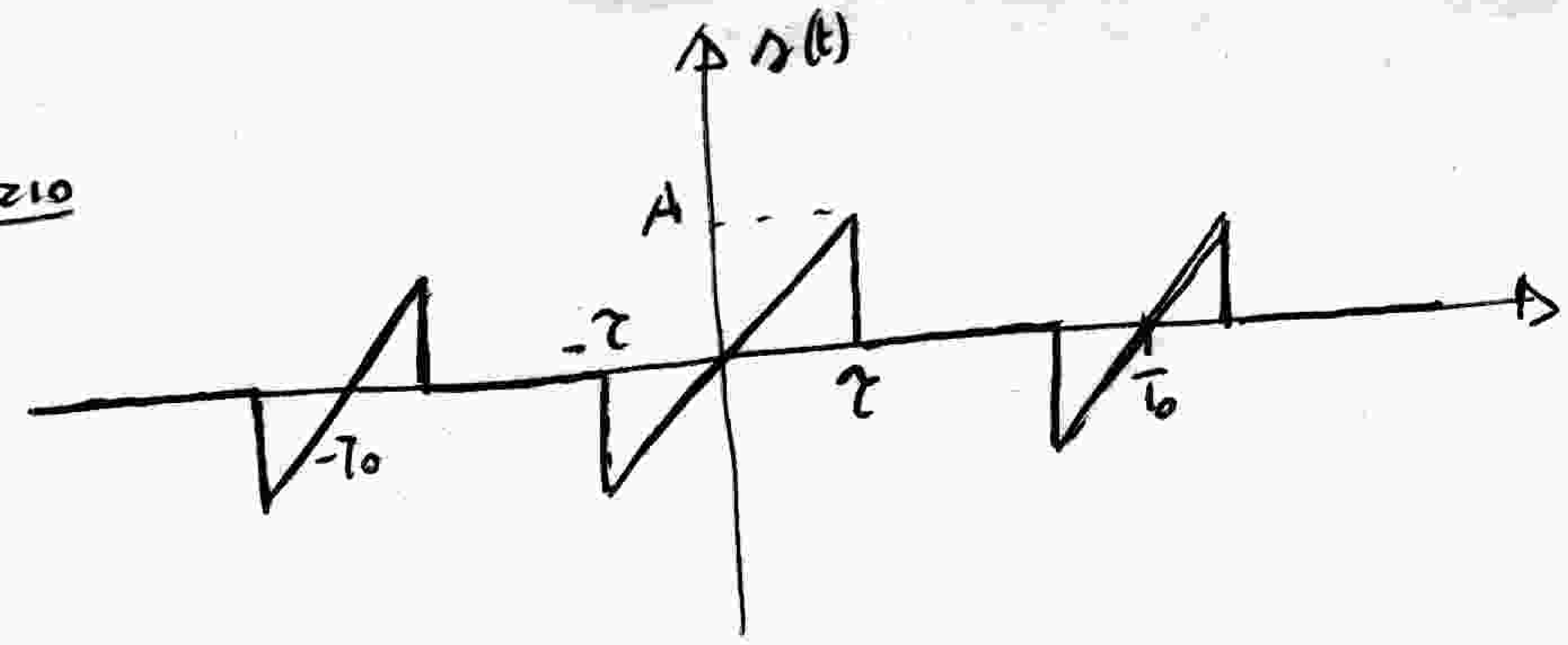
Esercizio



$s(t)$ dispari $\Rightarrow R_n = 0$

$$I_n = -\frac{2}{T_0}\int_0^{T_0/2} s(t)\sin 2\pi n\frac{t}{T_0}\,dt =$$

$$= -\frac{2}{T_0}\int_0^{\tau}\frac{At}{\tau}\sin 2\pi n\frac{t}{T_0}\,dt = -\frac{2A}{T_0\tau}\int_0^{\tau} t\sin 2\pi n\frac{t}{T_0}\,dt = -\frac{2A}{T_0\tau}\int_0^{\tau} t\,\frac{1}{\frac{2\pi n}{T_0}}\left(-\frac{d}{dt}\cos\frac{2\pi n t}{T_0}\right)dt$$

$$= +\frac{2A}{T_0\tau}\frac{1}{\frac{2\pi n}{T_0}}\left[t\cos\frac{2\pi n t}{T_0}\Big|_0^{\tau} - \int_0^{\tau}\cos\frac{2\pi n t}{T_0}\,dt\right] = \frac{A}{\pi n\tau}\left[\tau\cos\frac{2\pi n\tau}{T_0} - \right.$$

$$\left. -\frac{1}{\frac{2\pi n}{T_0}}\left[\sin\left(\frac{2\pi n t}{T_0}\right)\right]\Big|_0^{\tau}\right] = \frac{A}{\pi n\tau}\left[\tau\cos\frac{2\pi n\tau}{T_0} - \frac{T_0}{2\pi n}\sin\frac{2\pi n\tau}{T_0}\right]$$

se $\tau = T_0/4$

$$I_n = \frac{A4}{\pi n T_0}\left[\frac{T_0}{4}\cos\frac{\pi n}{2} - \frac{T_0}{2\pi n}\sin\frac{\pi n}{2}\right] = \frac{A}{\pi n}\cos\frac{\pi n}{2} - \frac{2A}{(\pi n)^2}\sin\frac{\pi n}{2}$$

$S_n = R_n + jI_n$

$S_0 = 0$

$S_1 = jI_1 = j\left(-\frac{2A}{\pi^2}\right) \Rightarrow S_{-1} = j\frac{2A}{\pi^2}$

$S_2 = jI_2 = j\cdot\left(-\frac{A}{2\pi}\right) \Rightarrow S_{-2} = j\frac{A}{2\pi}$

$S_3 = jI_3 = j\frac{2A}{9\pi^2} \qquad S_{-3} = -j\frac{2A}{9\pi^2}$

$S_4 = jI_4 = j\frac{A}{4\pi} \qquad S_{-4} = -j\frac{A}{4\pi}$

$\vdots$

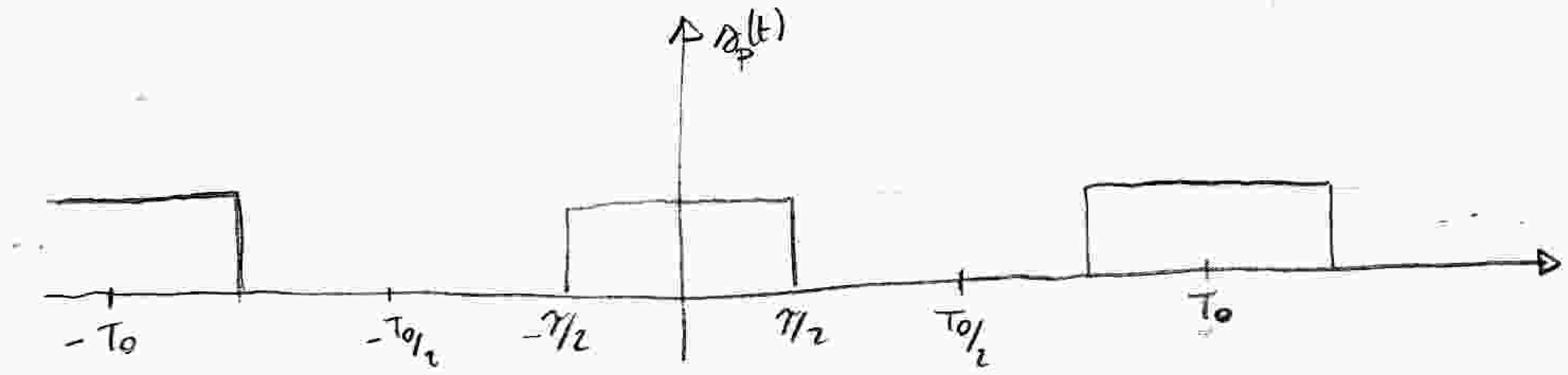
Trasformata continua di Fourier

Un segnale aperiodico $x(t)$, si può rappresentare come la sovrapposizione di componenti sinusoidali di ampiezza infinitesima e di frequenza variabile con continuità tra $-\infty$ e $+\infty$.

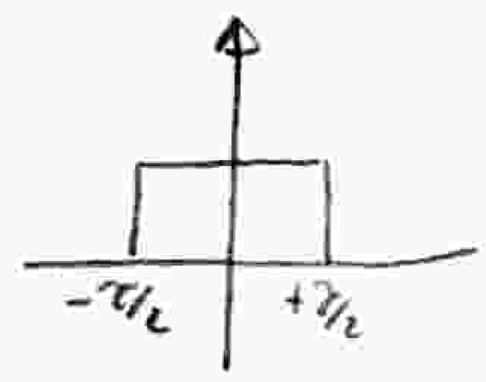
$$x(t) = \int_{-\infty}^{+\infty} X(f)\, e^{j2\pi f t}\, df$$

Si può dimostrare in modo intuitivo, andando a vedere come cambia lo spettro di un segnale periodico quando il periodo viene fatto tendere all'infinito, cioè diviene aperiodico.

Se consideriamo un treno di impulsi rettangolari



questo può essere scritto come $s_p(t) = \sum_{n=-\infty}^{\infty} rect\left(\frac{t-nT_0}{\tau}\right)$ dove $rect\left(\frac{t}{\tau}\right)$



nel limite $T_0 \to \infty$ i vari impulsi si allontanano e si ritrova il segnale

$$s(t) = rect\left(\frac{t}{\tau}\right) = \lim_{T_0 \to \infty} s_p(t)$$

Vediamo cosa succede ai coefficienti della serie di Fourier al tendere di T_0 all'infinito.

$$S_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} s_p(t)\, e^{-j2\pi n t/T_0}\, dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \sum_{q=-\infty}^{+\infty} s(t+qT_0)\, e^{-j2\pi n t/T_0}\, dt$$

visto che $e^{-j2\pi n q} = 1 \Rightarrow e^{-j2\pi n t/T_0} = e^{-j2\pi n (t+q/T_0) T_0}$

allora $S_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \sum_{q=-\infty}^{+\infty} s(t+qT_0)\, e^{-j2\pi n (t+q/T_0)T_0}\, dt = \frac{1}{T_0} \sum_{q=-\infty}^{+\infty} \int_{-T_0/2}^{T_0/2} s(t+qT_0)\, e^{-j2\pi n (t+qT_0)/T_0}\, dt =$
(si pone $t' = t+qT_0$)

$$= \frac{1}{T_0} \sum_{q=-\infty}^{+\infty} \int_{T_0(q-1/2)}^{T_0(q+1/2)} s(t')\, e^{-j2\pi n t'/T_0}\, dt' = \frac{1}{T_0} \int_{-\infty}^{+\infty} s(t')\, e^{-j2\pi n t'/T_0}\, dt'$$

se si confrontano

$$S_n = \frac{1}{T_0}\int_{-\infty}^{+\infty} s(t')\, e^{-j2\pi n t'/T_0}\, dt' \qquad S(f) \triangleq \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi f t}\, dt$$

si vede che i coefficienti S_n coincidono con $S(f)$ per $f = nf_0$, moltiplicati $1/T_0$

$$S_n = f_0\, S(nf_0)$$

i coefficienti sono definiti per valori successivi della frequenza

distanti $\Delta f = (n+1)f_0 - nf_0 = f_0 \Rightarrow S_n = S(nf_0)\,\Delta f$

Ritroviamo il segnale $s_p(t)$

$$s_p(t) = \sum_{n=-\infty}^{\infty} S_n\, e^{j2\pi n f_0 t} = \sum_{n=-\infty}^{\infty} S(nf_0)\,\Delta f\, e^{+j2\pi n f_0 t}$$

nel limite $T_0 \to \infty$

$s_p(t) = s(t)$, e la sommatoria si trasforma in un integrale

$$s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{j2\pi f t}\, df$$

$s(t)$ è la somma di infinite funzioni esponenziali del tipo

$$S(f)\, df\, e^{j2\pi f t}$$

rappresentabili come vettori di ampiezza infinitesima $|S(f)|df$

fase iniziale $\theta(f) = \angle S(f)$ e ruotanti con velocità angolare $\omega = 2\pi f$

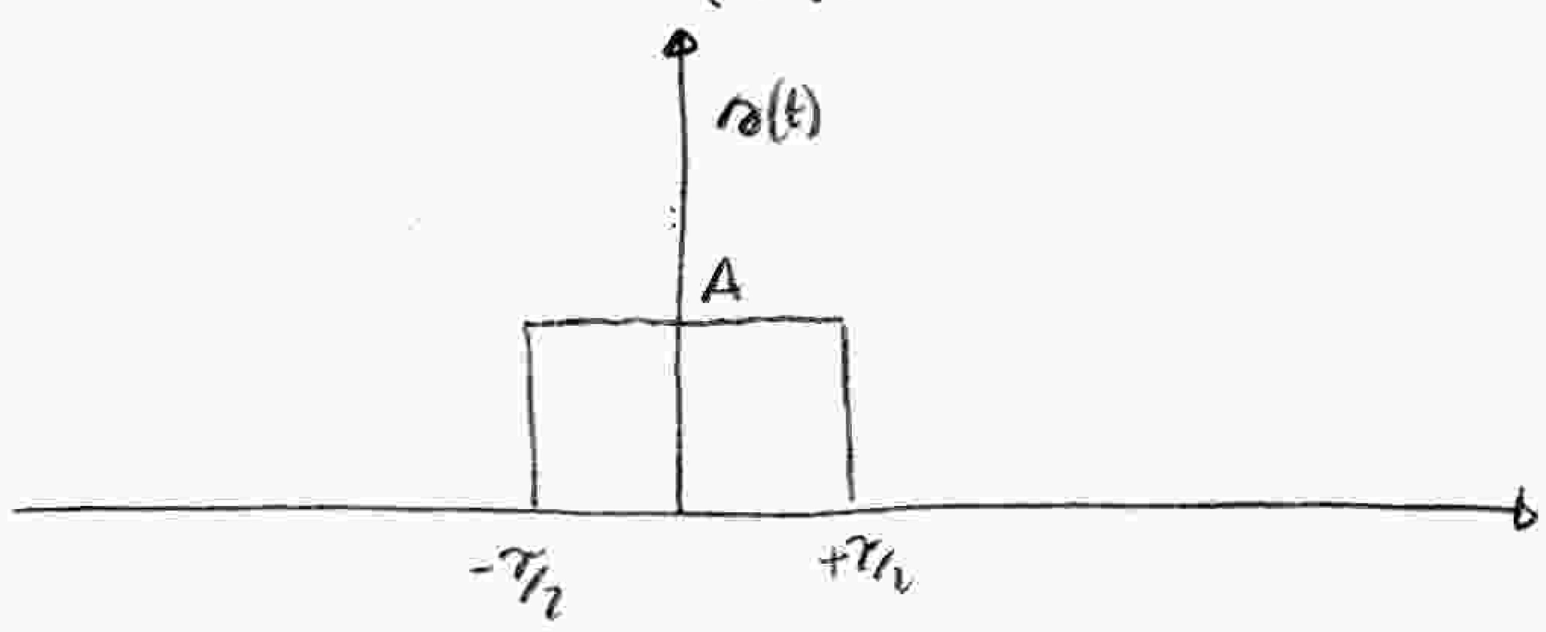
Se si pone $S(f) = |S(f)|\, e^{j\theta(f)}$

si ha che $|S(f)|$ è lo <u>spettro di ampiezza</u>

$\theta(f)$ è lo <u>spettro di fase</u>

Esempio

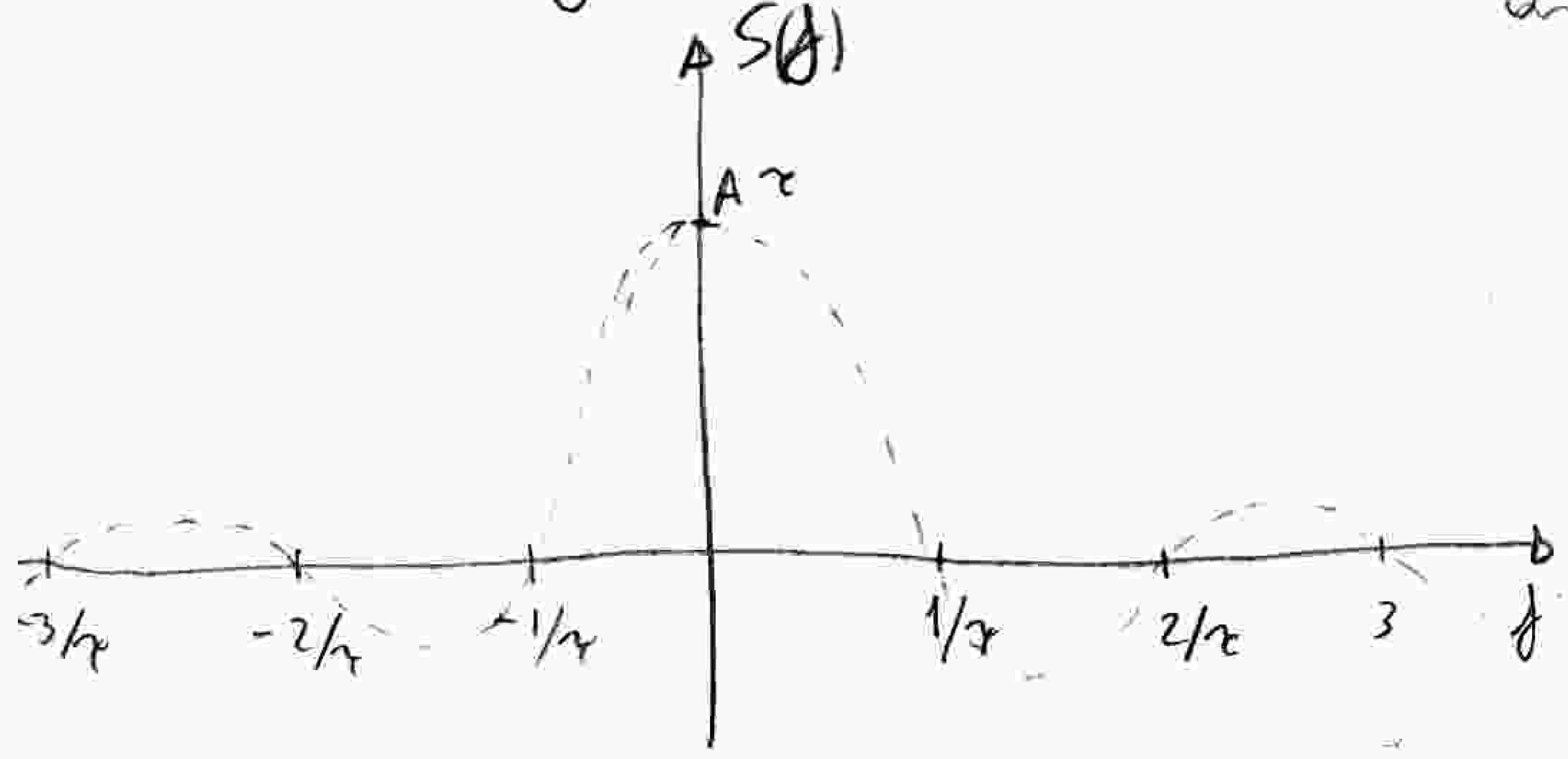
$$s(t) = A \operatorname{rect}(t/\tau)$$



$$S(f) = \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi f t}\, dt = A \int_{-\tau/2}^{\tau/2} e^{-j2\pi f t}\, dt = A \left(\frac{1}{-j2\pi f}\right)\left(e^{-j2\pi f t}\right)\Big|_{-\tau/2}^{\tau/2} =$$

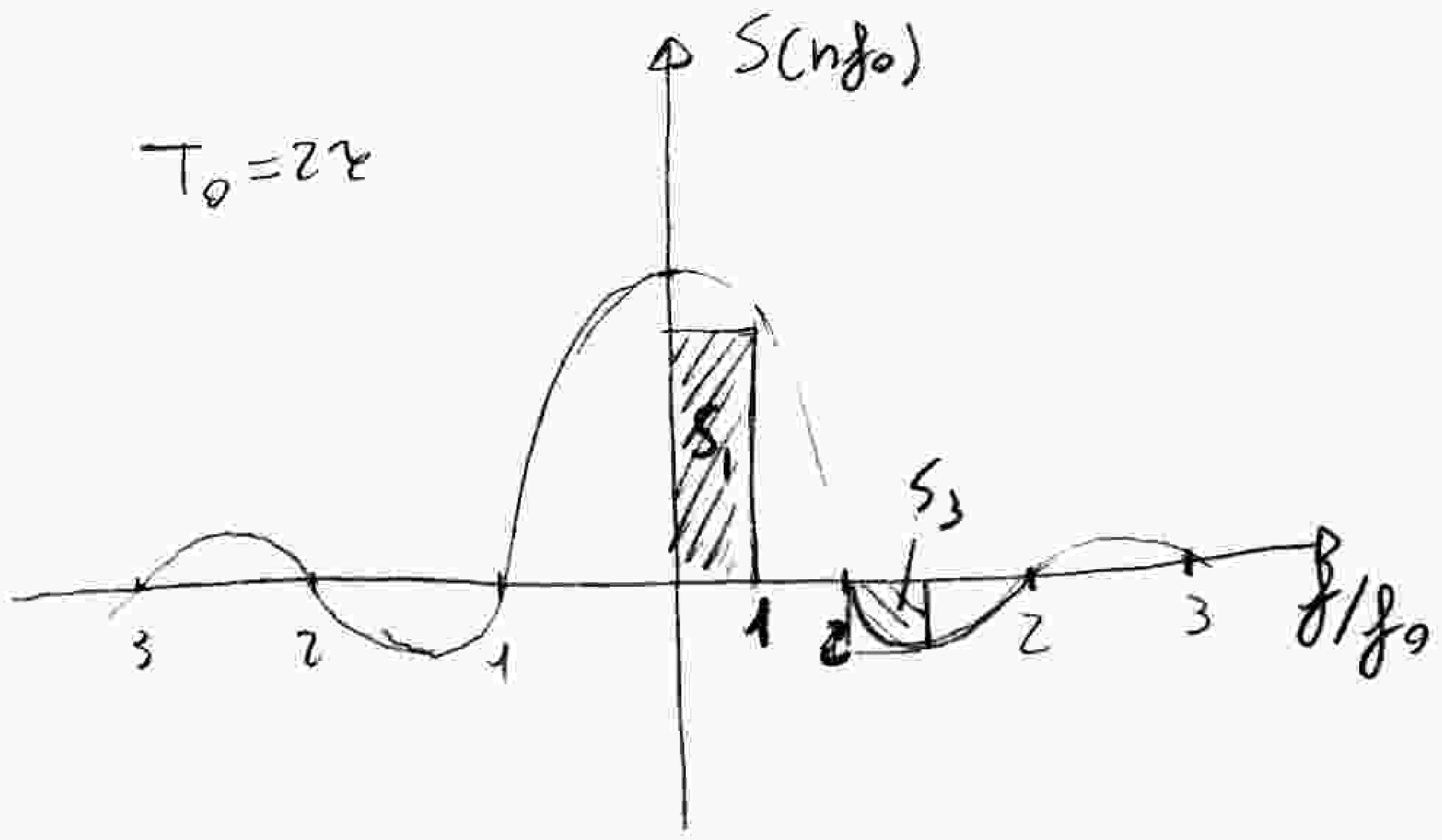
$$= A \frac{\sin \pi f \tau}{\pi f} = A\tau \operatorname{sinc}(f\tau)$$

è reale, si può usare anche un solo grafico

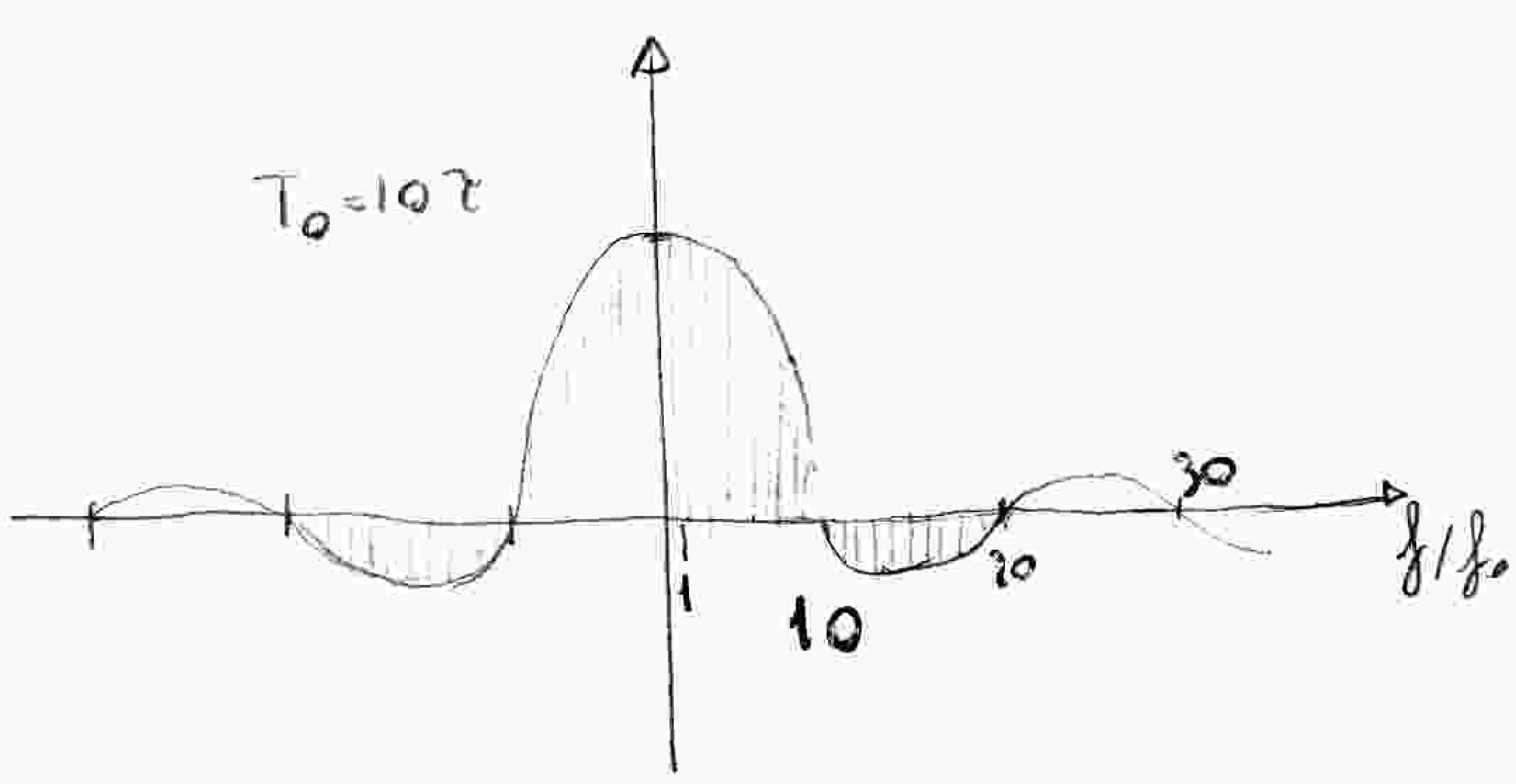


Visto che $S_n = f_0 S(nf_0)$ è possibile dal grafico ricavare i coefficienti della serie di Fourier del segnale periodico, ottenuto ripetendo $s(t)$

Vediamo il grafico normalizzato e consideriamo $T_0 = 2\tau$



Per trovare i coefficienti S_n basta tracciare le ordinate in $\pm nf_0$ e moltiplicarli per $\Delta f = f_0$ che è la distanza tra due righe successive.



al tendere di T_0 all'infinito le righe si infittiscono

Proprietà della Trasformata Continua di Fourier

- Cambiamento di scala

$\alpha \in \mathbb{R}$, costante $\qquad s(t) \Leftrightarrow S(f)$

$$s(\alpha t) \Leftrightarrow \frac{1}{|\alpha|} S(f/\alpha)$$

se indichiamo con $\mathcal{F}_c(s(t))$ la Trasformata continua di $s(t)$

$$\mathcal{F}_c(s(\alpha t)) = \int_{-\infty}^{\infty} s(\alpha t)\, e^{-j2\pi f t}\, dt$$

se si pone $\quad z = \alpha t$, con $\alpha > 0 \Rightarrow \mathcal{F}_c(s(z)) = \frac{1}{\alpha}\int_{-\infty}^{\infty} s(z)\, e^{-j2\pi f x/\alpha}\, dx = \frac{1}{\alpha} S(f/\alpha)$

per $\alpha < 0$ si trova $\quad \mathcal{F}_c(s(\alpha t)) = -\frac{1}{\alpha} S(f/\alpha)$

<u>NB</u> per $\alpha > 1$ $\quad s(\alpha t)$ rappresenta una "compressione" nella scala temporale
Quindi ad una compressione nella scala dei tempi corrisponde un espansione nel dominio temporale.

- Teorema del ritardo

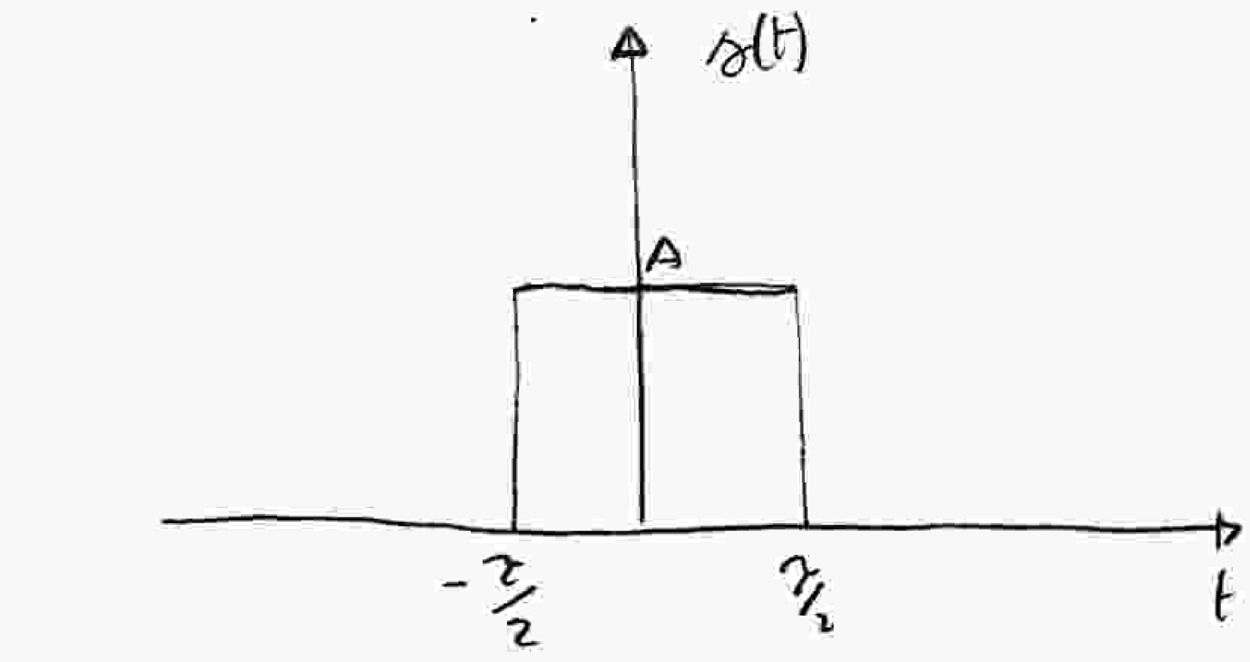
Se il segnale $s(t)$ ammette TCF $S(f)$, il segnale $s(t-t_0)$ ha come trasformata $S(f)\, e^{-j2\pi f t_0}$

$$s(t-t_0) \Leftrightarrow S(f)\, e^{-j2\pi f t_0}$$

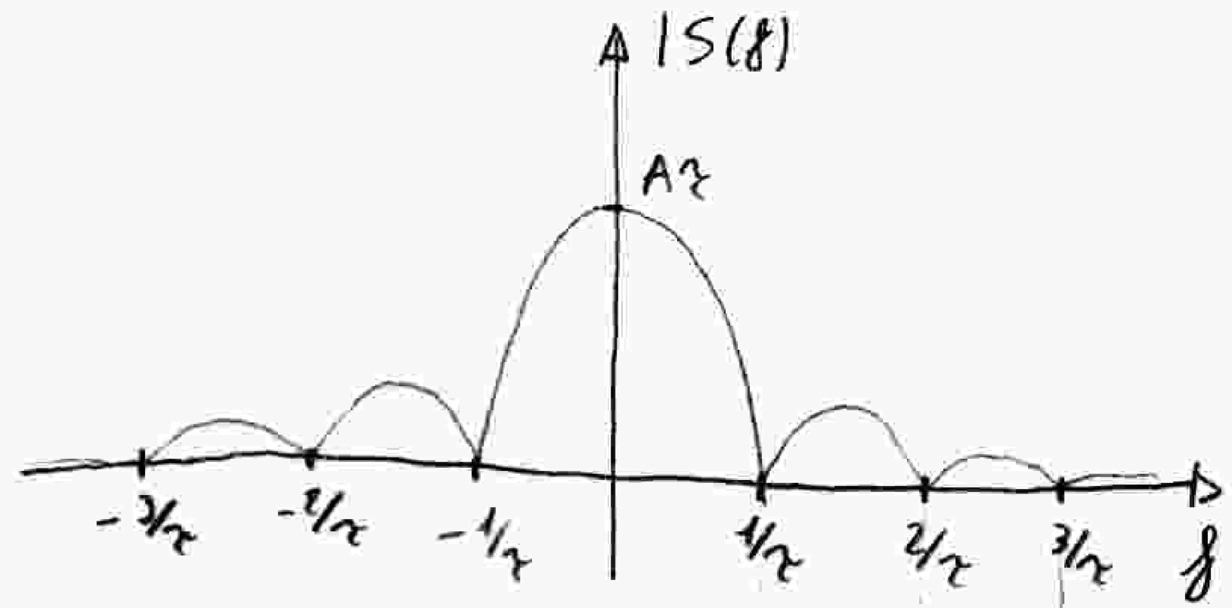
<u>DIM</u> $\quad \mathcal{F}_c[s(t-t_0)] = \int_{-\infty}^{\infty} s(t-t_0)\, e^{-j2\pi f t}\, dt =$ (si pone $t - t_0 = x$) $= \int_{-\infty}^{\infty} s(x)\, e^{-j2\pi f (x+t_0)}\, dx =$

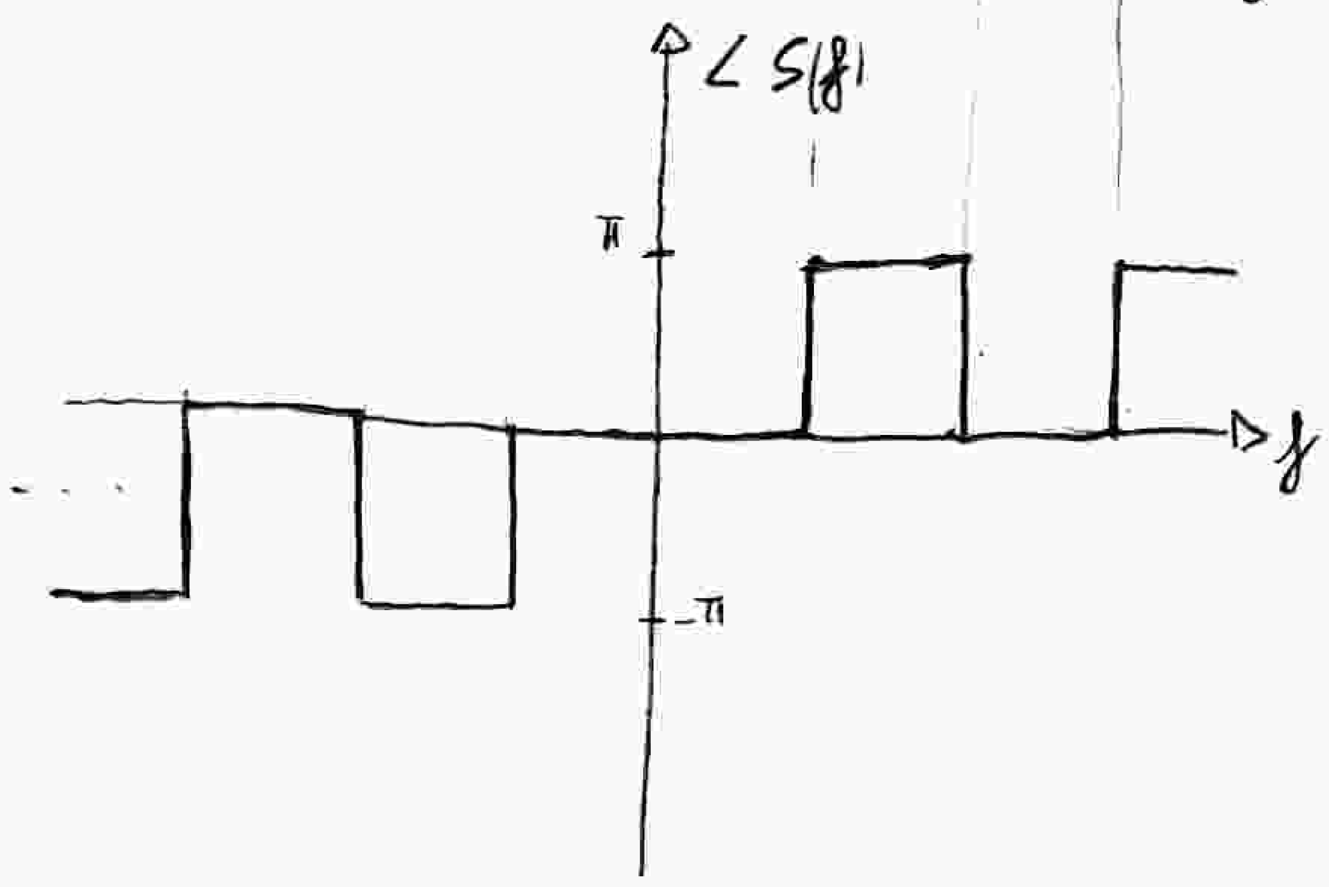
$$= S(f)\, e^{-j2\pi f t_0}$$

La traslazione temporale non altera il contenuto frequenziale del segnale ma ogni componente viene traslata di t_0 corrispondente ad uno sfasamento di $-2\pi f t_0$

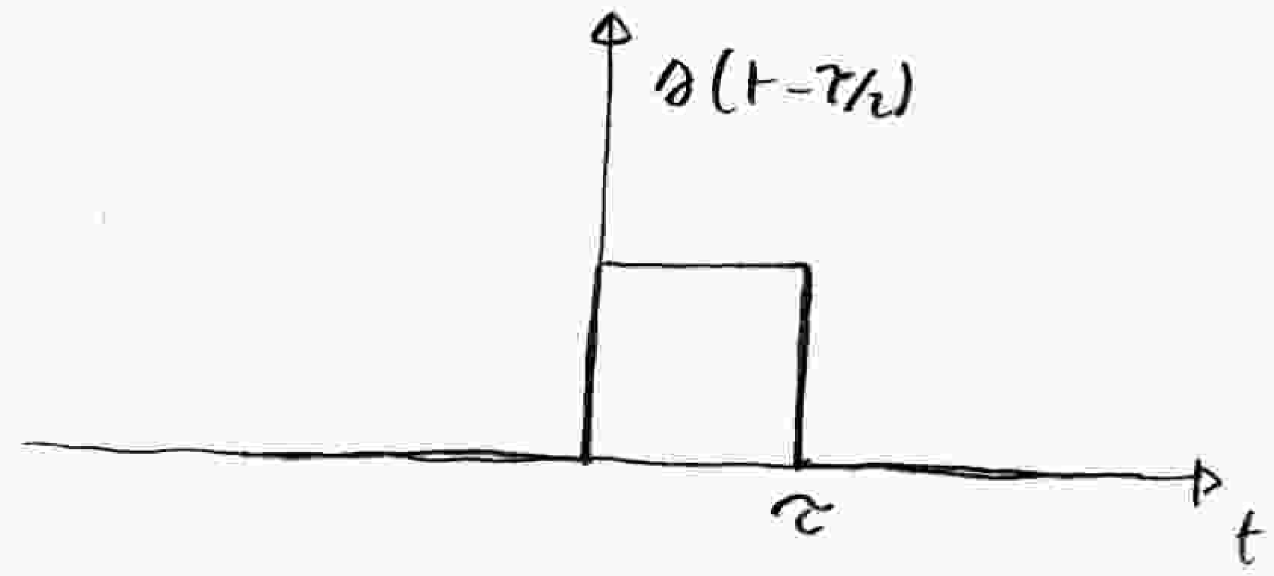


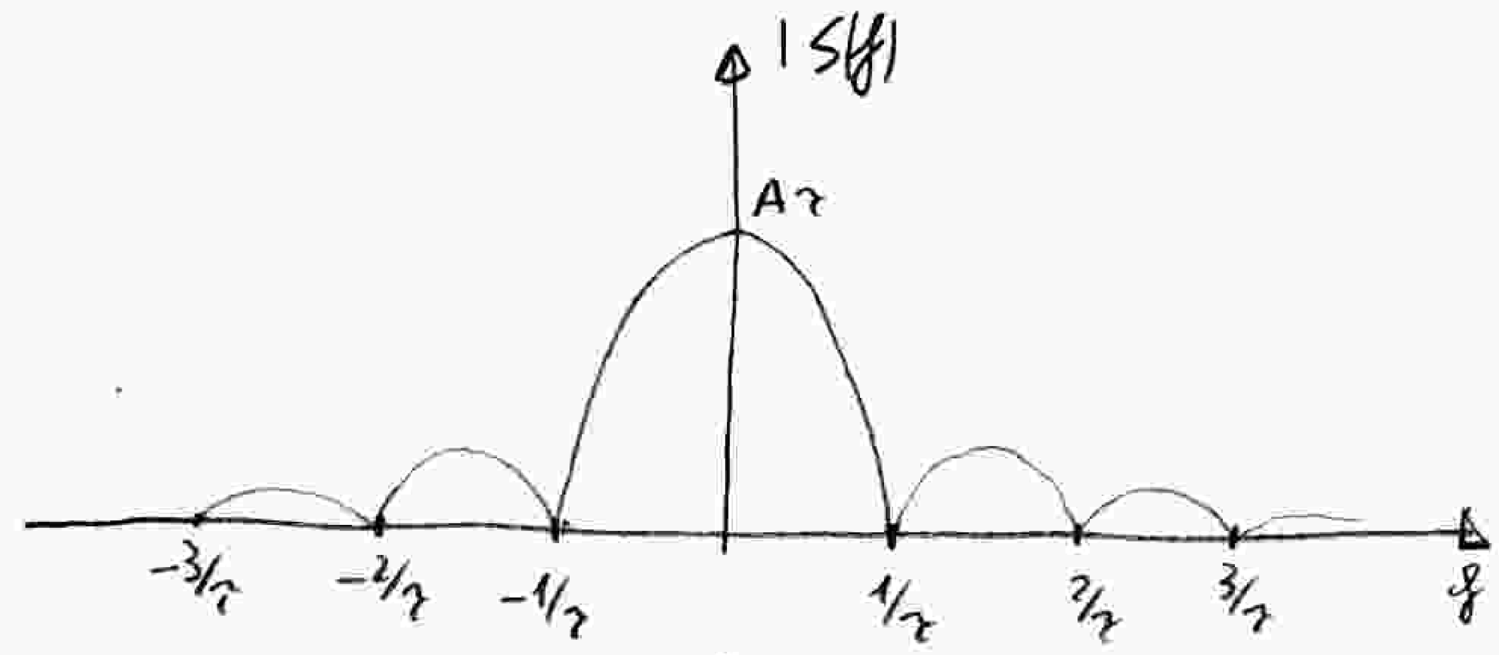
$$\boxed{\begin{aligned} s(t) &= A\,\mathrm{rect}\left(\frac{t}{\tau}\right) \\ S(f) &= A\tau\,\mathrm{sinc}(f\tau) \end{aligned}}$$

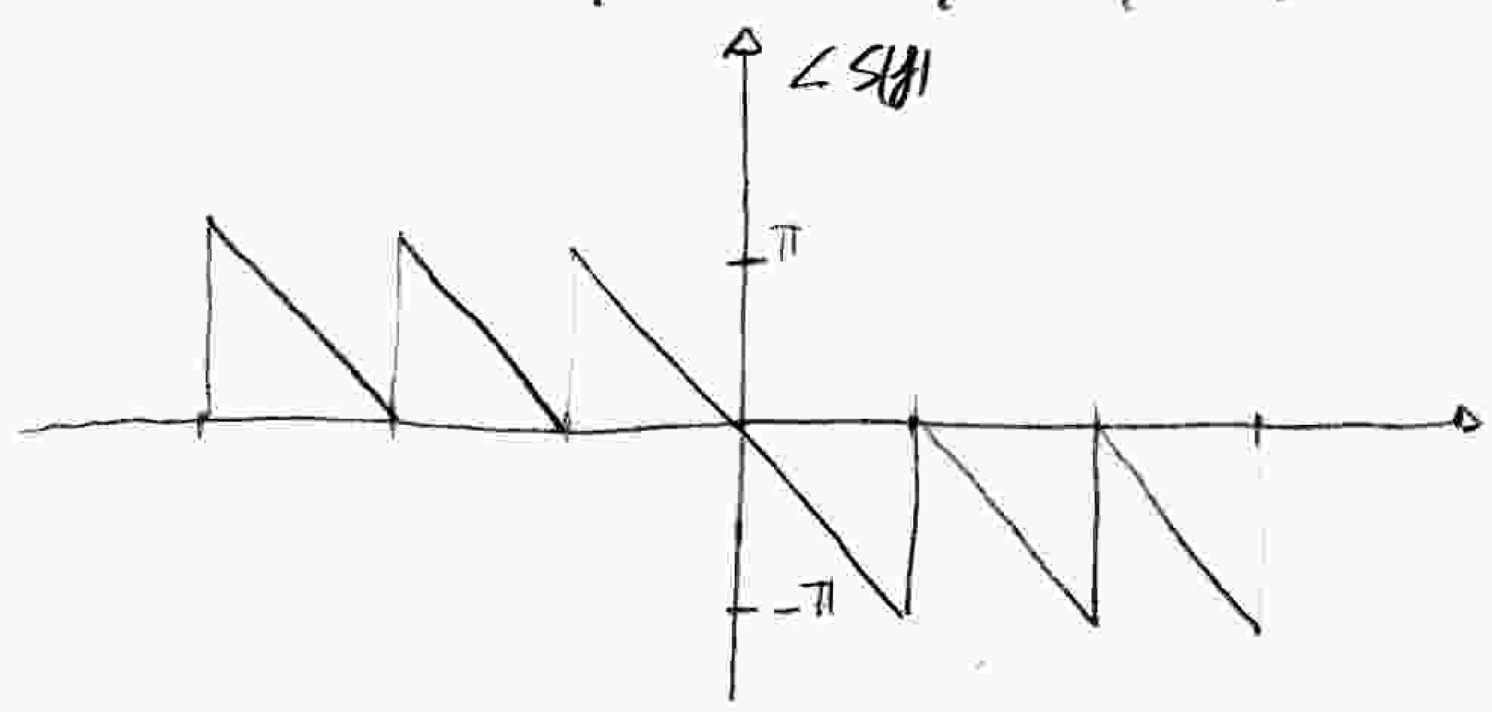




$$\underline{s(t-\tau/2) \Longleftrightarrow S(f)\,e^{-j2\pi f\frac{\tau}{2}}}$$

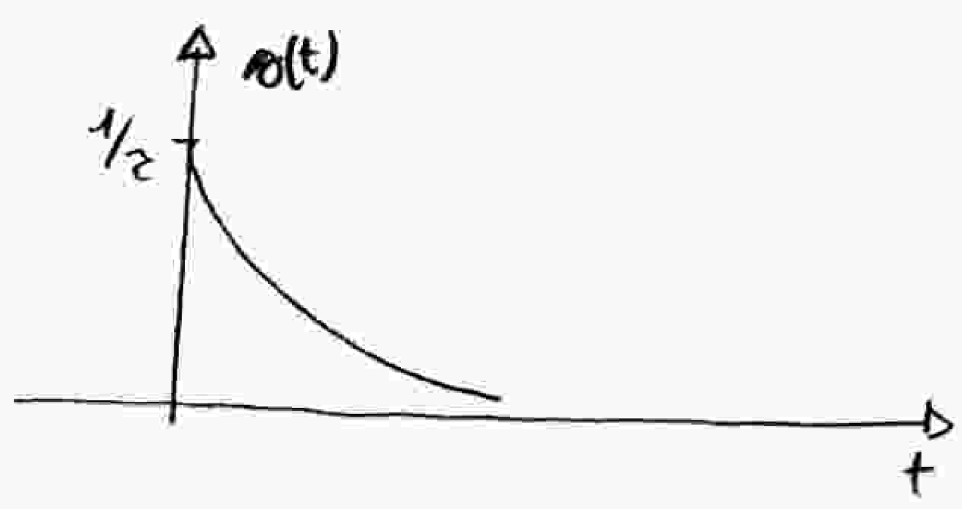






<u>esempio</u>

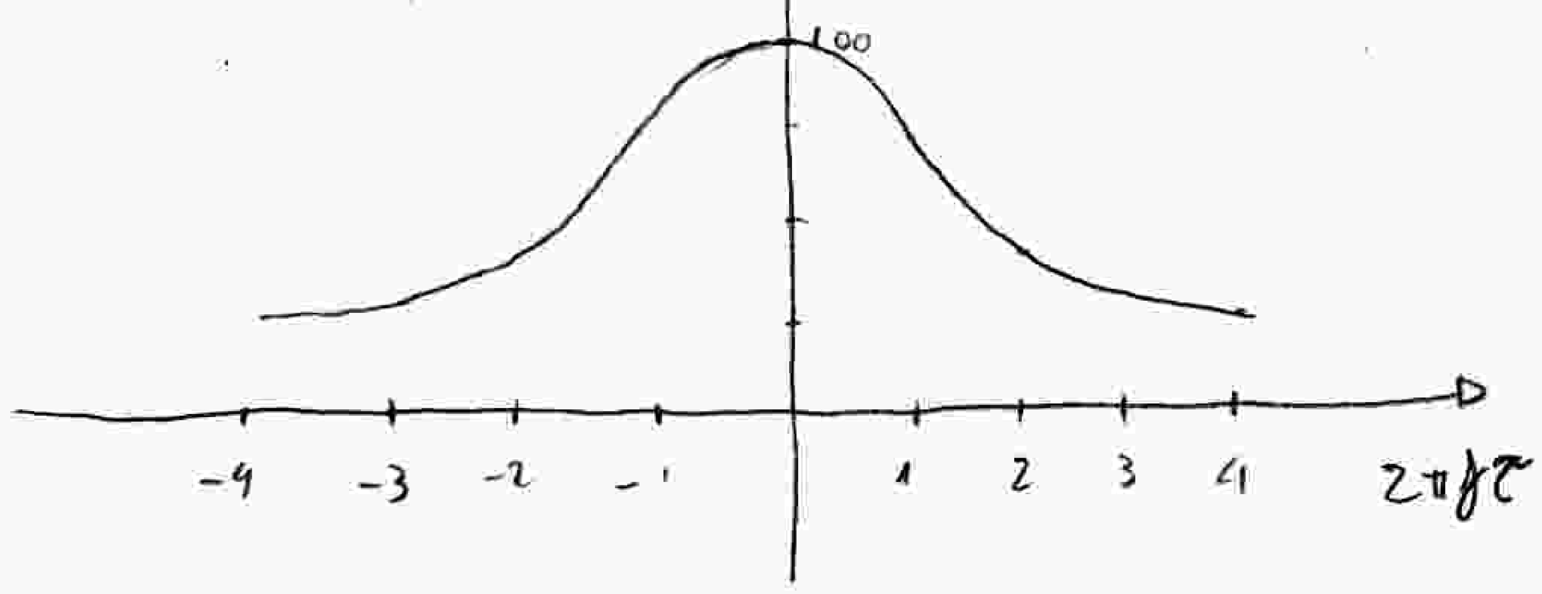
$$s(t) = \begin{cases} 1/\tau\, e^{-t/\tau} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

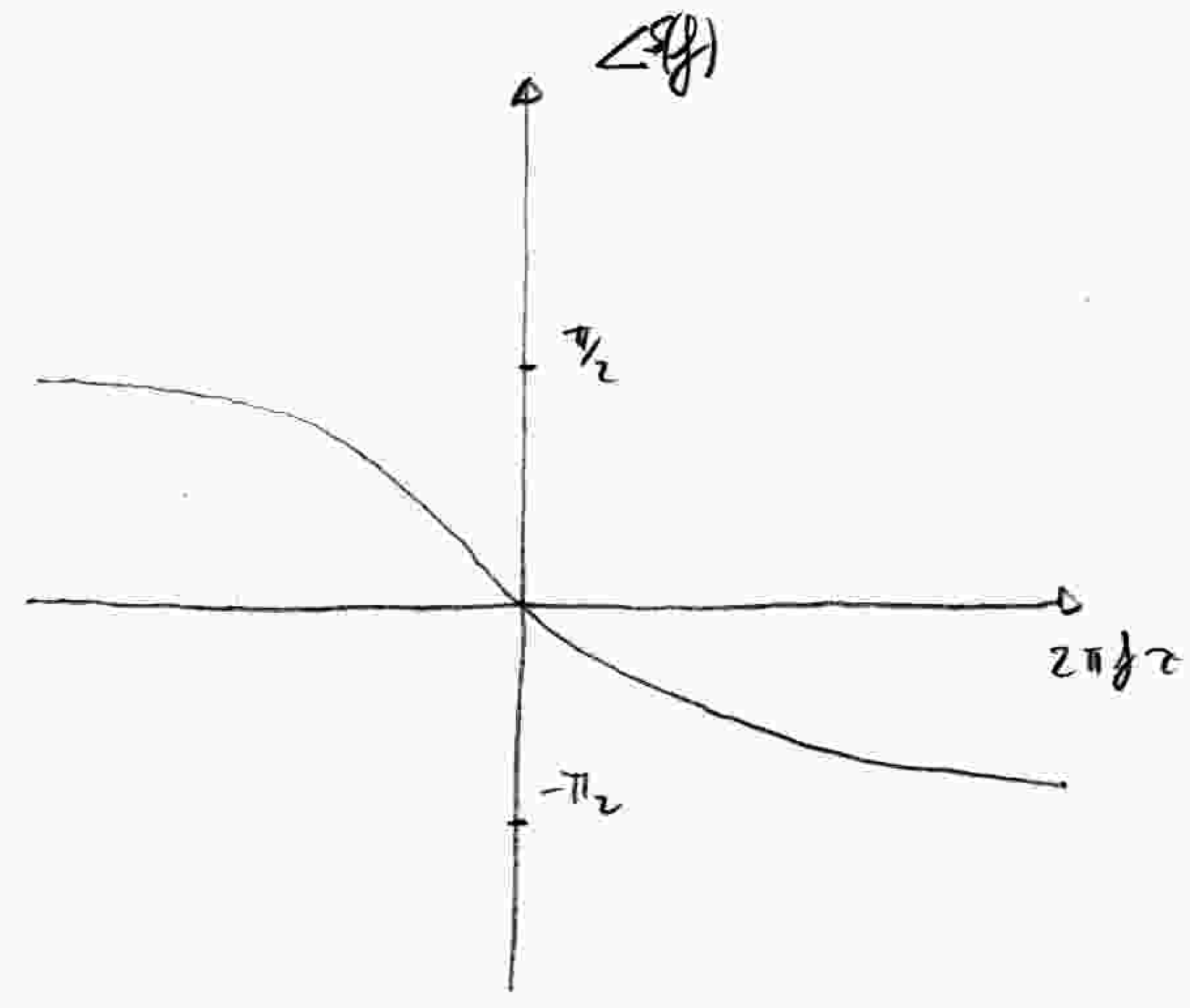


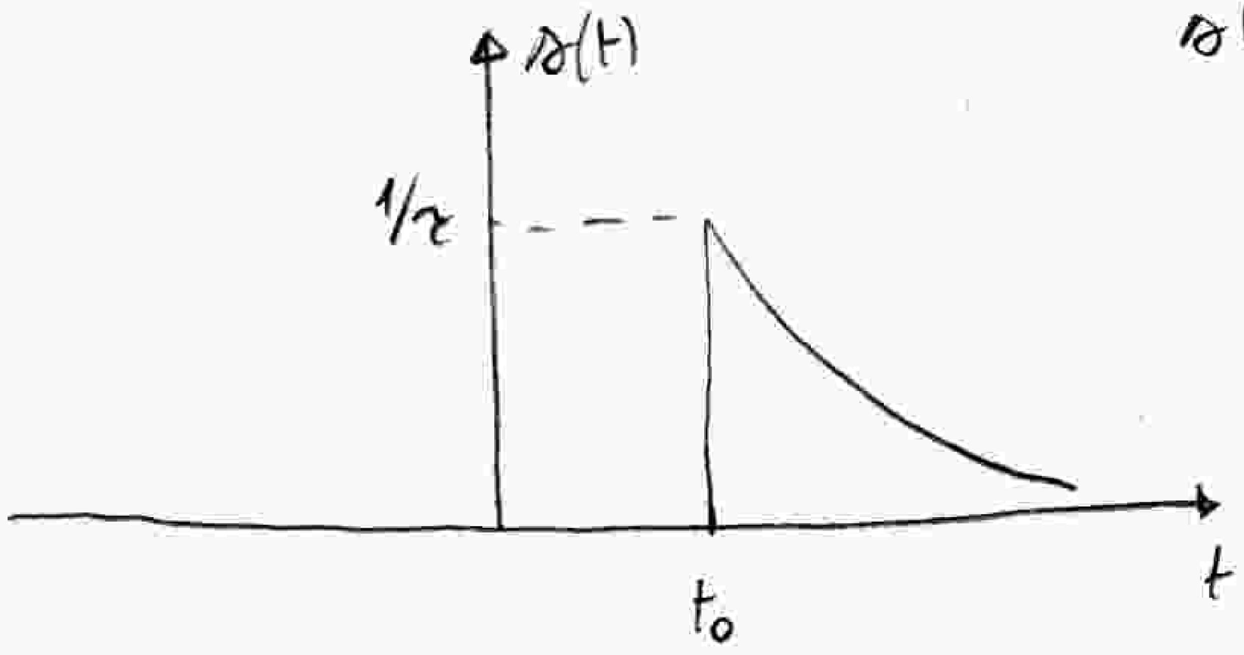
$$S(f) = \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi f t}\, dt = \int_0^{\infty} 1/\tau\, e^{-t/\tau} e^{-j2\pi f t}\, dt = \frac{1}{\tau} \int_0^{\infty} e^{-t(1/\tau + j2\pi f)}\, dt = \frac{1}{\tau}\left(-\frac{1}{1/\tau + j2\pi f}\right) e^{-t(1/\tau + j2\pi f)} \Big|_{t=0}^{t=\infty} =$$

$$= \frac{1}{1 + j2\pi f\tau}$$

$$|S(f)| = \frac{1}{\left[1 + (2\pi f\tau)^2\right]^{1/2}} \qquad \angle S(f) = -\,\mathrm{arctg}(2\pi f\tau)$$



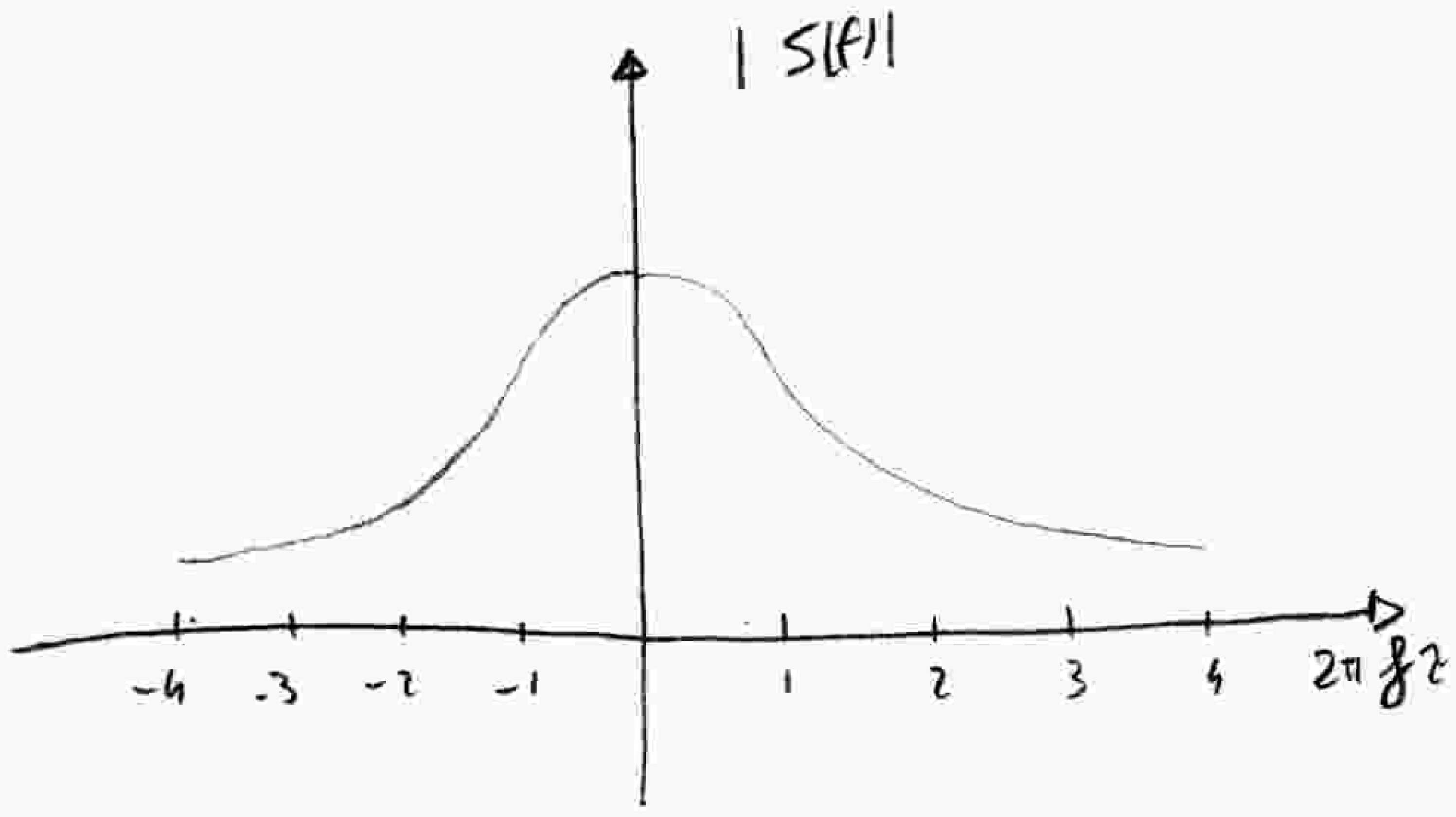


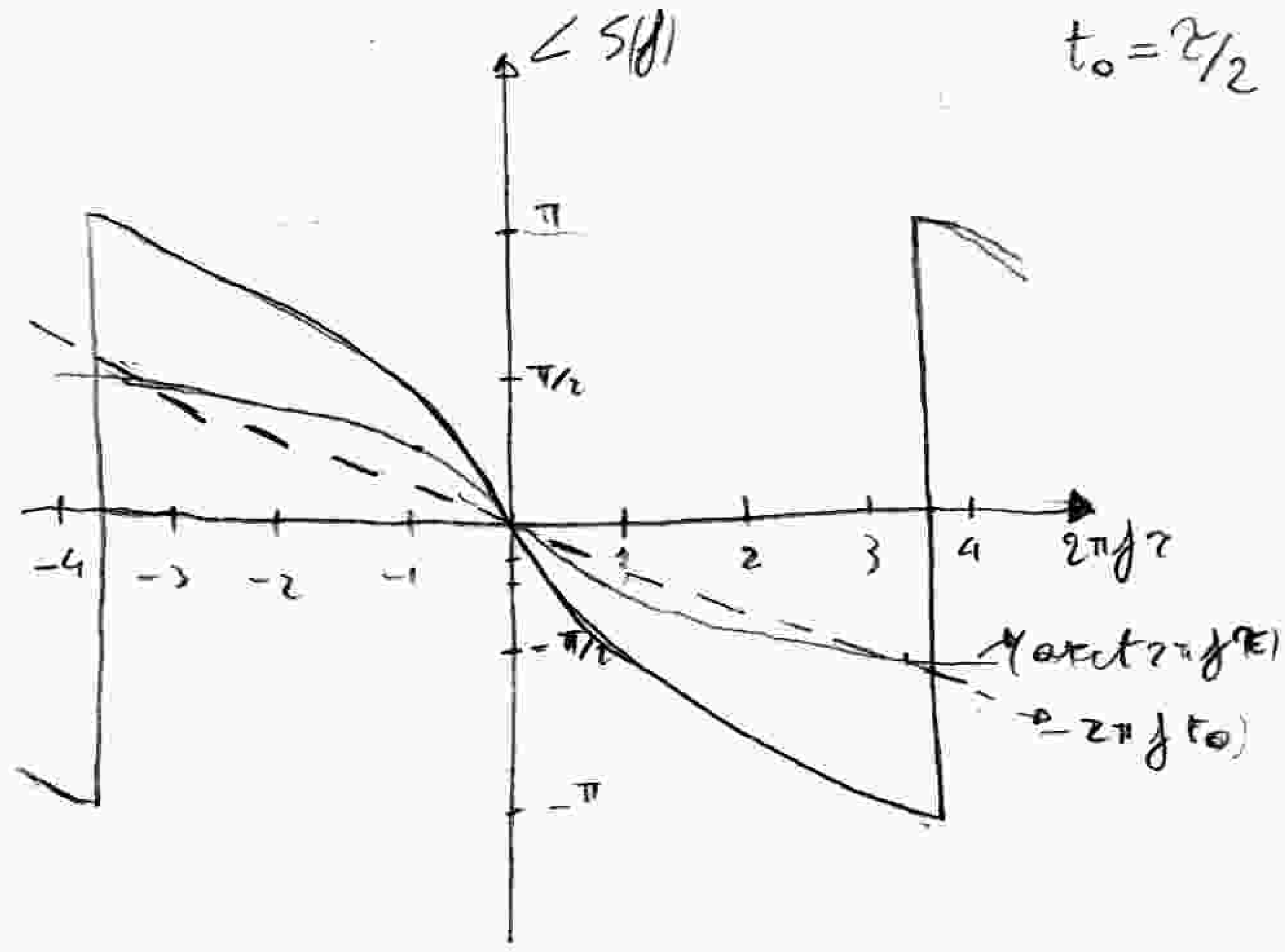


$$s(t) = \frac{1}{\tau} e^{-\frac{t-t_0}{\tau}} u(t-t_0)$$

$$\rightarrow \mathcal{F}_c[s(t)] = \frac{1}{1+j2\pi f\tau} \cdot e^{-j2\pi f t_0}$$

$$\angle S(f) = -\mathrm{arctg}(2\pi f\tau) - 2\pi f t_0$$





- <u>Derivazione (nel tempo)</u>

$$s(t) \Leftrightarrow S(f) \quad \Rightarrow \quad \frac{ds(t)}{dt} \Leftrightarrow j2\pi f S(f)$$

DIM

$$\frac{ds(t)}{dt} = \frac{d}{dt}\left[\int_{-\infty}^{\infty} S(f) e^{j2\pi f t} df\right] = \int_{-\infty}^{\infty} j2\pi f S(f) e^{j2\pi f t} df \Leftarrow \text{antitrasformata di } j2\pi f S(f)$$

- <u>Integrazione (nel tempo)</u>

$$s(t) \Leftrightarrow S(f) \quad e \quad S(0) = 0$$

$$\int_{-\infty}^{t} s(\alpha)\, d\alpha \Leftrightarrow \frac{S(f)}{j2\pi f}$$

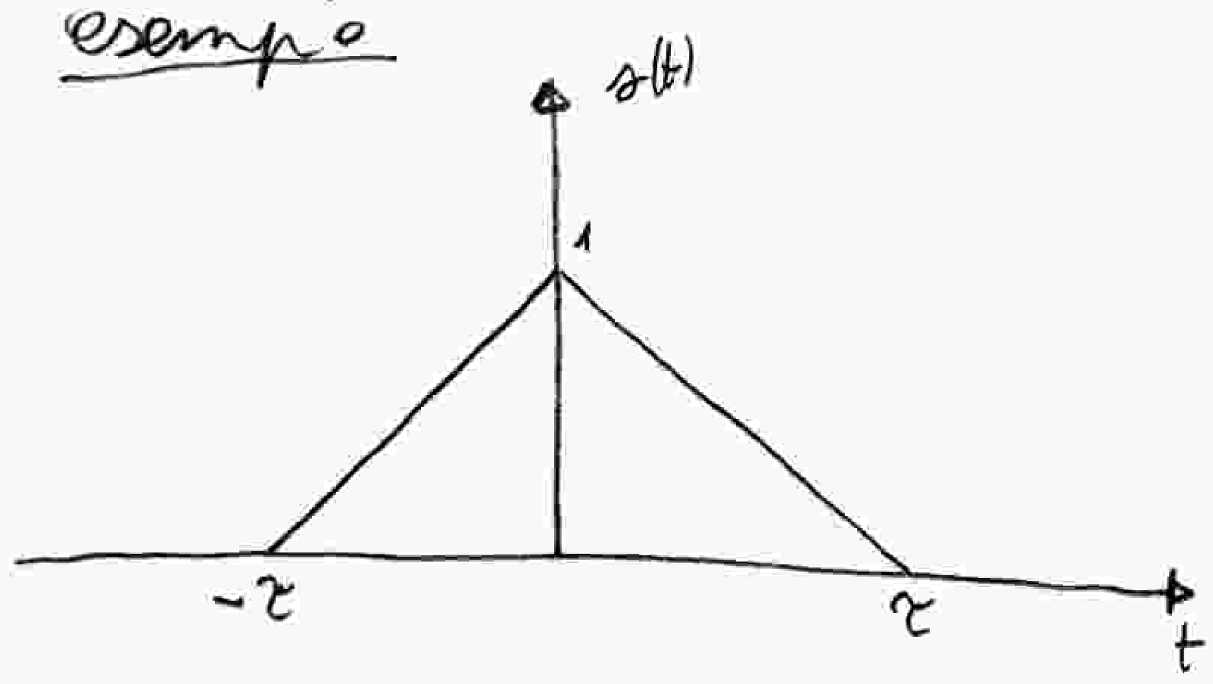
DIM si pone $\varphi(t) = \int_{-\infty}^{t} s(\alpha)\, d\alpha$ e $\mathcal{F}_c[\varphi(t)] = \Phi(f)$

$$s(t) = \frac{d\varphi(t)}{dt} \Leftrightarrow j2\pi f \Phi(f) \Rightarrow S(f) = j2\pi f \Phi(f)$$

$$\Phi(f) = \frac{1}{j2\pi f} S(f)$$

visto che $\Phi(f) \Leftrightarrow \int_{-\infty}^{t} s(\alpha)\, d\alpha$

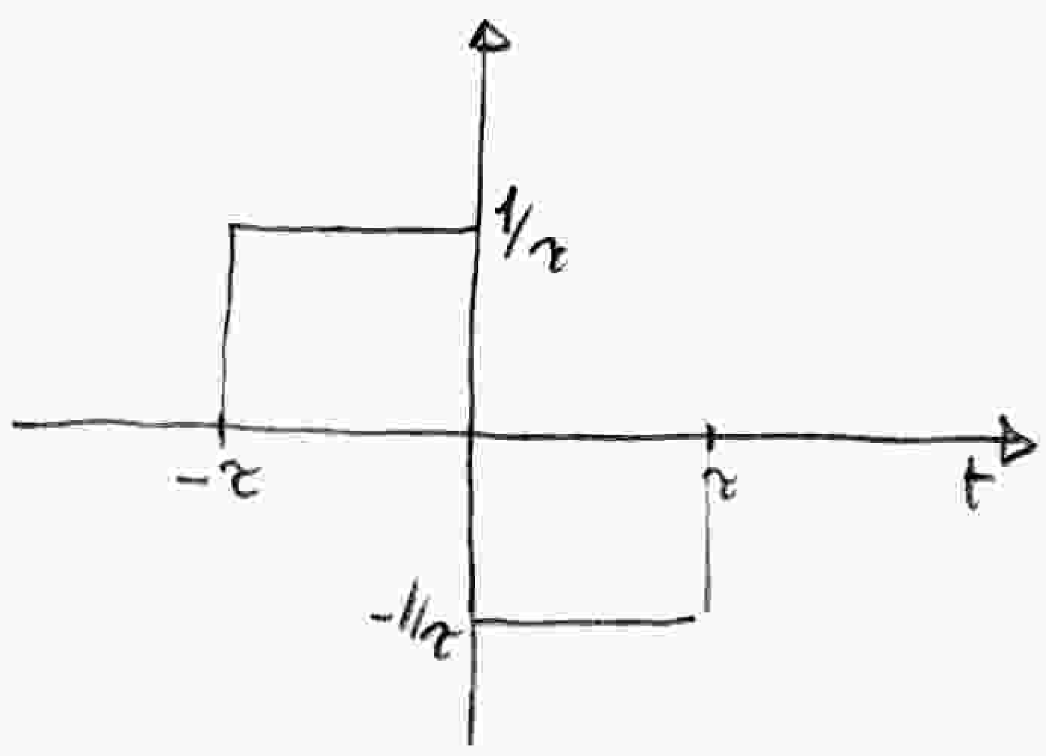
N.B.! La condizione $S(0) = \int_{-\infty}^{\infty} s(t) dt$ assicura che $\lim_{t\to\infty} \varphi(t) = 0$; ciò è necessario per l'esistenza di $\Phi(f)$. Con l'introduzione della $\delta(t)$ la condizione non è più necessaria esempio



$S(f) = ?$

noi possiamo facilmente calcolare la derivata di $s(t)$, la cui trasformata è nota e poi applicare la regola di integrazione

$\dot{s}(t)$



$$\dot{s}(t) = \frac{1}{\tau}\left[\mathrm{rect}\left(\frac{t+\frac{\tau}{2}}{\tau}\right) - \mathrm{rect}\left(\frac{t-\tau/2}{\tau}\right)\right]$$

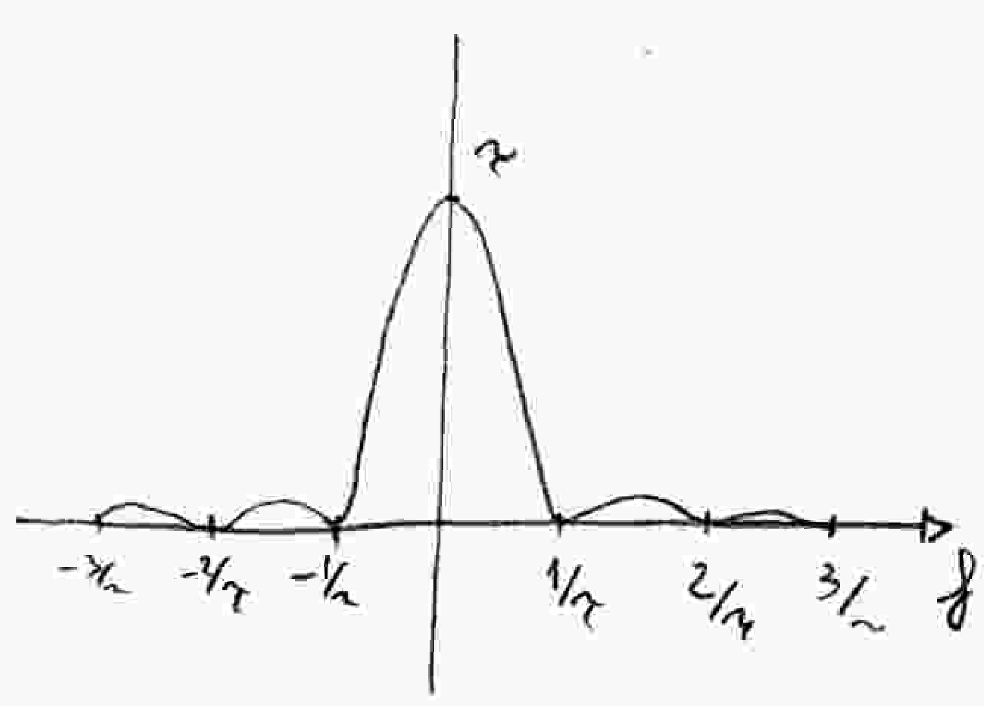
$$\Downarrow$$

$$\mathcal{F}_c[\dot{s}(t)] = \frac{1}{\tau}\left[\tau \frac{\sin(\pi f \tau)}{\pi f \tau} e^{j2\pi f \frac{\tau}{2}} - \tau \frac{\sin \pi f \tau}{\pi f \tau} e^{-j2\pi f \tau/2}\right] =$$

$$= \mathrm{sinc}(f\tau)\left(e^{j\pi f \tau} - e^{-j\pi f \tau}\right) =$$

$$= \mathrm{sinc}(f\tau)\ 2j\sin(\pi f \tau) = j2\pi f\tau\ \mathrm{sinc}^2(f\tau)$$

visto che $s(t) = \int_{-\infty}^{t} \dot{s}(\alpha)\,d\alpha \Rightarrow \mathcal{F}_c[s(t)] = \frac{1}{j2\pi f}\mathcal{F}_c[\dot{s}(t)] = \tau\,\mathrm{sinc}^2(f\tau)$



- Traslazione in frequenza

$$s(t) \Longleftrightarrow S(f)$$

$$s(t)\,e^{j2\pi f_0 t} \Longleftrightarrow ?$$

$$\mathcal{F}_c[s(t)e^{j2\pi f_0 t}] = \int_{-\infty}^{\infty} s(t)\,e^{j2\pi f_0 t}e^{-j2\pi f t}\,dt = \int_{-\infty}^{\infty} s(t)\,e^{-j2\pi(f-f_0)t}\,dt = S(f-f_0)$$

$$s(t)\,e^{j2\pi f_0 t} \Longleftrightarrow S(f-f_0)$$

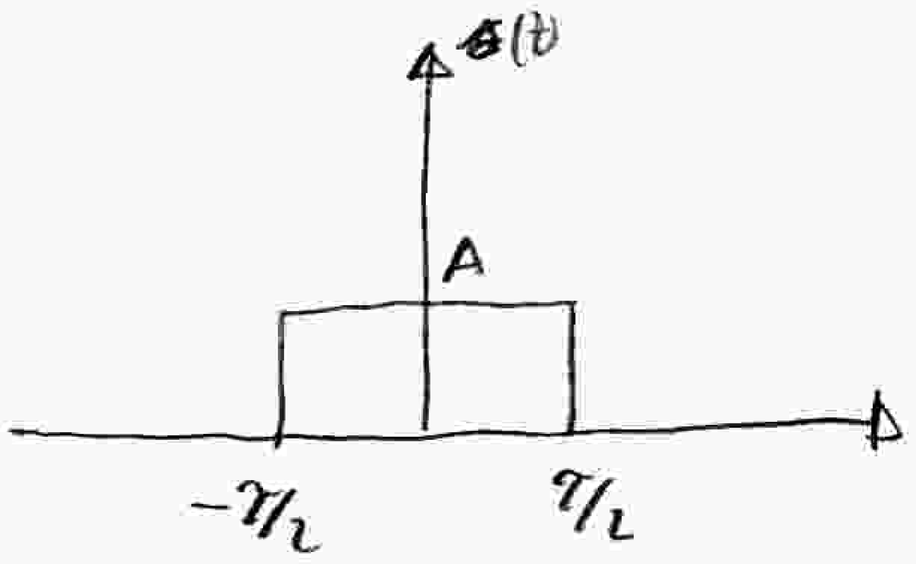
- Dualità

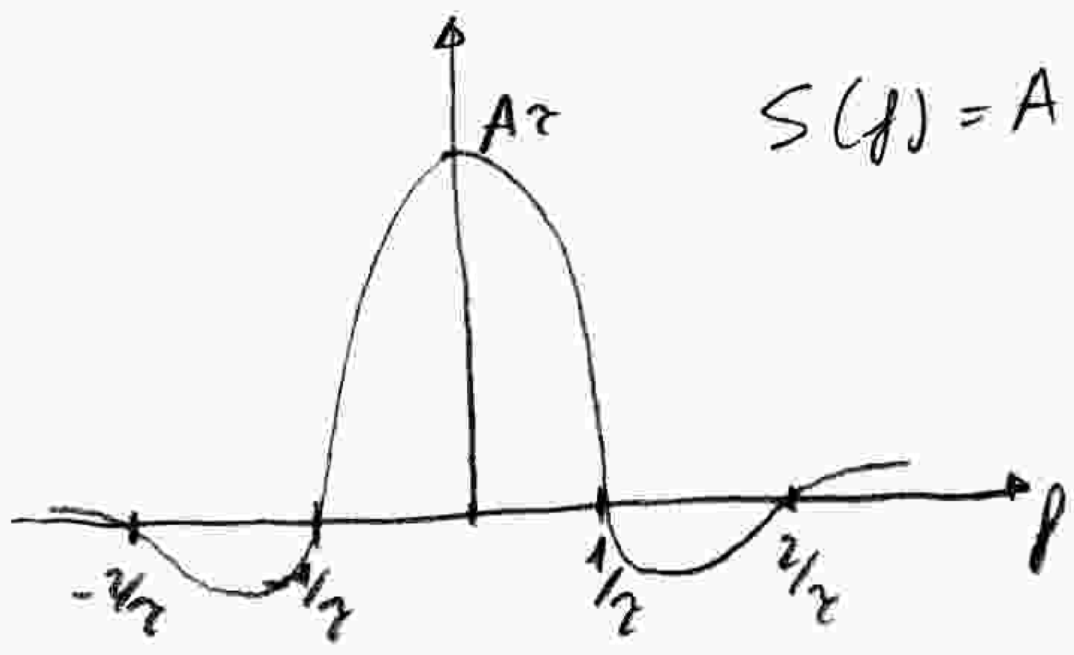
Se $s(t)$ ha come TCF $S(f)$, allora se si considera quest'ultima come funzione del tempo

$$S(t) \Longleftrightarrow s(-f)$$

$$s(-t) = \int_{-\infty}^{\infty} S(f)\, e^{-j2\pi f t}\, df \qquad \text{pongo } f \equiv x$$

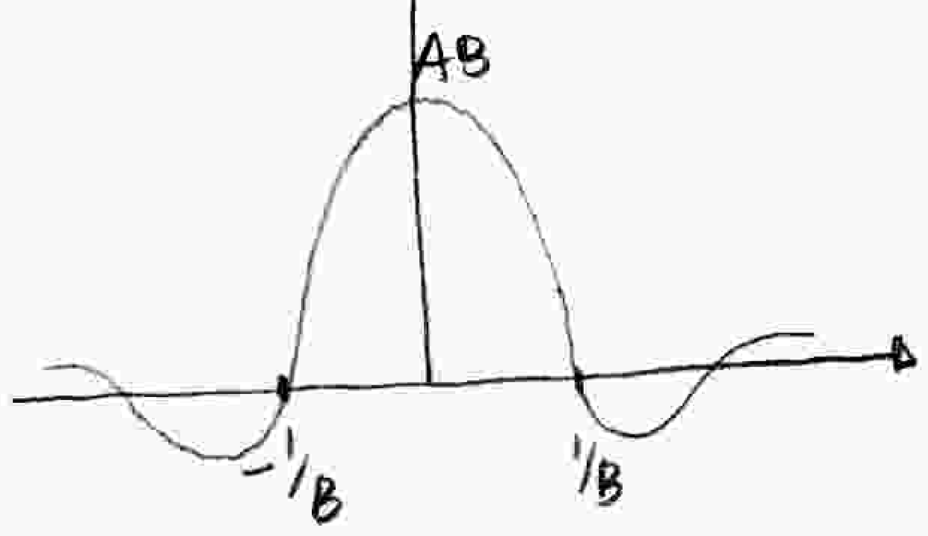
$$s(-t) = \int_{-\infty}^{\infty} S(x)\, e^{-j2\pi t x}\, dx \quad \text{e quindi} \quad s(-f) = \int_{-\infty}^{\infty} S(x)\, e^{-j2\pi f x}\, dx$$



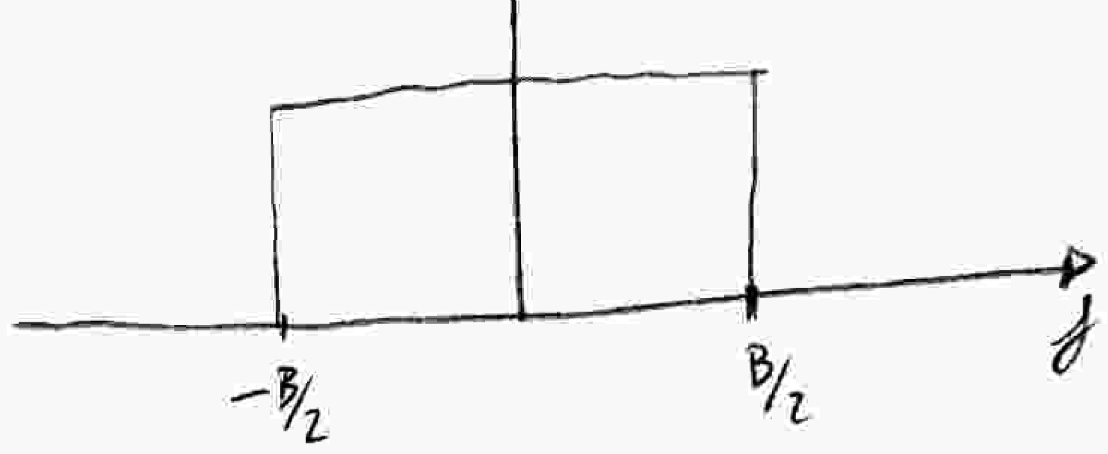


$S(f) = A\tau\, \mathrm{sinc}(f\tau)$

$s(t) = AB\, \mathrm{sinc}(Bt)$



$S(f) = A\, \mathrm{rect}\left(\frac{f}{B}\right)$



- convoluzione temporale

$x(t)$ $y(t)$ continui e ad energia finita e tali che $x(t) \Leftrightarrow X(f)$, $y(t) \Leftrightarrow Y(f)$

il segnale continuo $s(t) = \int_{-\infty}^{\infty} x(\alpha)\, y(t-\alpha)\, d\alpha$

si indica con $s(t) = x(t) \otimes y(t)$ ed è detto prodotto di convoluzione

allora

$$x(t) \otimes y(t) \Leftrightarrow X(f)\, Y(f)$$

DIM

$$\mathcal{F}_c[x(t)\otimes y(t)] = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x(\alpha)\, y(t-\alpha)\, d\alpha\right] e^{-j2\pi f t}\, dt = \int_{\alpha=-\infty}^{\infty} x(\alpha)\left[\int_{t=-\infty}^{\infty} y(t-\alpha)\, e^{-j2\pi f t}\, dt\right] d\alpha =$$

applicando il teorema del ritardo $= \int_{\alpha=-\infty}^{\infty} x(\alpha)\, Y(f)\, e^{-j2\pi f\alpha}\, d\alpha = Y(f)\int_{\alpha=-\infty}^{\infty} x(\alpha)\, e^{-j2\pi f\alpha}\, d\alpha = Y(f)\, X(f)$

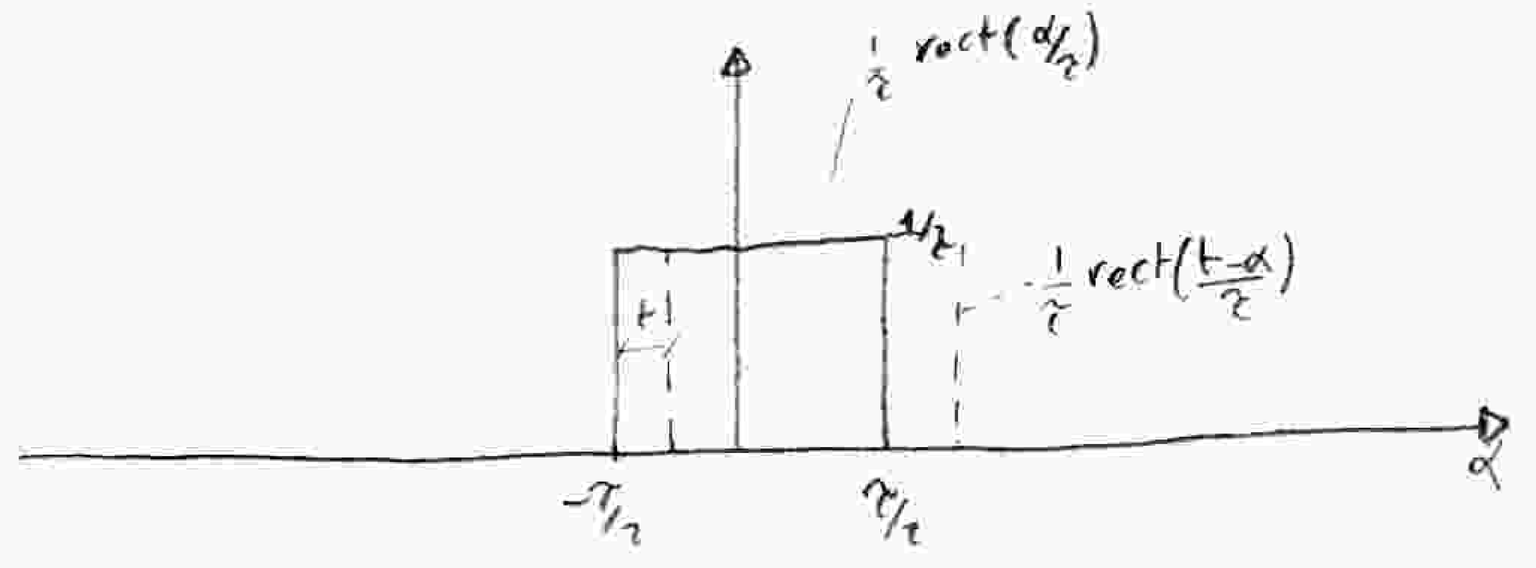
es antitrasformata di $S(f) = \text{sinc}^2(f\tau)$ $\quad \mathcal{F}_c^{-1}[S(f)] = ?$

$S(f)$ può essere scritta come $S(f) = \text{sinc}(f\tau)\cdot\text{sinc}(f\tau)$ con $X(f) = Y(f) = \text{sinc}(f\tau)$

da cui $s(t) = \int_{-\infty}^{\infty} x(\alpha)\, y(t-\alpha)\, d\alpha$

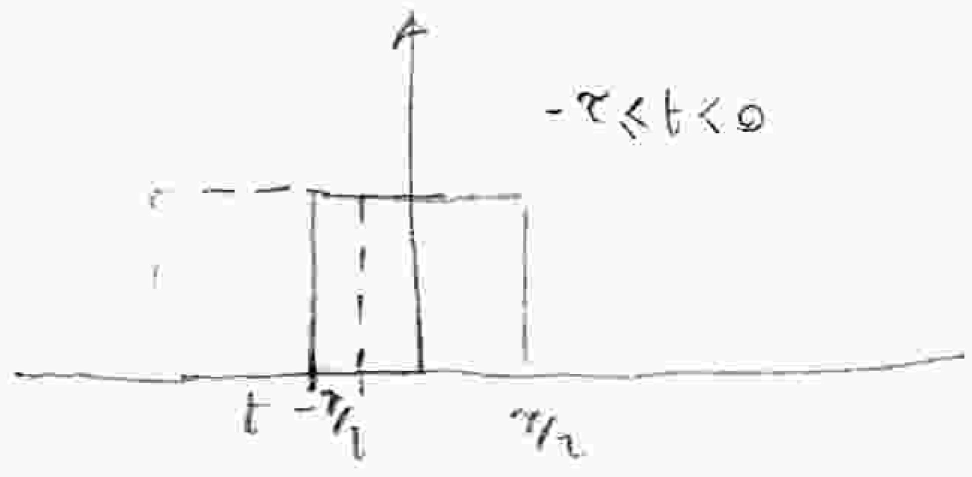
ma $X(f) \Leftrightarrow \frac{1}{\tau}\,\text{rect}\left(\frac{t}{\tau}\right) \Rightarrow x(t) = \mathcal{F}_c^{-1}[X(f)] = \frac{1}{\tau}\,\text{rect}(t/\tau)$

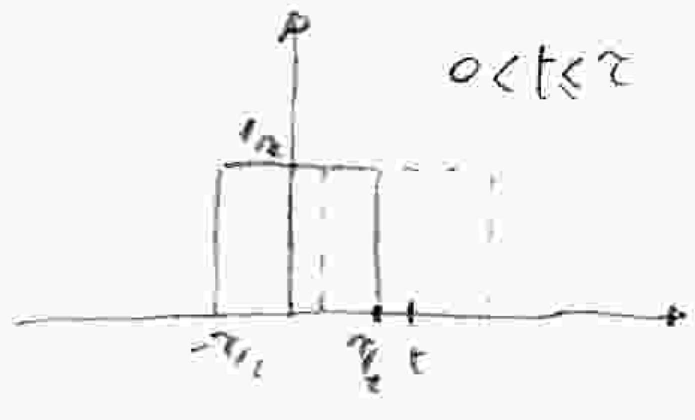
$$s(t) = \int_{-\infty}^{\infty} \frac{1}{\tau}\cdot\text{rect}\left(\frac{\alpha}{\tau}\right)\cdot\frac{1}{\tau}\,\text{rect}\left(\frac{t-\alpha}{\tau}\right) d\alpha$$

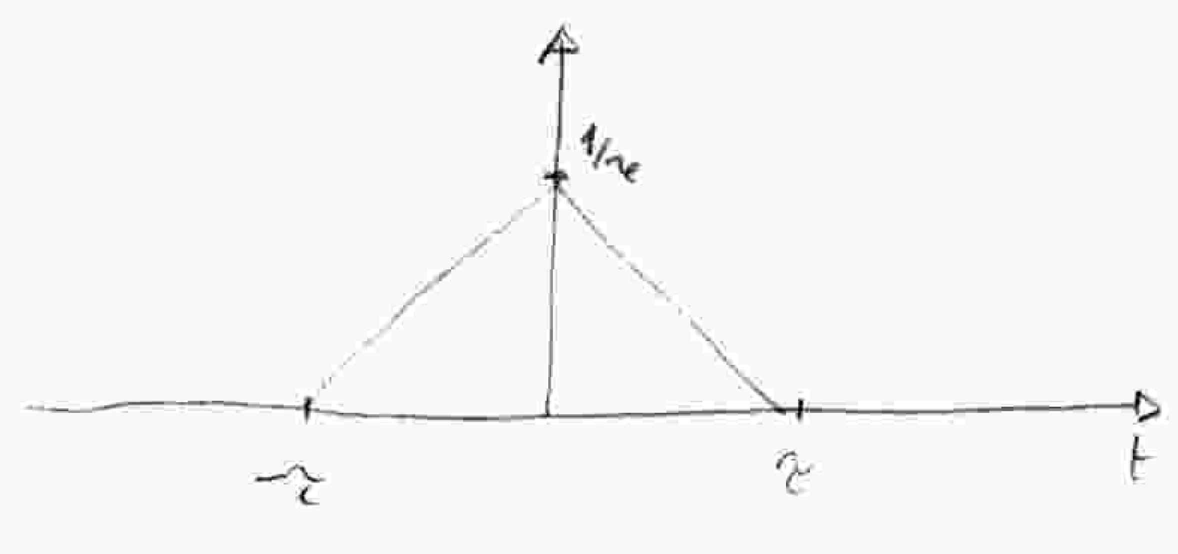


$$s(t) = \begin{cases} 0 & \text{per } t < -\tau \\ \frac{1}{\tau^2}\cdot\left[t+\frac{\tau}{2}-\left(-\frac{\tau}{2}\right)\right] = \frac{1}{\tau}\left[1+\frac{t}{\tau}\right] & -\tau \le t < 0 \\ \frac{1}{\tau^2}\left[\tau/2-\left(t-\frac{\tau}{2}\right)\right] = \frac{1}{\tau}\left[1-\frac{t}{\tau}\right] & 0 < t \le \tau \\ 0 & \text{per } t > \tau \end{cases}$$

$$s(t) = \begin{cases} 1/\tau\left[1-\frac{|t|}{\tau}\right] & |t| \le \tau \\ 0 & \text{per } |t| > \tau \end{cases}$$







Funzione generalizzata (o impulsiva di Dirac) $\delta(t)$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = \int_{-\infty}^{\infty} f(t)\,\delta(t_0-t)\,dt = f(t_0)$$

N.B

$$\int_{-\infty}^{\infty} \delta(t-t_0)\,dt = 1$$

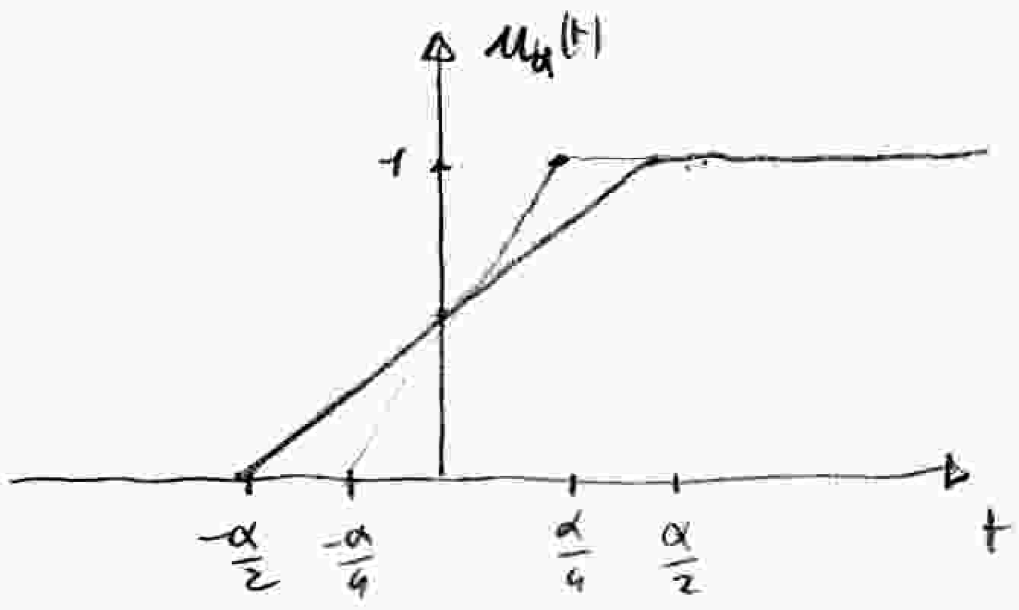
- Se $f(t)$ è un impulso rettangolare centrato in t_0, di ampiezza A e durata 2ε

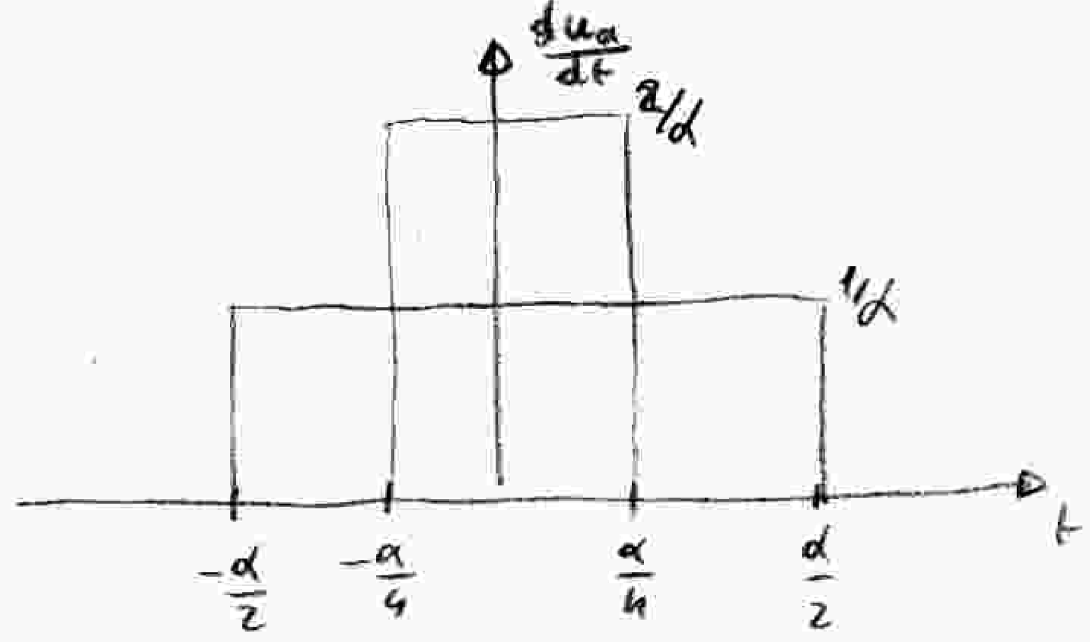
$$\int_{-\infty}^{\infty} A \,\mathrm{rect}\left(\frac{t-t_0}{2\varepsilon}\right)\delta(t-t_0)\,dt = A\int_{t_0-\varepsilon}^{t_0+\varepsilon} \delta(t-t_0)\,dt = A$$

Quindi si ha

$$\int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t) \quad \Rightarrow \quad \frac{d}{dt}u(t) = \delta(t)$$

- La relazione precedente va intesa nel limite





nel limite $\alpha \to 0$ la $\frac{du_\alpha}{dt}$ tende alla $\delta(t)$

- Se si pone $f(t) = e^{-j2\pi f t}$

$$\int_{-\infty}^{\infty} e^{-j2\pi f t}\,\delta(t)\,dt = 1 \quad \Rightarrow \quad \Delta(f) = 1$$

visto che la relazione precedente equivale alla $\mathcal{F}_c[\delta(t)]$

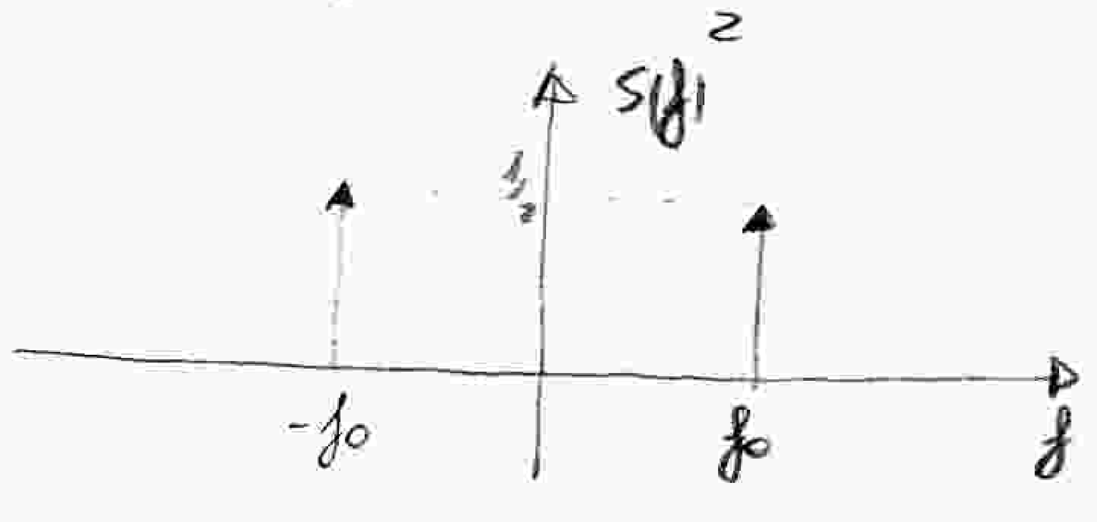
Una volta definita la funzione generalizzata $\delta(t)$ si può esprimere la TCF di alcuni segnali a potenza media finita.

a) $A \Leftrightarrow A\,\delta(f)$ deriva dalla proprietà di simmetria

b) $e^{j2\pi f_0 t} \Leftrightarrow \delta(f-f_0)$ deriva dalla traslazione in frequenza

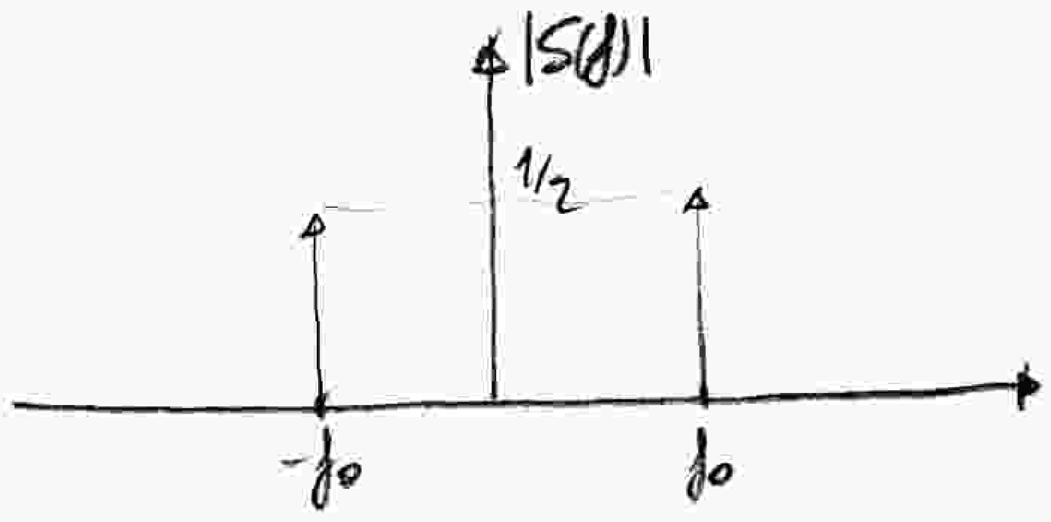
c) $s(t) = \cos 2\pi f_0 t$

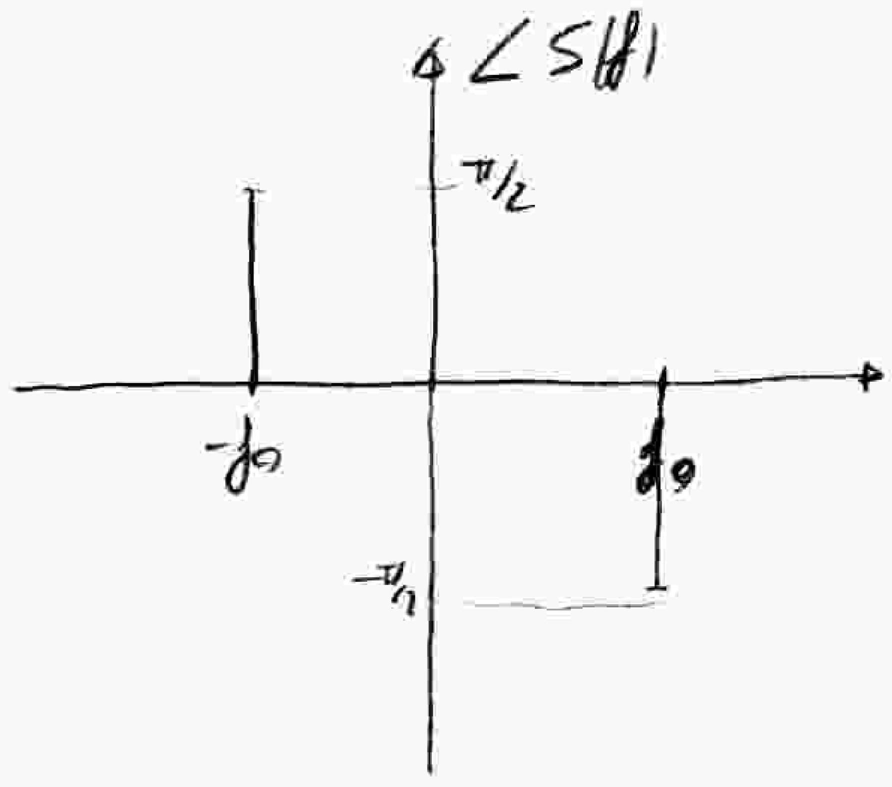
$$\mathcal{F}_c[s(t)] = \mathcal{F}_c\left[\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right] = \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$$



d) $s(t) = \sin(2\pi f_0 t)$

$$\mathcal{F}_c[\sin(2\pi f_0 t)] = \mathcal{F}_c\left[\frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j}\right] = \frac{1}{2j}[\delta(f-f_0) - \delta(f+f_0)]$$

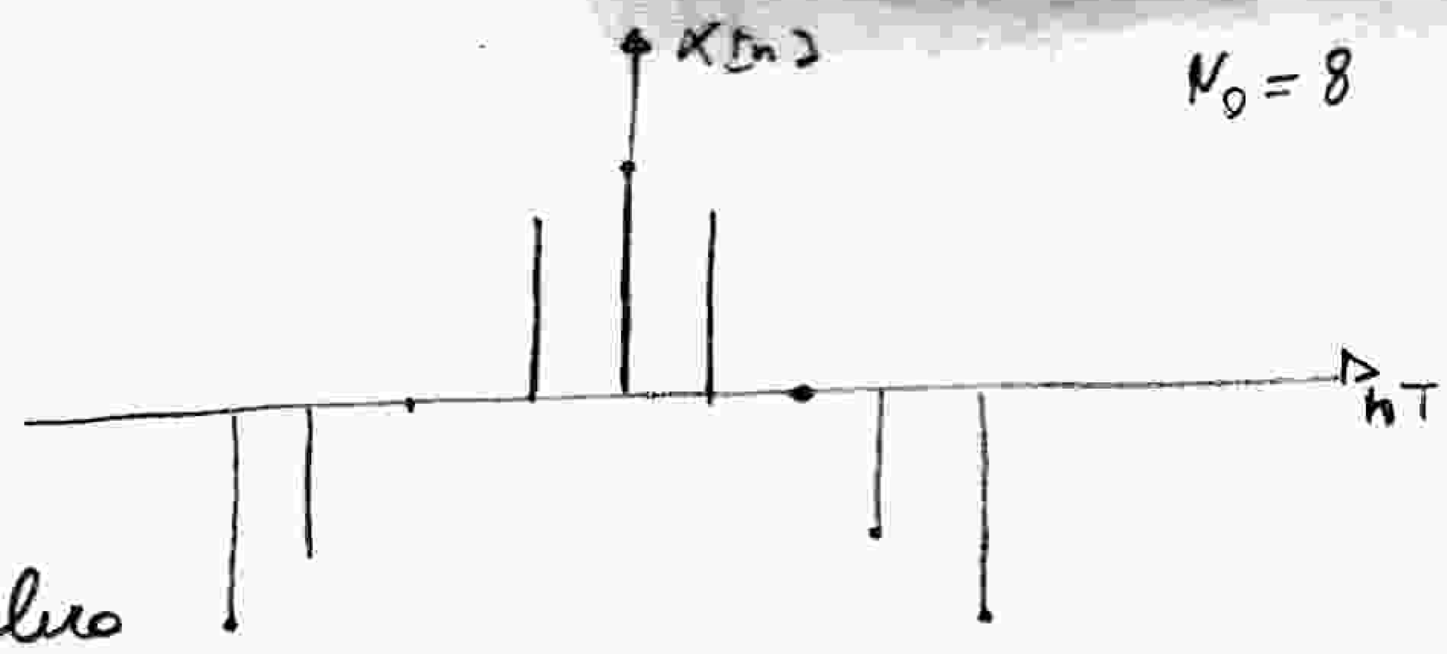




Esercizi sulla TDF

$N_0 = 8$

$$x[n] = \cos\left(\frac{2\pi nT}{N_0 T}\right)$$

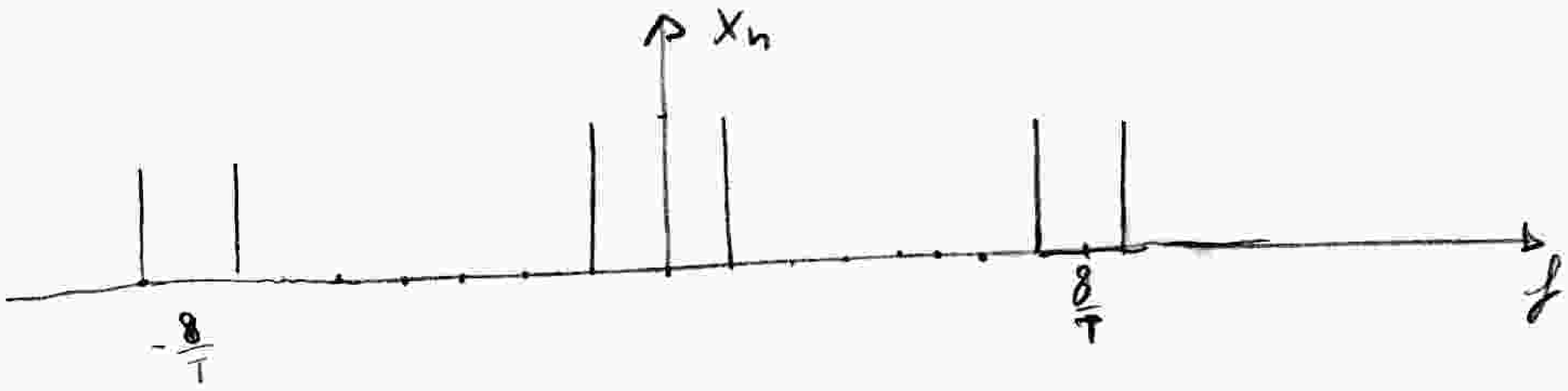


utilizzando le formule di Eulero

$$x[n] = \frac{1}{2}\left(e^{j2\pi n/N_0} + e^{-j2\pi n/N_0}\right) =$$

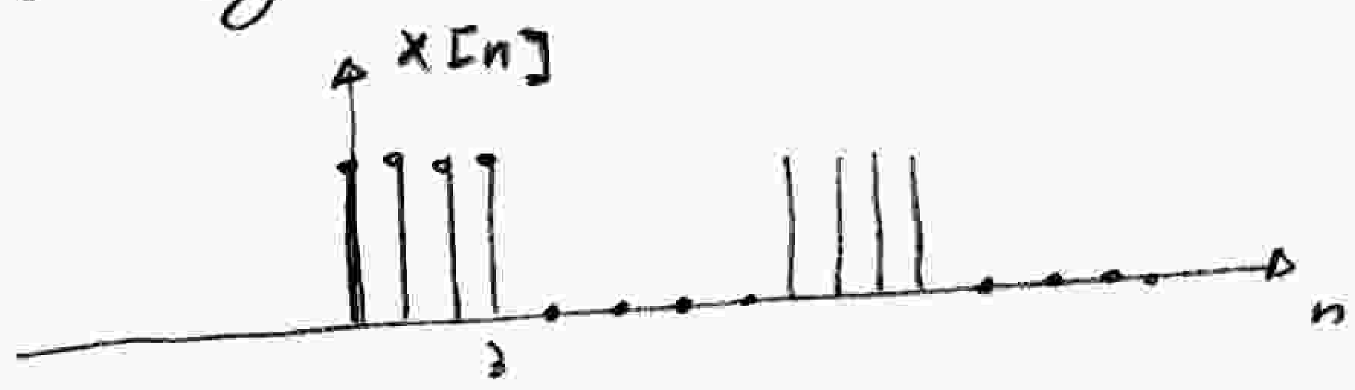
confrontando con $x[n] = \sum_{K=0}^{N_0-1} \bar{X}_K \, e^{j\frac{2\pi K n}{N_0}}$

si trova $\bar{X}_1 = \frac{1}{2}$ $\quad \bar{X}_{-1} = \frac{1}{2}$ $\quad \bar{X}_n = 0 \quad n = 0, \pm 2, \ldots, \pm \frac{N_0}{2}$



- $x[n]$ periodica di periodo $N_0 = 8$ definita nel periodo come

$$x[n] = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & 4 \le n \le 7 \end{cases}$$



$$\bar{X}_K = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n] e^{-j\frac{2\pi K n}{N_0}} = \frac{1}{8} \sum_{n=0}^{3} e^{-j\frac{2\pi K n}{8}} = \frac{1}{8} \sum_{n=0}^{3} \left(e^{-j\frac{\pi K}{4}}\right)^n$$

si usa la formula

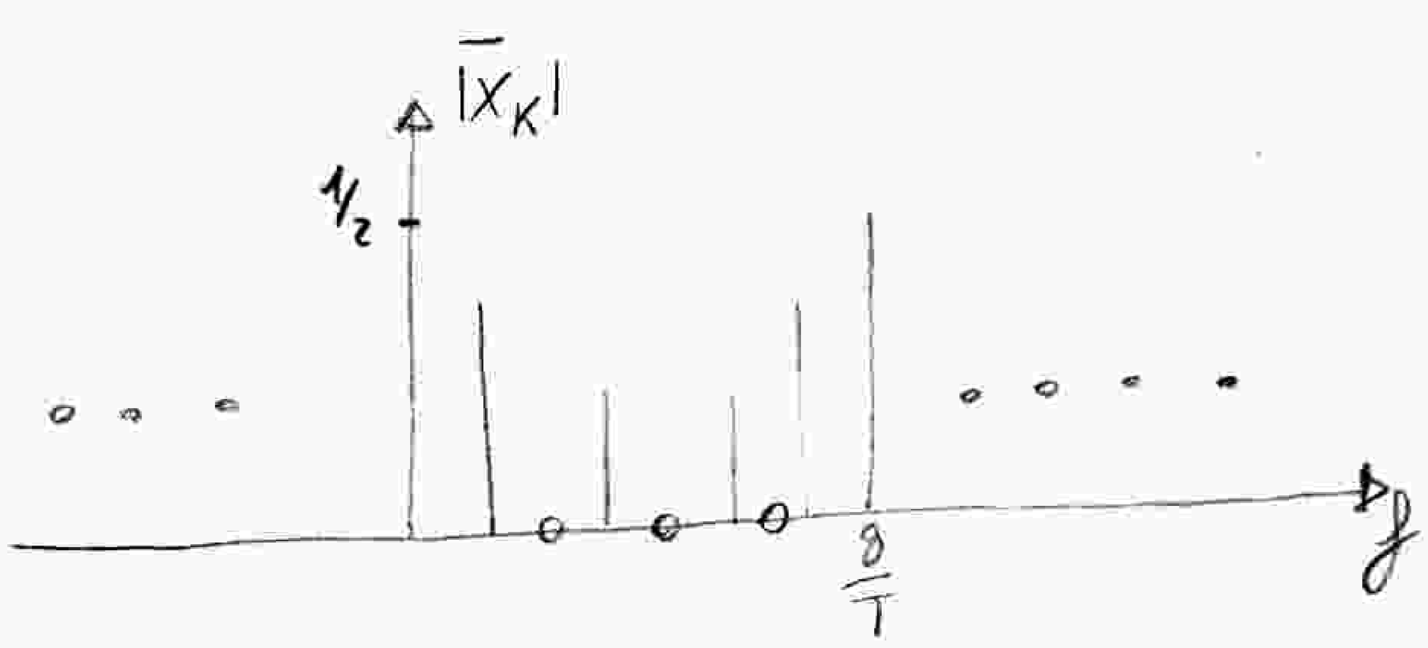
$$\sum_{n=0}^{N_0-1} a^n = \begin{cases} \frac{1-a^{N_0}}{1-a} & \text{per } a \neq 1 \\ N_0 & \text{per } a = 1 \end{cases}$$

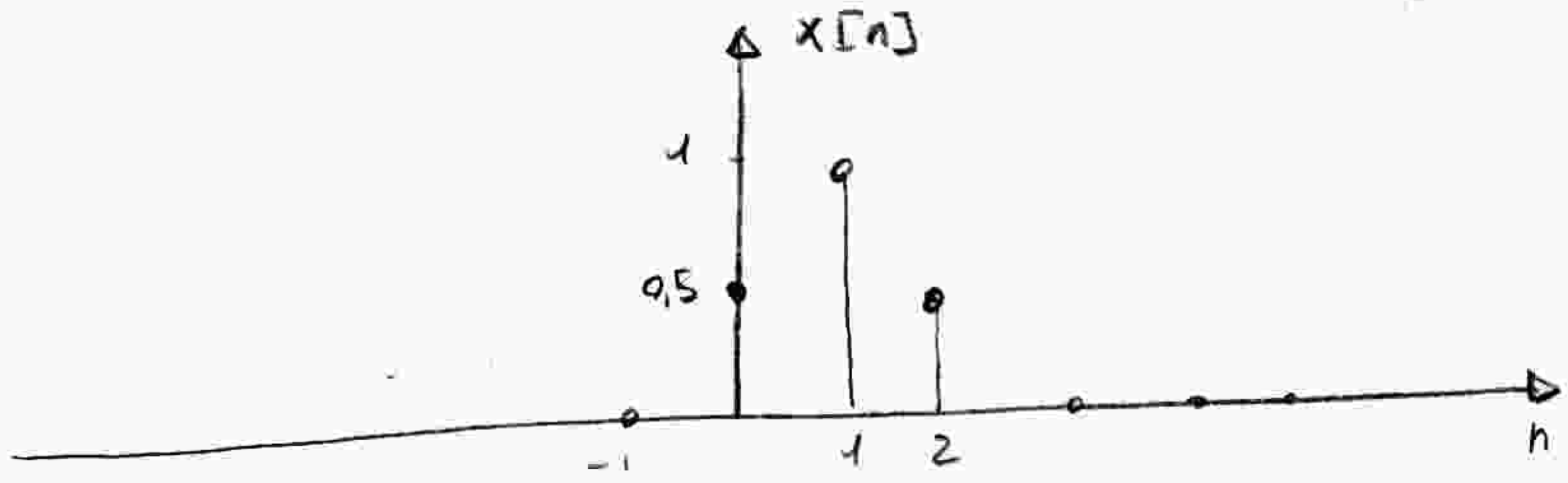
$$\bar{X}_K = \begin{cases} 1/2 & \text{per } K = 0 \\ \frac{1}{8} \, \frac{1-e^{-jK\pi}}{1-e^{-j\pi K/4}} & K \neq 0 \end{cases}$$

$\bar{X}_2 = \bar{X}_4 = \bar{X}_6 = 0$

$|\bar{X}_1| = |\bar{X}_7| = 1/8 \sin \pi/8 = 0{,}327$

$|\bar{X}_3| = |\bar{X}_5| = 1/8 \sin 3\pi/8 = 0{,}135$

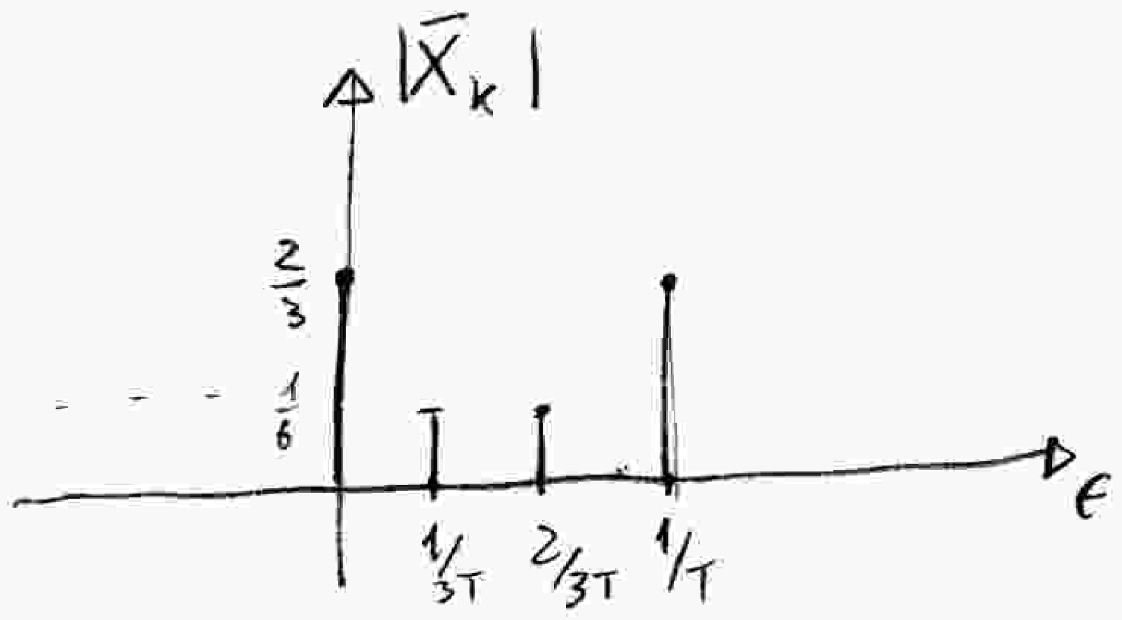


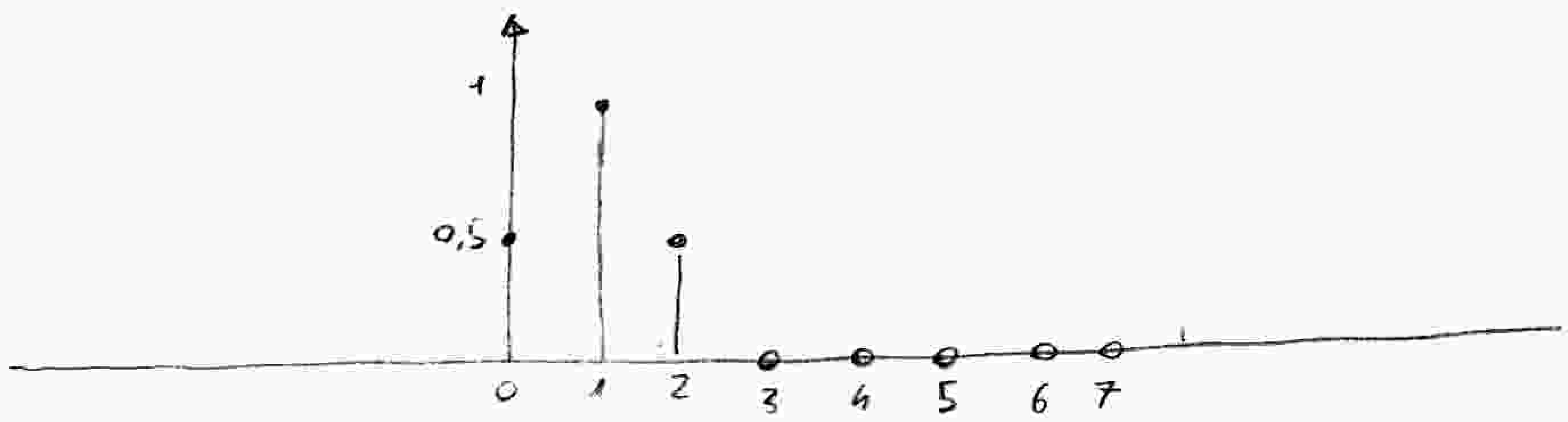


$$\bar{X}_K = \frac{1}{N_0}\sum_{n=0}^{N_0-1} x[n]\, e^{-j\frac{2\pi k n}{N_0}} = \frac{1}{3}\left[x[0] + x[1]\, e^{-j\frac{2\pi K}{3}} + x[2]\, e^{-j\frac{4\pi K}{3}}\right] =$$

$$= \frac{1}{3}\, e^{-j\frac{2\pi K}{3}}\left[\frac{1}{2}\, e^{j\frac{2\pi K}{3}} + 1 + \frac{1}{2}\, e^{-j\frac{2\pi K}{3}}\right] = \frac{1}{3}\, e^{-j\frac{2\pi K}{3}}\left[1 + \cos\frac{2\pi K}{3}\right] \quad K = 0, 1, \ldots, N-1$$

cioè $K = 0, 1, 2$





$$\bar{X}_K = \frac{1}{8}\, e^{-j\frac{2\pi K}{8}}\left[1 + \cos\frac{2\pi K}{8}\right] \qquad K = 0, \ldots, 7$$