Consideriamo $x[n]$ sequenza aperiodica

$$\overline{X}(f) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j2\pi n f T}$$

$$x[n] = \frac{1}{F}\int_0^F \overline{X}(f)\, e^{j2\pi n f T} df$$

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n-rN] \qquad \text{periodica di periodo } N$$

troviamo la relazione tra $\overline{X}(f)$ e $\tilde{X}[k]$

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n-rN] = \sum_{r=-\infty}^{\infty} \frac{1}{F}\int_0^F \overline{X}(f)\, e^{j2\pi(n-rN)fT} df =$$

$$= T\int_0^{1/T} \overline{X}(f) \sum_{r=-\infty}^{\infty} e^{j2\pi(n-rN)fT} df =$$

$$\circledast \quad \sum_{r=-\infty}^{\infty} e^{-j2\pi n f T} = \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta\left(f-\frac{k}{T}\right) \Rightarrow \sum_{r=-\infty}^{\infty} e^{-j2\pi r N f T} = \frac{1}{NT}\sum_{k=-\infty}^{\infty} \delta\left(f-\frac{k}{NT}\right)$$

$$= T\int_0^F \overline{X}(f)\frac{1}{NT}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{NT}\right)\cdot e^{j2\pi n f T} df = \frac{1}{N}\sum_{k=-\infty}^{\infty}\int_0^F \overline{X}(f)\,\delta\left(f-\frac{k}{NT}\right) e^{j2\pi n f T} df =$$

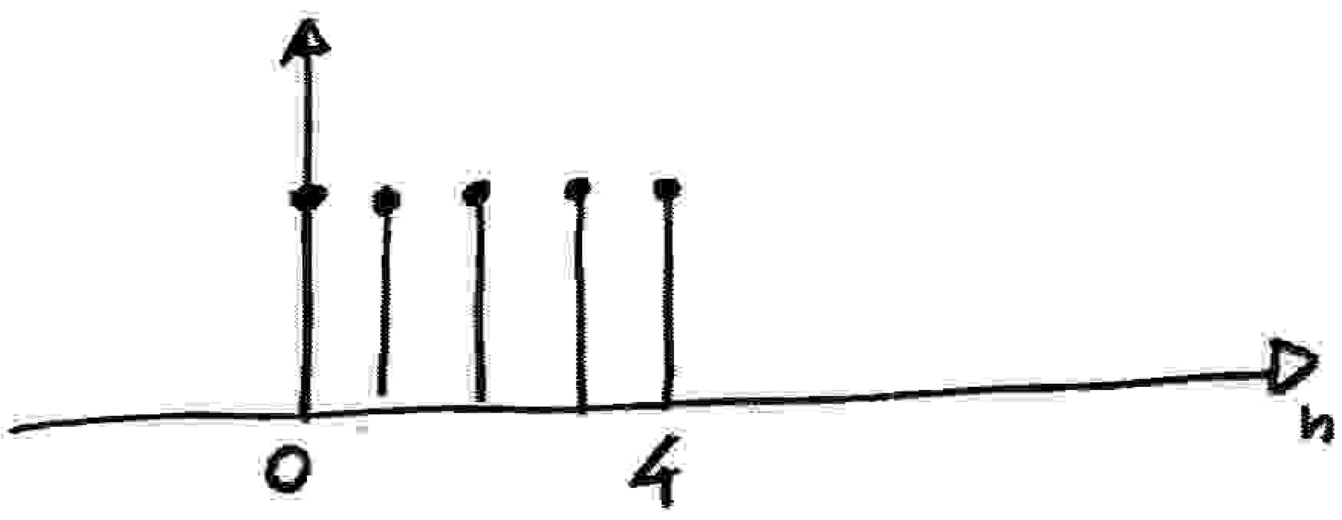
$$= \frac{1}{N}\sum_{k=0}^{N-1} \overline{X}\left(\frac{k}{NT}\right) e^{j2\pi \frac{nk}{N}}$$

$$\tilde{x}[n] = \sum_{k=-\infty}^{\infty} \tilde{X}[k]\, e^{j2\pi\frac{nk}{N}} \qquad \tilde{X}[k] = \frac{1}{N}\overline{X}\left(\frac{k}{NT}\right)$$

$\circledast$ si ottiene da sviluppo in Serie di Fourier di pettine di δ, e poi $\mathcal{F}(\cdot)$

esempio

Analisi spettrale



TF

$$\bar{X}(f) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j2\pi n f T} = \sum_{n=0}^{4} \left(e^{-j2\pi f T}\right)^n = \qquad \sum_{n=0}^{N-1} a^n = \begin{cases} \frac{1-a^N}{1-a} & a \neq 1 \\ N & a=1 \end{cases}$$

$$= \frac{1 - e^{-j2\pi f T 5}}{1 - e^{-j2\pi f T}} = \frac{e^{-j5\pi f T}}{e^{-j\pi f T}} \, \frac{e^{j5\pi f T} - e^{-j\pi 5 f T}}{e^{j\pi f T} - e^{-j\pi f T}} = \frac{\sin 5\pi f T}{\sin \pi f T}\, e^{-4\pi f T}$$

periodo $N=5$

$$\bar{X}\left(\frac{K}{NT}\right) = N\, \tilde{X}(k)$$

$$\tilde{X}[K] = \frac{1}{N}\, \bar{X}\left(\frac{K}{NT}\right) = \frac{1}{N}\, \frac{\sin \pi K}{\sin \frac{\pi K}{5}}\, e^{-j\frac{4}{5}\pi K} \qquad K = 0, 1, \dots, 4$$

periodo $N=10$

$$\tilde{X}[k] = \frac{1}{N}\, \frac{\sin \frac{\pi K}{2}}{\sin \frac{\pi K}{10}}\, e^{-j\frac{2}{5}\pi K} \qquad K = 0, 1, \dots, 9$$