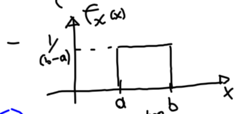
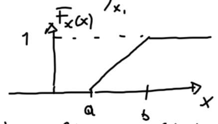
$F_X(x) = P\{X \le x\}$ $\qquad f_X(x) = \frac{dF_X(x)}{dx}$

$P\{x_1 < X \le x_2\} = F_X(x_2) - F_X(x_1) = \int_{x_1}^{x_2} f_X(x)\,dx$

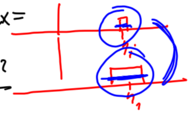




$\eta_x = E\{x\} = \int_{-\infty}^{+\infty} x\, f_X(x)\,dx \Rightarrow \mathcal{U}(a,b) \quad E\{x\} = \frac{b+a}{2}$

$E\{x^2\} = \int_{-\infty}^{+\infty} x^2 f_X(x)\,dx = \frac{1}{3}\cdot\frac{b^3-a^3}{b-a}$

$E\{(x-\eta_x)^2\} = \int_{-\infty}^{+\infty} (x-\eta_x)^2 f_X(x)\,dx =$

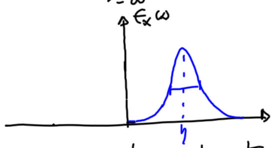


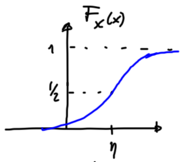
$= E\{x^2\} - \eta_x^2 = \frac{(b-a)^2}{12}$

$x \in N(\eta, \sigma^2)$

$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\eta)^2}{2\sigma^2}}$ $\qquad \eta, \sigma^2$

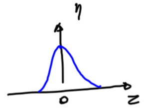
$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\eta)^2}{2\sigma^2}}\, dx$





variabile standardizzata

$N(0,1)$ $\qquad z = \frac{x-\eta}{\sigma}$



$\varphi(z)$ densità di prob. di z

$z = g(x) = \frac{x-\eta}{\sigma}$

$\varphi(z) = \left.\frac{f_X(x)}{|g'(x)|}\right|_{x=g^{-1}(z)} =$

$= \frac{1}{|1/\sigma|}\, f_X(\sigma z + \eta) =$ $\qquad x = \sigma z + \eta$

$= \sigma \cdot \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(\sigma z + \cancel{\eta} - \cancel{\eta})^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z^2}{2}}$

z	$\varphi(z)$
⋮	#
⋮	#
⋮	#

$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{z^2}{2}}\, dz$

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{z^2}{2}} dz$$

z	$\Phi(z)$
$\vdots$	$\vdots$

$$P\{a < z \le b\} = \Phi(b) - \Phi(a)$$

$$P\{c_1 < x \le c_2\} = P\{c_1 < \sigma z + \eta \le c_2\} =$$

$$= P\left\{\frac{c_1 - \eta}{\sigma} < z \le \frac{c_2 - \eta}{\sigma}\right\} = \Phi\left(\frac{c_2 - \eta}{\sigma}\right) - \Phi\left(\frac{c_1 - \eta}{\sigma}\right)$$

es $\quad x$ peso delle barbabietole

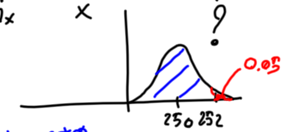
$\eta_x = 250\,g$ $\quad$ $f_x(x)$ x gaussiano normale

$\mathcal{N}(\eta_x, \sigma_x)$

?

5% barb.

5% peso > 252 g



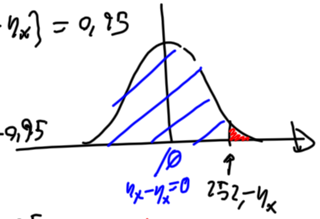
$$P\{x > 252\} = 0{,}05$$

$\int_{252}^{\infty} f_x(x)\,dx$ $\quad$ visto che $\int_{-\infty}^{+\infty} f_x(x)\,dx = 1$

$$P\{x \le 252\} = 1 - P\{x > 252\} = 95\% \quad (0{,}95)$$

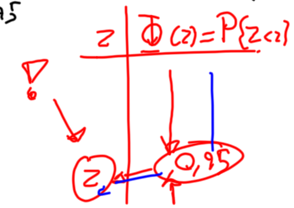
$$P\{x \le 252\} = 0{,}95 \qquad z = \frac{x - \eta_x}{\sigma_x}$$

$$P\{x - \eta_x \le 252 - \eta_x\} = 0{,}95$$



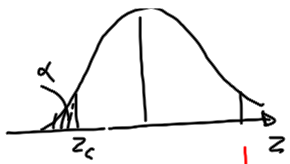
$$P\left\{\frac{x - \eta_x}{\sigma_x} \le \frac{252 - \eta_x}{\sigma_x}\right\} = 0{,}95$$

$$P\left\{z \le \frac{2}{\sigma_x}\right\} = 0{,}95$$

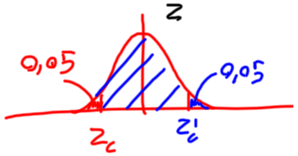


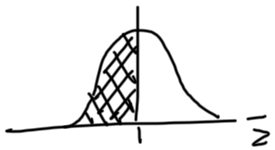
z	$\Phi(z)$
1,6449	0,95

$$\frac{2}{\sigma_x} = 1{,}6449 \longrightarrow \sigma_x = 1{,}215\,g$$



z_c	$\Phi(z_c)$
−1,6449	0,05
−2,3263	0,01
0	1/2





MATLAB

$F_x(x) \longrightarrow$ normcdf(·)

$f_x(x) \longrightarrow$ normpdf(·)

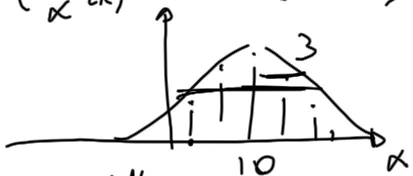
$\Phi(z_c) = \Phi(1,6449) =$

$F_x^{-1}(x)$ norminv(·)

norminv(0,01, 0,1)

Es

Disegnare $f_x(x)$ $N(10, 3)$



calcolare l'area della curva $x \in [-\infty, 5]$

$P\{X < 5\}$

prove ripetute

- ogni prova ha 2 esiti (successo / insuccesso)
- prob. succ. (o insucc.) sia uguale $\forall$ prova

probabilità di avere K successi su n prove

- ES.

T successo
C insuccesso

$\forall$ singola prova $P(T) = 1/2$

10 prove $\to$ calcolare la prob. di avere k successi

$P(0) = ?$ .... $P(10) = ?$
$P(1) = ?$

$$P(k) = \binom{n}{k} p^k q^{n-k}$$

p : prob. successo
$q = 1-p \to$ prob. insuccesso

MATLAB

```
>> p=0.5;
>> q=1-p;
>> k=[0:10]
>> cb = factorial(n)./(factorial(k).*
   factorial(n-k))
```

$>> n=10 \quad \binom{n}{k} = \frac{n!}{k!(n-k)!}$

$pk = [p(0) \; p(1) \; .... \; p(10)]$

$p(0) = \binom{n}{0} p^0 q^{n-0} = \binom{10}{0} 1 \cdot q^{10}$
$p(1) = \binom{n}{1} p^1 q^9 ....$

ES

105 donne
100 uomini

K donne su 5 figli?

$p = \frac{105}{205} = 0.5122$
$q = 1-p = 0,4878$

$p(0) = \binom{5}{0} p^0 q^5 = \frac{5!}{0!\,5!} 1 \cdot 0,4878^5 = 0.0276$

$p(1) = \binom{5}{1} p^1 q^4 = \frac{5!}{4!\,1!} 0,5122 \cdot 0.4878^4 = 0,14$

$p(2)$

.

$p(5)$ ....

$$p(K<4) = \sum_{K=0}^{3} p(K) = p(0)+p(1)+p(2)+p(3)$$

$n \quad p \quad q \qquad X_i \leftarrow$ singola prova

successo $X_i = 1$
insuccesso $X_i = 0$

$$E\{X_i\} = \sum_i x_i P\{X = x_i\} =$$
$$= 1 \cdot p + 0 \cdot q = p$$

$$E\{(X_i - \eta_x)^2\} = \sum_i (x_i - \eta_x)^2 P\{X = x_i\} =$$
$$= (1-p)^2 p + p^2(1-p) = (1-p)(p - p^2 + p^2) =$$

$\uparrow x_i = 0$

$$= p(1-p) = pq$$

$$X = \sum_{i=1}^{n} x_i$$

$$E\{X\} = E\left\{\sum_{i=1}^{n} x_i\right\} = \sum_{i=1}^{n} E\{x_i\} = np$$

$$\sigma_X^2 = \underset{E\{(X-\eta_x)^2\}}{\mathrm{var}(X)} = \mathrm{var}\left(\sum_{i=1}^{n} x_i\right) \overset{\triangle!}{=} \sum_{i=1}^{n} \mathrm{var}(X_i) = \sum_{i=1}^{n} pq = npq$$

MATLAB

$p = 0{,}2 \qquad q = 0{,}8 \qquad n = 10$

$p(K) \longrightarrow ?$

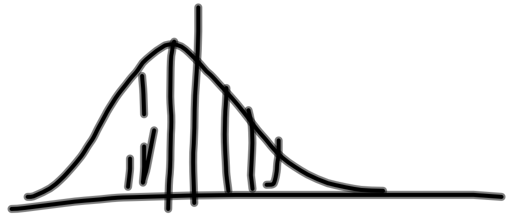
$p = 0{,}8 \qquad q = 0{,}2 \qquad n = 10$

$p(K) \longrightarrow ?$

teorema di Moivre - Laplace

$npq \gg$ bino $\rightarrow$ gaussiana

$\mathcal{N}(np, npq)$



?

a parità di n

quali sono i valori di p e q migliori?

(migliori: binomiale $\Rightarrow$ gaussiana)

$npq \gg$