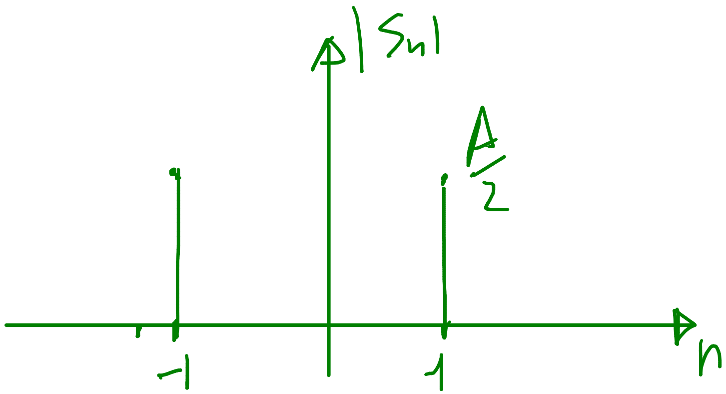
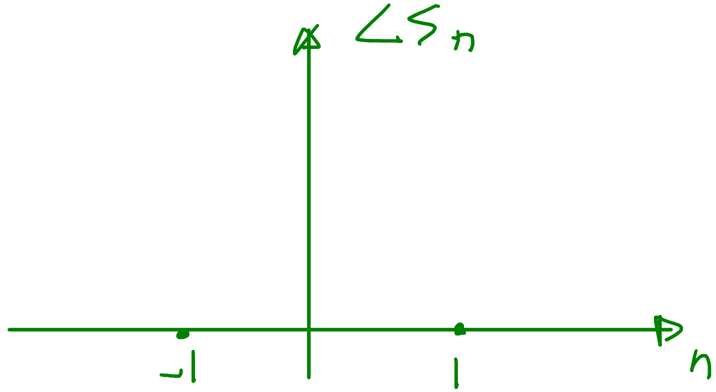
$s(t) = A\cos 2\pi f_1 t \qquad T_1 = 1/f_1$



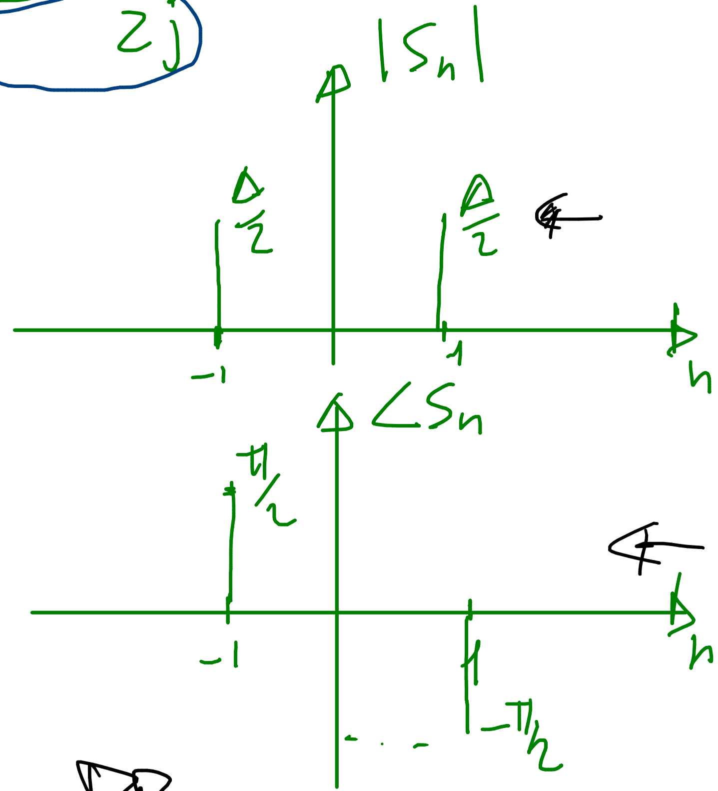


$$x(t) = A \sin(2\pi f_1 t) = A \, \frac{e^{j2\pi f_1 t} - e^{-j2\pi f_1 t}}{2j}$$

$$T_0 = \frac{1}{f_1} \qquad n=1 \quad , \quad n=-1$$

$$S_1 = \frac{A}{2j} = \frac{A}{2} e^{-j\frac{\pi}{2}}$$

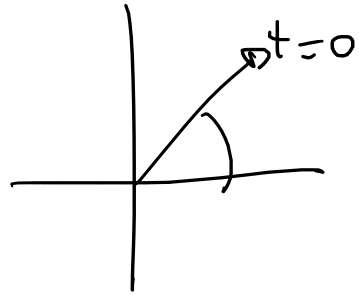
$$S_{-1} = -\frac{A}{2j} = e^{j\pi} \frac{A}{2} e^{-j\frac{\pi}{2}} = \frac{A}{2} e^{j\frac{\pi}{2}}$$



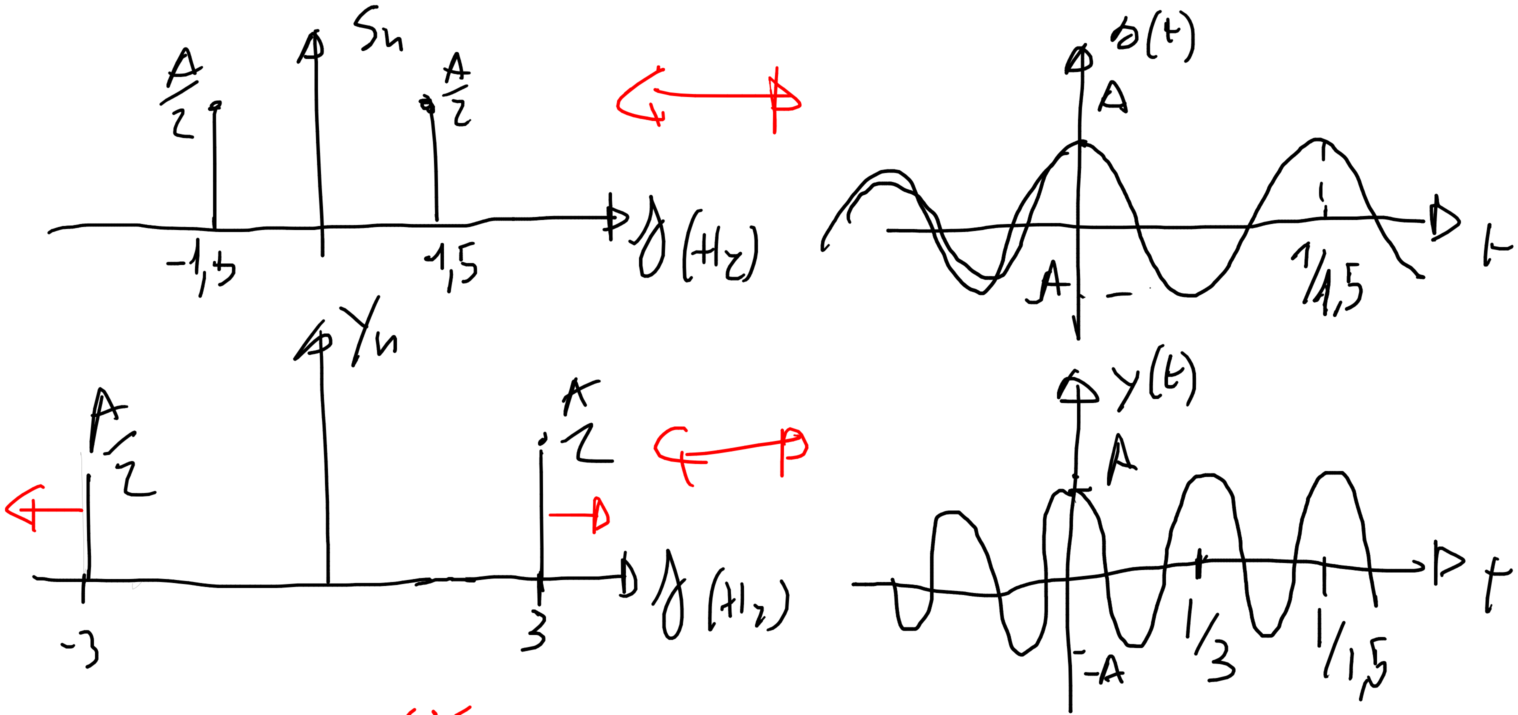
! $x(t) \in \mathbb{R} \Leftrightarrow S_n = S^*_{-n}$!

!! confrontare con spettro coseno !!

$S\, e^{-j2\pi f_1 t}$

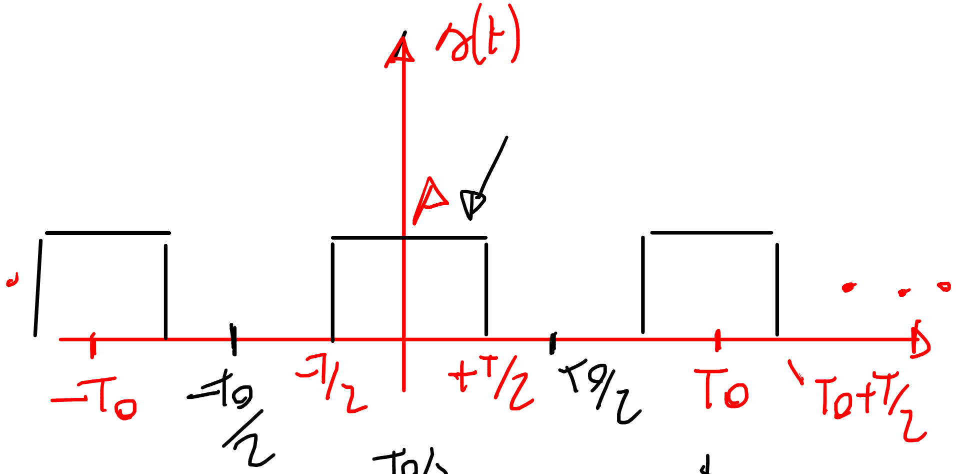


$$s(t) = A\cos\left(2\pi f_1 t - \frac{\pi}{2}\right)$$



N.B. $\angle S_n = 0$, $\angle Y_n = 0$

onda quadra $s(t) = A \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t-kT_0}{T}\right) \qquad T \le T_0$



$$S_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} s(t)\, e^{-j2\pi n \frac{t}{T_0}}\, dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A\, \text{rect}\left(\frac{t}{T}\right) e^{-j2\pi n \frac{t}{T_0}}\, dt =$$

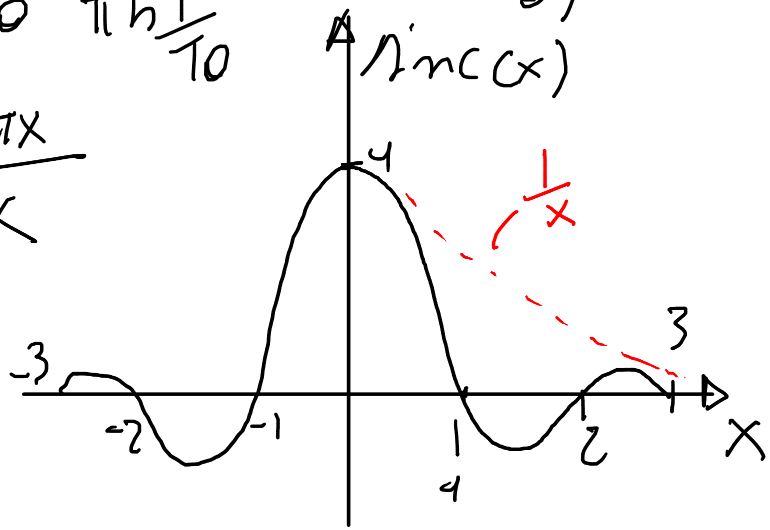
$$= \frac{1}{T_0}\int_{-T/2}^{T/2} A\, e^{-j2\pi n \frac{t}{T_0}}\, dt = \frac{A}{T_0}\, \frac{1}{\left(-j\frac{2\pi n}{T_0}\right)}\, e^{-j2\pi n \frac{t}{T_0}} \Bigg|_{-T/2}^{T/2} =$$

$$= \frac{A}{T_0}\,\frac{T_0}{(-j2\pi n)}\left(e^{-j\frac{2\pi n}{T_0}T/2} - e^{+j\frac{2\pi n T/2}{T_0}}\right) =$$

$$= \frac{A}{T_0}\,\frac{T_0}{\cancel{-j}\,\cancel{2}\pi n}\left(\cancel{-2j}\sin \frac{\cancel{2}\pi n T}{\cancel{2}T_0}\right) = \frac{A}{T_0}\,\frac{1}{\pi n \frac{1}{T_0}}\sin\left(\pi n \frac{T}{T_0}\right) =$$

$$\lim_{|n|\to\infty} |S_n| = 0 \qquad = \frac{AT}{T_0}\,\frac{1}{\pi n \frac{T}{T_0}}\sin\left(\pi n \frac{T}{T_0}\right) =$$

$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$



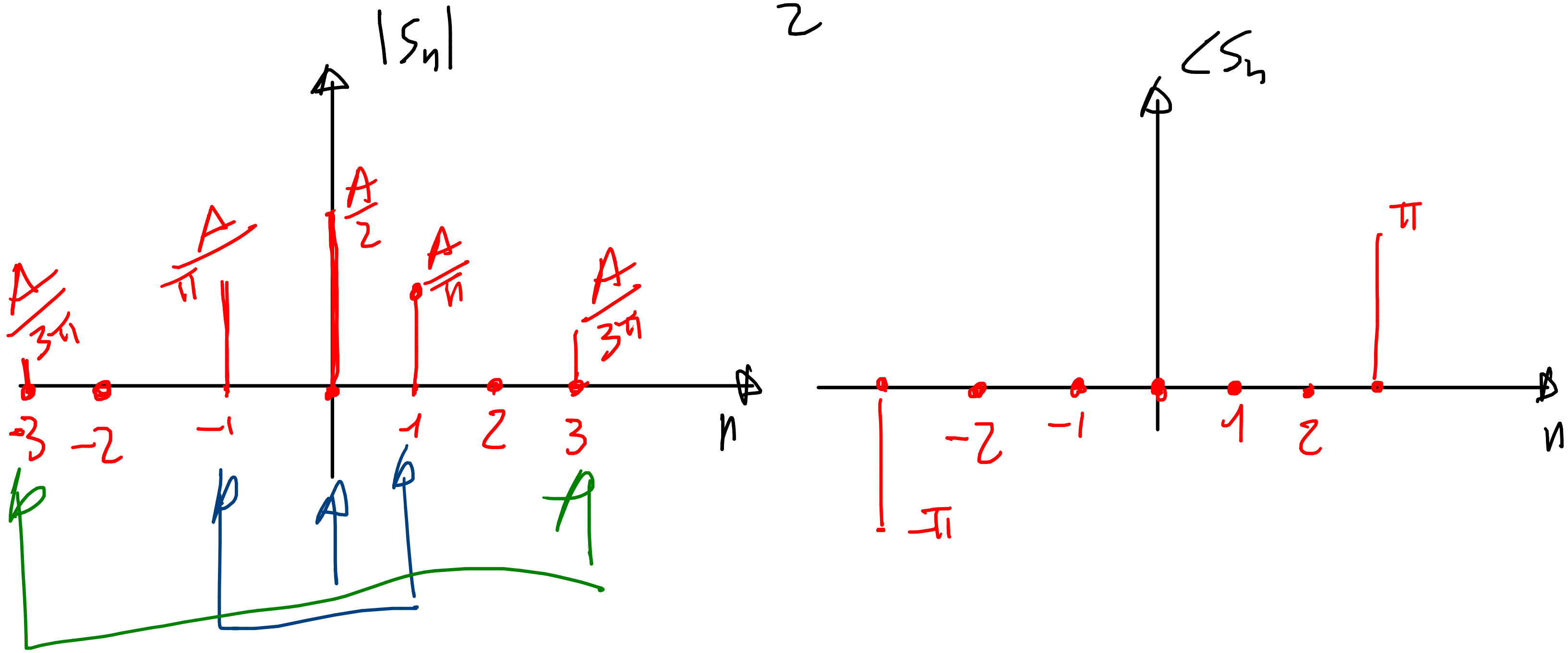
$$= \frac{AT}{T_0}\,\mathrm{sinc}\left(n\frac{T}{T_0}\right)$$

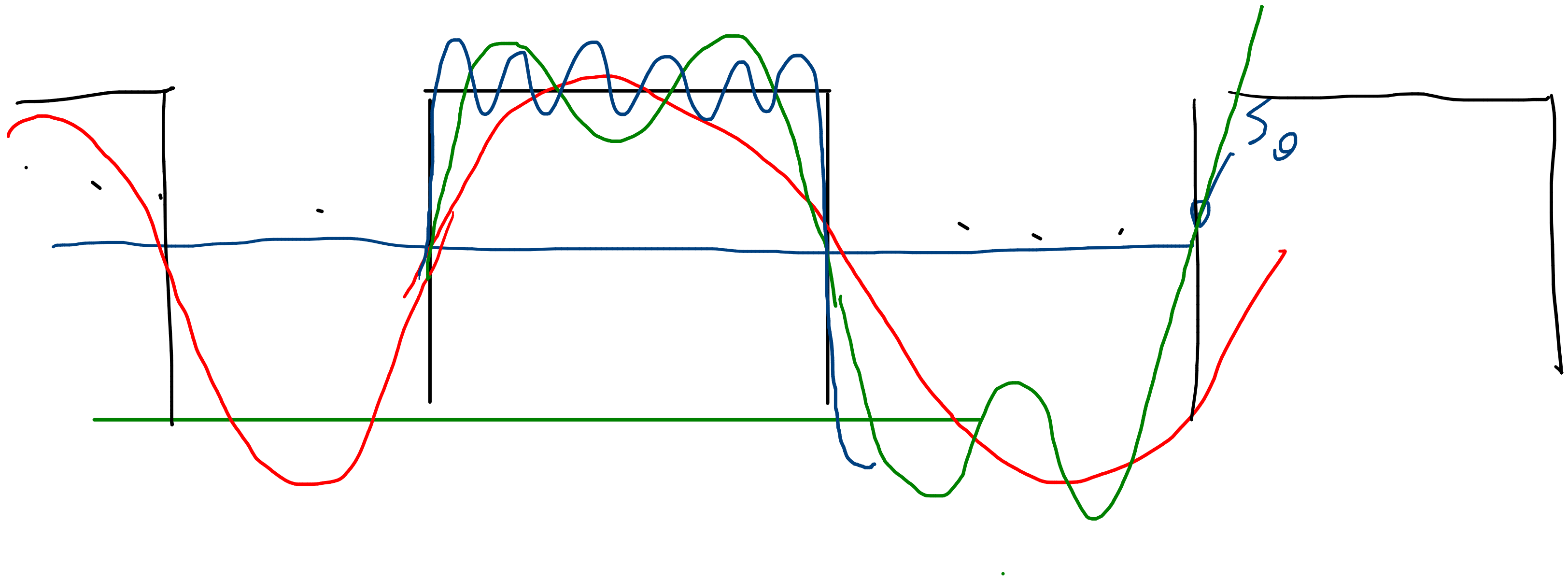
$$S_2 = \frac{A}{2}\,\frac{\sin \pi}{\pi} = \emptyset = S_{-2}$$

$$S_3 = \frac{A}{2}\,\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}} = -\frac{A}{3\pi} = \frac{A}{3\pi}e^{j\pi}$$

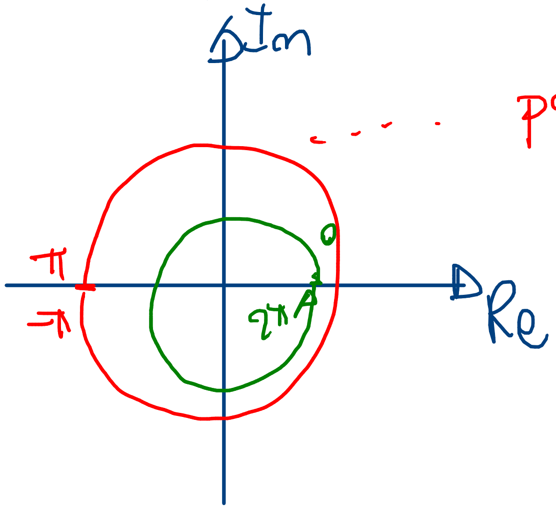
$$S_{-3} = \frac{A}{3\pi}e^{-j\pi}$$

$$S_n = \frac{A}{2}\,\mathrm{sinc}\left(\frac{n}{2}\right) = \frac{A}{2}\,\frac{\sin\frac{\pi n}{2}}{\frac{\pi n}{2}}$$




S_9

Come rappresentare la fase degli S_n?



possiamo avere dei grafici dove si evidenziano le simmetrie di fase

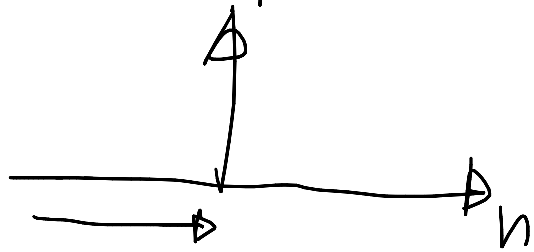
$s(t) \rightarrow S_n$ analisi di Fourier

$S_n \rightarrow s(t)$ operazione di ricostruzione o sintesi consideriamo $|n| \leq 3$

$$s(t) = \sum_{n=-\infty}^{\infty} S_n \, e^{j 2\pi n \frac{t}{T_0}}$$

se ricostruiamo il segnale con un numero finito di componenti

$$s(t) = \sum_{n=-3}^{3} S_n \, e^{j 2\pi n \frac{t}{T_0}}$$



$$s(t) = S_{-3} \, e^{-j 6\pi \frac{t}{T_0}} + S_{-1} \, e^{-j 2\pi \frac{t}{T_0}} + S_0 + S_1 e^{j 2\pi \frac{t}{T_0}} + S_3 e^{j 6\pi \frac{t}{T_0}} =$$

$$= \frac{A}{3\pi} e^{-j\pi} e^{-j 6\pi \frac{t}{T_0}} + \underline{\frac{A}{\pi} e^{-j 2\pi \frac{t}{T_0}}} + \overset{\frac{A}{2}}{S_0} + \underline{\frac{A}{\pi} e^{j 2\pi \frac{t}{T_0}}} + \frac{A}{3\pi} e^{j\pi} e^{j 6\pi \frac{t}{T_0}} =$$

$$= \frac{A}{2} + \frac{2A}{\pi} \cos 2\pi \frac{t}{T_0} + \frac{2}{3} \frac{A}{\pi} \cos\left(6\pi \frac{t}{T_0} + \pi\right) =$$

$$= \frac{A}{2} + \frac{2A}{\pi}\cos\left(2\pi\frac{t}{T_0}\right) - \frac{2A}{3\pi}\cos\left(6\pi\frac{t}{T_0}\right)$$