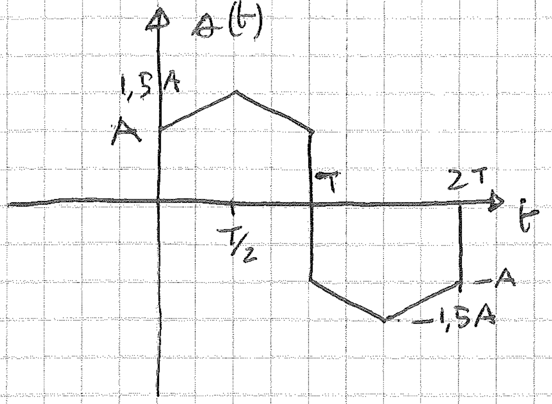
A.1

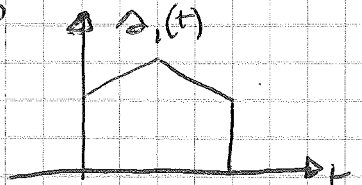
$A = 2V$

$T = 1s$

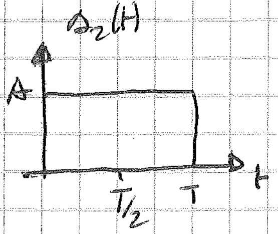


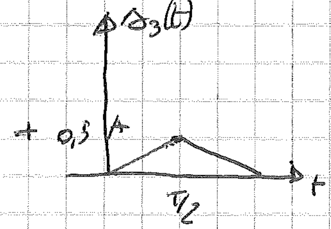
posso vedere $s(t)$ come $s(t) = s_1(t) - s_1(t-T)$

con



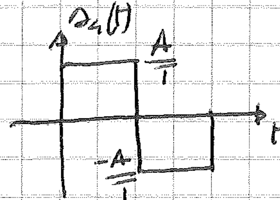
inoltre $s_1(t) = s_2(t) + s_3(t)$.





$S_2(f) = AT\,\mathrm{sinc}(fT)\,e^{-j\pi fT}$

$S_3(f) = \frac{S_4(f)}{j2\pi f} =$ $\qquad s_4(t) = \frac{d\,s_3(t)}{dt}$



$$= \frac{1}{j2\pi f}\left(\frac{A}{T}\frac{T}{2}\,\mathrm{sinc}\left(f\frac{T}{2}\right)e^{-j2\pi f\frac{T}{4}} - \frac{A}{T}\frac{T}{2}\,\mathrm{sinc}\left(f\frac{T}{2}\right)e^{-j2\pi f\frac{3}{4}T}\right) =$$

$$= \frac{A}{2}\,\mathrm{sinc}\left(f\frac{T}{2}\right)\frac{1}{j2\pi f}\left(e^{-j\pi f\frac{T}{2}} - e^{-j\pi f\frac{3}{2}T}\right) =$$

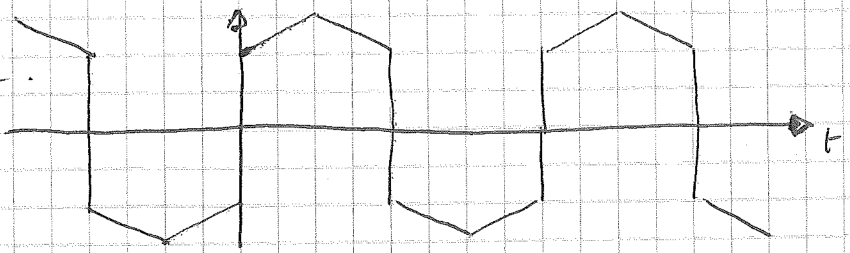
$$= \frac{A}{2}\,\mathrm{sinc}\left(f\frac{T}{2}\right)\frac{e^{-j\pi fT}}{j2\pi f}\left(e^{j\pi f\frac{T}{2}} - e^{-j\pi f\frac{T}{2}}\right) = \frac{A}{2}\,\mathrm{sinc}\left(f\frac{T}{2}\right)\frac{2j\sin(\pi f\frac{T}{2})e^{-j\pi fT}}{j2\pi f} =$$

$$= \frac{AT}{4}\,\mathrm{sinc}^2\left(f\frac{T}{2}\right)e^{-j\pi fT}$$

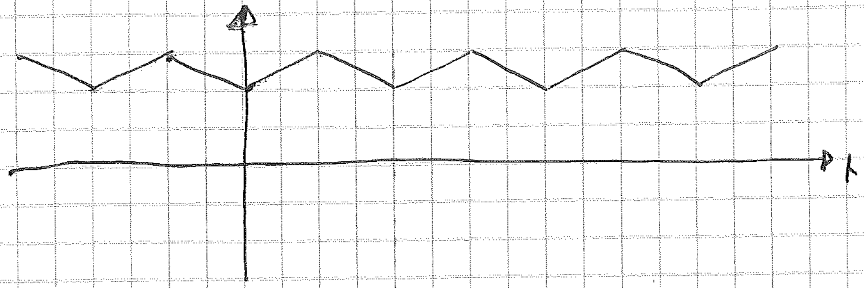
$$S_1(f) = S_2(f) + S_3(f) = \left[AT\,\mathrm{sinc}(fT) + A\frac{T}{4}\,\mathrm{sinc}^2\left(f\frac{T}{2}\right)\right]e^{-j\pi fT}$$

$$S(f) = S_1(f) - S_1(f)e^{-j2\pi fT} = \left[AT\,\mathrm{sinc}(fT) + \frac{AT}{4}\,\mathrm{sinc}^2\left(f\frac{T}{2}\right)\right]e^{-j2\pi fT}\,2j\sin\pi fT$$

$s_p(t) = rep_{2T}[s(t)]$



$s_2(t) = |s_p(t)|$



$s_p(t)$ è dispari $\Rightarrow$ $S_n = -S_{-n}$ visto che è anche reale $S_n = jI_n$

l'andamento del modulo dei coeff $n \to \infty$ $\quad \propto \frac{1}{n}$

il periodo è $2T$ $\Rightarrow$ $f_n = \frac{n}{2T}$

il valore medio è nullo $S_0 = 0$

$s_2(t)$ è pari $\Rightarrow S_n = S_{-n}$ visto che è anche reale $S_n = R_n$

l'andamento del modulo dei coeff per $n \to \infty$ $\quad \propto \frac{1}{n^2}$

il periodo è T $\Rightarrow$ $f_n = \frac{n}{T}$

il valore medio è diverso da 0 $S_0 = \frac{5}{4}A$

$$y(t) = x(t) + \sqrt{2}\, x(t-t_0) + x(t-2t_0)$$

Linearità

$$y_1(t) = x_1(t) + \sqrt{2}\, x_1(t-t_0) + x_1(t-2t_0)$$

$$y_2(t) = x_2(t) + \sqrt{2}\, x_2(t-t_0) + x_2(t-2t_0)$$

$$x_3(t) = a\, x_1(t) + b\, x_2(t)$$

$$y_3(t) = x_3(t) + \sqrt{2}\, x_3(t-t_0) + x_3(t-2t_0) =$$

$$= a\, x_1(t) + b x_2(t) + \sqrt{2}\, a\, x_1(t-t_0) + \sqrt{2}\, a\, x_2(t-t_0) + a\, x_1(t-2t_0) + b\, x_2(t-2t_0) =$$

$$= a\left(x_1(t) + \sqrt{2}\, x_1(t-t_0) + x_1(t-2t_0)\right) + b\left(x_2(t) + \sqrt{2}\, x_2(t-t_0) + x_2(t-2t_0)\right) =$$

$$= a\, y_1(t) + b\, y_2(t) \quad \Rightarrow \text{Lineare}$$

tempo invarianza

$$x_1(t) = x(t-t_A)$$

$$y_1(t) = x_1(t) + \sqrt{2}\, x_1(t-t_0) + x_1(t-2t_0) = x(t-T_A) + \sqrt{2}\, x(t-T_A-t_0) + x(t-2t_0-T_A)$$

questo è uguale a $y(t-T_A) \Rightarrow$ T. invariante

$$Y(f) = X(f) + \sqrt{2}\, X(f)\, e^{-j2\pi f t_0} + X(f)\, e^{-j4\pi f t_0} =$$

$$= X(f)\left(1 + \sqrt{2}\, e^{-j2\pi f t_0} + e^{-j4\pi f t_0}\right)$$

$$H(f) = \frac{Y(f)}{X(f)} = 1 + \sqrt{2}\, e^{-j2\pi f t_0} + e^{-j4\pi f t_0} = e^{-j2\pi f t_0}\left(e^{j2\pi f t_0} + \sqrt{2} + e^{-j2\pi f t_0}\right) =$$

$$= e^{-j2\pi f t_0}\left(\sqrt{2} + 2\cos(2\pi f t_0)\right)$$

$$|H(f)| = |\sqrt{2} + 2\cos 2\pi f t_0|$$

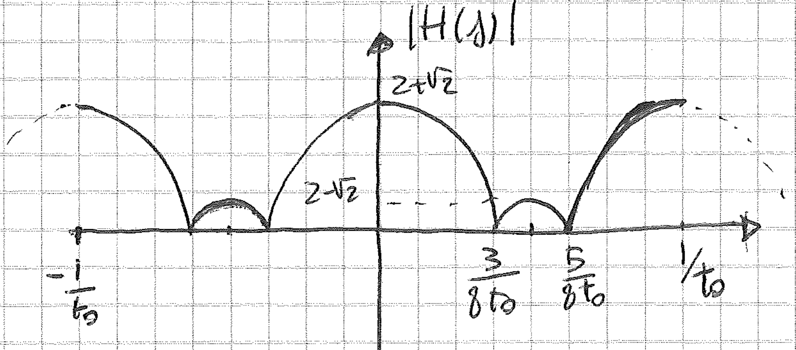
il modulo si annulla per

$$\cos 2\pi f t_0 = -\frac{\sqrt{2}}{2}$$

$$2\pi f t_0 = \begin{cases} \frac{3}{4}\pi + 2K\pi \\ \frac{5}{4}\pi + 2K\pi \end{cases}$$

$$f_{1,K} = \frac{3}{8t_0} + \frac{K}{t_0}$$

$$f_{2,K} = \frac{5}{8t_0} + \frac{K}{t_0}$$



$$x(t) = \sin\left(\frac{3}{4}\frac{\pi t}{t_0}\right) + \cos\left(\frac{\pi t}{t_0}\right)$$

$$X(f) = \frac{1}{2j}\,\delta\left(f - \frac{3}{8t_0}\right) - \frac{1}{2j}\,\delta\left(f + \frac{3}{8t_0}\right) + \frac{1}{2}\,\delta\left(f - \frac{1}{2t_0}\right) + \frac{1}{2}\,\delta\left(f + \frac{1}{2t_0}\right)$$

$$Y(f) = \frac{1}{2j} H\left(\frac{3}{8t_0}\right) \delta\left(f - \frac{3}{8t_0}\right) - \frac{1}{2j} H\left(-\frac{3}{8t_0}\right) \delta\left(f + \frac{3}{8t_0}\right) +$$
$$+ \frac{1}{2} H\left(\frac{1}{2t_0}\right) \delta\left(f - \frac{1}{2t_0}\right) + \frac{1}{2} H\left(-\frac{1}{2t_0}\right) \delta\left(f + \frac{1}{2t_0}\right)$$

vista la simmetria coniugata di $H(f)$ per la quale $H(f) = H^*(-f)$

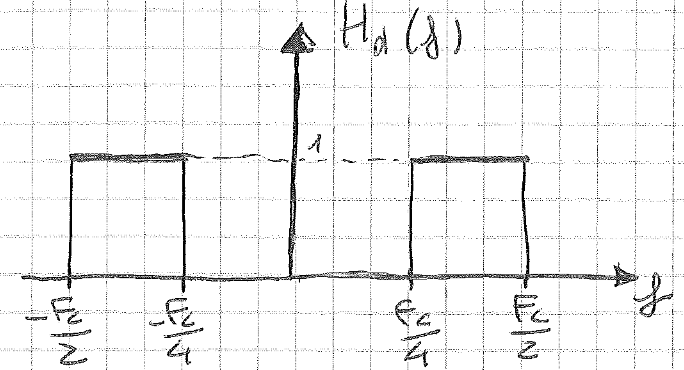
$$Y(f) = \frac{1}{2j} \left|H\left(\frac{3}{8t_0}\right)\right| e^{j\angle H\left(\frac{3}{8t_0}\right)} \delta\left(f - \frac{3}{8t_0}\right) - \frac{1}{2j} \left|H\left(+\frac{3}{8t_0}\right)\right| e^{-j\angle H\left(\frac{3}{8t_0}\right)} \delta\left(f + \frac{3}{8t_0}\right) +$$
$$+ \frac{1}{2} \left|H\left(\frac{1}{2t_0}\right)\right| e^{j\angle H\left(\frac{1}{2t_0}\right)} \delta\left(f - \frac{1}{2t_0}\right) + \frac{1}{2} \left|H\left(\frac{1}{2t_0}\right)\right| e^{-j\angle H\left(\frac{1}{2t_0}\right)} \delta\left(f + \frac{1}{2t_0}\right)$$

$$y(t) = \left|H\left(\frac{3}{8t_0}\right)\right| \sin\left(\frac{3\pi t}{4t_0} + \angle H\left(\frac{3}{8t_0}\right)\right) + \left|H\left(\frac{1}{2t_0}\right)\right| \cos\left(\frac{\pi t}{t_0} + \angle H\left(\frac{1}{2t_0}\right)\right)$$

visto che $\left|H\left(\frac{3}{8t_0}\right)\right| = 0$ e $H\left(\frac{1}{2t_0}\right) = (\sqrt{2} - 2)e^{-j\pi} = 2 - \sqrt{2}$

$$y(t) = (2 - \sqrt{2}) \cos\left(\frac{\pi t}{t_0}\right)$$

C.1



$\bar{H}_d(f)$ Risposta in freq. desiderata

antitrasformo per trovare $h_d[n]$ risp. imp.

$$h_d[n] = T_c \int_{-\frac{F_c}{2}}^{\frac{F_c}{2}} \bar{H}_d(f)\, e^{j2\pi n f T_c}\, df =$$

$$= T_c \int_{-\frac{F_c}{2}}^{-\frac{F_c}{4}} e^{+j2\pi n f T_c}\, df + T_c \int_{+\frac{F_c}{4}}^{\frac{F_c}{2}} e^{j2\pi n f T_c}\, df =$$

$$= \frac{T_c}{j2\pi n T_c}\, e^{j2\pi n f T_c}\Big|_{-\frac{F_c}{2}}^{-\frac{F_c}{4}} + \frac{T_c}{j2\pi n T_c}\, e^{j2\pi n f T_c}\Big|_{\frac{F_c}{4}}^{\frac{F_c}{2}} =$$

$$= \frac{e^{-j\frac{\pi n}{2}} - e^{-j\pi n}}{j2\pi n} + \frac{e^{j\pi n} - e^{+j\frac{\pi n}{2}}}{j2\pi n} =$$

$$= \frac{2j \sin \pi n}{2j\pi n} - \frac{2j \sin \frac{\pi n}{2}}{2j\pi n} = \mathrm{sinc}(n) - \frac{1}{2}\mathrm{sinc}\left(\frac{n}{2}\right)$$

Per ottenere un filtro FIR causale bisogna troncare, in modo simmetrico rispetto a $n=0$ in modo da garantire fase lineare, e traslare

il filtro FIR avrà $2M+1$ campioni, per cui

$$h[n] = \mathrm{sinc}(n-M) - \frac{1}{2}\mathrm{sinc}\left(\frac{n-M}{2}\right) \quad \text{per } n = 0, 1, 2, \dots, 2M$$

$$y(t) = x(t) - \frac{1}{2\pi}\frac{d}{dt}y(t)$$

trasf. ingresso-uscita di un sistema LTI

$x(t)$ è l'ingresso

Calcolare la risp. in frequenza

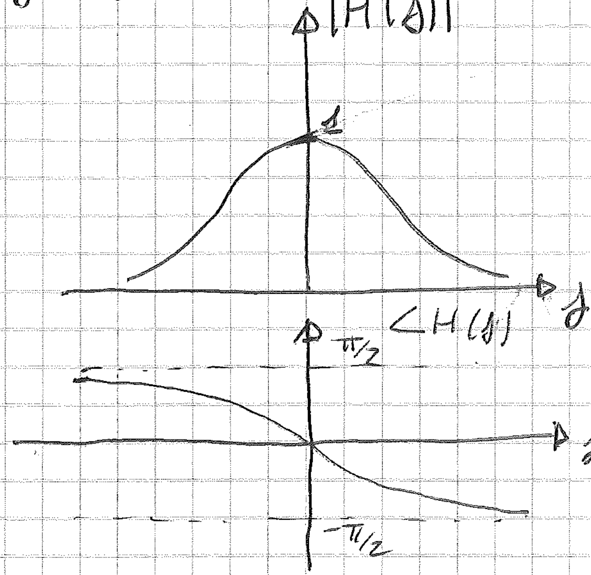
- un modo consiste nel definire $H(f) = \dfrac{Y(f)}{X(f)}$

applico la TCF ad entrambi i membri dell'equazione

$$Y(f) = X(f) - \frac{1}{2\pi} j2\pi f\, Y(f)$$

$$\Downarrow$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{1}{1+jf}$$



$$|H(f)| = \frac{1}{\sqrt{1+f^2}}$$

$$\angle H(f) = -\mathrm{atg}(f)$$

Se in ingresso abbiamo

$$x(t) = 1 + \sin(10\pi t)$$

sfrutto la rappr. in frequenza $\quad X(f) = \delta(f) + \frac{1}{2j}\delta(f-5) - \frac{1}{2j}\delta(f+5)$

allora $\quad Y(f) = H(f)\,X(f) =$

$$= H(f)\left[\delta(f) + \frac{1}{2j}\delta(f-5) - \frac{1}{2j}\delta(f+5)\right] =$$

$$= H(0)\,\delta(f) + \frac{H(5)}{2j}\delta(f-5) - \frac{H(-5)}{2j}\delta(f+5) =$$

sapendo che il sistema è reale e quindi $H(f) = H^*(-f)$

posso scrivere

$$= H(0)\,\delta(f) + \frac{H(5)}{2j}\delta(f-5) - \frac{H^*(5)}{2j}\delta(f+5) =$$

$$= H(0)\,\delta(f) + \frac{|H(5)|}{2j}e^{j\angle H(5)}\delta(f-5) - \frac{|H(5)|}{2j}e^{-j\angle H(5)}\delta(f+5)$$

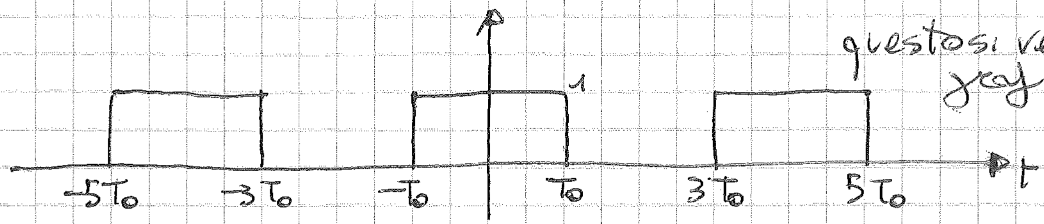
da cui antitrasformando $\quad y(t) = 1 + |H(5)|\sin(10\pi t + \angle H(5)) =$

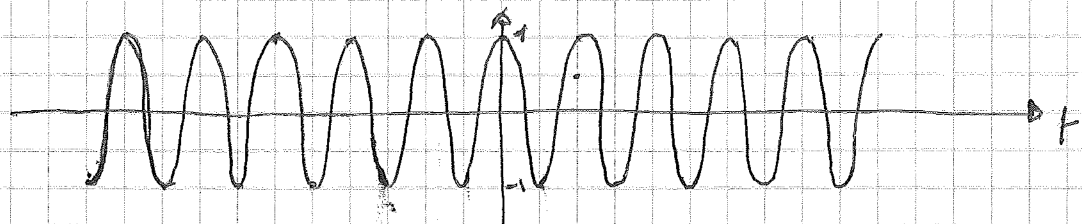
$$= 1 + \frac{1}{\sqrt{26}}\sin(10\pi t - \mathrm{atg}(5))$$

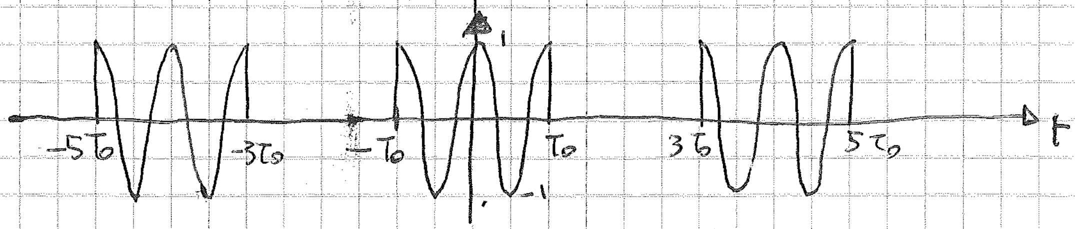
E.1

$s(t) = \cos\frac{2\pi t}{T_0} \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t-4kT_0}{2T_0}\right)$ ← è periodico di periodo $4T_0$

questo si vede bene dai grafici







un modo per trovare i coeff. dello sviluppo di $s(t)$ è partire dalla TCF del segnale aperiodico

$$s_1(t) = \cos\frac{2\pi t}{T_0}\,\text{rect}\left(\frac{t}{2T_0}\right)$$

$$S_1(f) = 2T_0\,\text{sinc}(2T_0 f) \otimes \frac{\delta(f-f_0)+\delta(f+f_0)}{2} =$$

$$= \frac{2T_0}{2}\text{sinc}\left(2T_0(f-f_0)\right) + \frac{2T_0}{2}\text{sinc}\left(2T_0(f+f_0)\right) =$$

$$= T_0\,\text{sinc}(2T_0 f - 2) + T_0\,\text{sinc}(2T_0 f + 2)$$

i coeff. dello sviluppo di $s(t)$ si trovano come

$$S_n = \frac{1}{4T_0} S_1\left(\frac{n}{4T_0}\right)$$

$$S_n = \frac{1}{4}\text{sinc}\left(\frac{n}{2}-2\right) + \frac{1}{4}\text{sinc}\left(\frac{n}{2}+2\right)$$

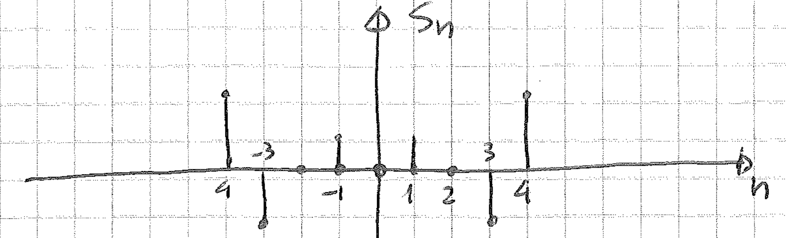
$S_1 = \frac{1}{4}\text{sinc}\left(-\frac{3}{2}\right) + \frac{1}{4}\text{sinc}\left(\frac{5}{2}\right) = \frac{1}{4}\left(-\frac{2}{3\pi} + \frac{2}{5\pi}\right) = -0,0849$ $\Rightarrow S_{-1} = S_1$

$S_2 = \frac{1}{4}\text{sinc}(-1) + \frac{1}{4}\text{sinc}(3) = 0$ $\Rightarrow S_2 = S_{-2}$

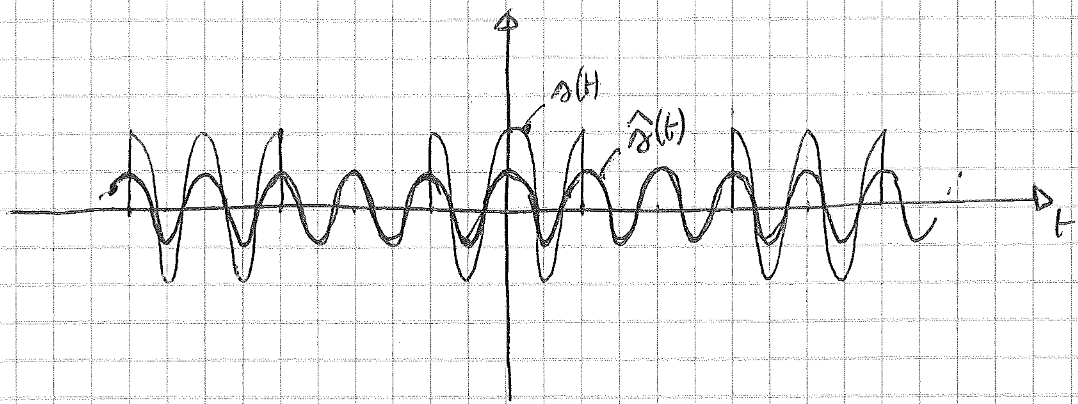
$S_3 = \frac{1}{4}\text{sinc}\left(-\frac{1}{2}\right) + \frac{1}{4}\text{sinc}\left(\frac{7}{2}\right) = \cancel{0,1416}$ $0,1364$ $\Rightarrow S_{-3} = \cancel{0,1416}$ $-0,1364$

$S_4 = \frac{1}{4}\text{sinc}(0) + \frac{1}{4}\text{sinc}(8) = \frac{1}{4}$ $\Rightarrow S_{-4} = \frac{1}{4}$

E.2



$$\hat{s}(t) = S_4 e^{j\frac{8\pi t}{4T_0}} + S_{-4} e^{-j\frac{8\pi t}{4T_0}} = \frac{1}{2}\cos\frac{8\pi t}{4T_0} = \frac{1}{2}\cos\frac{2\pi t}{T_0}$$



- $s(t) = \sum_{k=-\infty}^{\infty} \mathrm{rect}(t-k6) + \sum_{h=-\infty}^{\infty} S_h \, e^{j\frac{2\pi t h}{15}}$

il periodo è il m.c.m. tra 6 e 15 $\Rightarrow T_0 = 30$

- Campionamento passa banda se $f_{min} = 20$ e $f_{max} = 42$ Hz

$B = 22 \qquad n = \left\lfloor \frac{f_{max}}{B} \right\rfloor = \left\lfloor \frac{42}{22} \right\rfloor = 1$

$F_c = \frac{2F_{max}}{n} = 84$ Hz

- $s(t) = 1 + \sin\left(\frac{\pi}{5}t\right) + \sin\left(\frac{\pi t}{6}\right)$

max tempo di campionamento?

$f_{max} = \frac{1}{10} \qquad (f_{min} = 0)$

$F_c \geq 2 f_{max} \qquad t_c \leq \frac{1}{2 f_{max}} = 5$

- $x[n] = [u[n] - u[n-2]] \otimes \sum_{h=-\infty}^{\infty} \delta[n-h6] \qquad \tilde{X}_1 = ?$

$\tilde{X}_k = \frac{1}{6} \sum_{n=0}^{5} x[n] e^{-j\frac{2\pi n k}{6}} = \frac{1}{6}\left[1 + e^{-j\frac{2\pi k}{6}}\right]$

$\tilde{X}_1 = \frac{1}{6}\left[1 + e^{-j\frac{2\pi}{6}}\right] = \frac{1}{6}\left[1 + \cos\frac{\pi}{3} - j\sin\frac{\pi}{3}\right] = \frac{1}{6}\left[\frac{3}{2} - j\frac{\sqrt{3}}{2}\right] =$

$= \frac{1}{4} - j\frac{1}{4\sqrt{3}}$