Esercizi di calcolo combinatorio I

Esercizio A1

4. num. colori $=7$

1) $D_{7,6} = \frac{7!}{1!} = 7!$

2) $7 \times D^{(r)}_{6,3} = 7 \cdot 6^3$

3) $p = 0.8$

$P_{10}(7) = \binom{10}{7} 0{,}8^7 0{,}2^3 = \frac{10!}{7!\,3!} \, 0{,}8^7 \, 0{,}2^3 \cong 0{,}2$

Esercizio A2

10 palline Rosse

5 palline nere

a) I)[1] $P = \frac{C_{5,2}}{C_{15,2}} = \frac{5!}{3!\,2!} \, \frac{13!\,2!}{15!} = \frac{5 \cdot 4}{2} \cdot \frac{2}{15 \cdot 14} = \frac{5 \cdot 4}{15 \cdot 14} = 0.095$

II) $p_1 = \frac{5}{15}$ $\quad p_2 = \frac{4}{14}$

p_1 = prob. estrarre 1 pallina nera (1° estrazione)

p_2 = prob. estrarre 1 pallina nera, dopo averne estratta 1 nera

$p = p_1 \cdot p_2$

b) I) $P = \frac{C_{10,5}}{C_{15,5}} = \frac{10!}{5!\,5!} \cdot \frac{5!\,10!}{15!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11} = 0{,}0839$

II) $p_1 = \frac{10}{15}$ $\quad p_2 = \frac{9}{14}$ $\quad p_3 = \frac{8}{13}$ $\quad p_4 = \frac{7}{12}$ $\quad p_5 = \frac{6}{11}$

$p = p_1 p_2 p_3 p_4 p_5$

c) $P(\text{almeno 1 rosso}) = 1 - P(\text{tutte palline nere}) = 1 - \frac{C_{5,5}}{C_{15,5}} = 1 - \frac{10!\,5!}{15!} =$

$= 1 - \frac{5 \cdot 4 \cdot 3 \cdot 2}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11} = 0{,}9997$

d) $p = \frac{5}{15} = \frac{1}{3}$ $\quad q = \frac{2}{3}$ $\qquad n = 5$ num. estrazioni $\qquad$ 3 successi

$P_5(3) = \binom{5}{3} p^3 q^2 = \frac{5!}{3!\,2!} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 = \frac{5 \cdot 4}{2} \cdot \frac{1}{27} \cdot \frac{4}{9} = 0{,}165$

<u>II° modo c</u>

casi favorevoli $\quad F = C_{10,5} + C_{10,4} \cdot C_{5,1} + C_{10,3} \cdot C_{5,2} + C_{10,2} \cdot C_{5,3} + C_{10,1} \cdot C_{5,4}$

casi possibili $\quad F + C_{5,5}$

$P = \frac{F}{F + C_{5,5}}$

[1] con I e II si intendono due possibili modi per risolvere l'esercizio

Esercizio A3

$p=0,7 \quad q=1-p=0,3$

a) $P_5(2)=\binom{5}{2}p^2q^3=\frac{5!}{3!\,2!}(0,7)^2(0,3)^3=10\cdot 0,7^2\, 0,3^3=0,13$

b) $P\{\text{almeno 3 valide}\}=P_5(3)+P_5(4)+P_5(5)=\binom{5}{3}(0,7)^3\,0,3^2+\binom{5}{4}\,0,7^4\,0,3+\binom{5}{5}\,0,7^5=$

$=\frac{5!}{3!\,2!}\,0,7^3\,0,3^2+\frac{5!}{4!\,1!}\,0,7^4\,0,3+\;0,7^5=10\cdot 0,7^3\cdot 0,3^2+5\cdot 0,7^4\cdot 0,3+0,7^5=0,837$

$\boxed{N.B.}$ $P\{X\geq x_i\}=\sum_{X\geq x_i} p_i=P_5(3)+P_5(4)+P_5(5)$

$P\{X\geq x_i\}=1-P\{X<x_i\}=1-\sum_{X<x_i} p_i=$

$=1-P_5(1)-P_5(2)$

Esercizio A4

$Prev=1/20=0.05$

$spec=0,99$

$$P_{m,tp}=\frac{sens\cdot prev}{sens\cdot prev+(1-spec)\cdot(1-prev)}$$

$$0,83=\frac{sens\cdot 0,05}{sens\cdot 0,05+0,01\cdot 0,95}$$

$0,83\cdot 0,05\cdot sens+0,83\cdot 0,01\cdot 0,95=sens\cdot 0,05$

$$sens=\frac{0,83\cdot 0,01\cdot 0,95}{0,17\cdot 0,05}=0,93$$

Esercizio A5

malati 1000
sani 1000

	P	N
m	VP	FN
s	FP	VN

$FN=50 \Rightarrow VP=950$
$FP=5 \Rightarrow VN=995$

$$\text{Sensibilità}=\frac{VP}{VP+FN}=\frac{950}{1000}=95\%$$

$$Spec=\frac{VN}{FP+VN}=\frac{995}{1000}=99,5\%$$

$$P_{m,tp}=\frac{sens*prev}{[sens*prev+(1-prev)*(1-spec)]}=\frac{0,95\cdot 0,1}{0,1\cdot 0,95+0,9\cdot 0,005}=\frac{0,095}{0,095+0,0045}$$

$$=\frac{0,095}{0,0995}=0,95477$$

Esercizio A6

$p = 0,02$

$n = 30$

a) $k = 5$ $\quad P_{30}(5) = \binom{5}{30} p^5 q^{30-5} = \binom{5}{30} 0,02^5 \, 0,98^{25} = \frac{5!}{30!\,25!} 0,02^5 \, 0,98^{25}$

b) $P = P_{30}(5) + P_{30}(6) + P_{30}(7) + P_{30}(8) + P_{30}(9) + P_{30}(10) =$

$\binom{5}{30} 0,02^5 \, 0,98^{25} + \binom{6}{30} 0,02^6 \, 0,98^{24} + \binom{7}{30} 0,02^7 \, 0,98^{23} + \binom{8}{30} 0,02^8 \, 0,98^{22} +$

$+ \binom{9}{30} 0,02^9 \, 0,98^{21} + \binom{10}{30} 0,02^{10} \, 0,98^{20}$

Esercizio A7

Parte 2

Si utilizzi la binomiale per calcolare la probabilità che un giocatore di pallacanestro, con una percentuale sul tiro da 6 metri del 60%, realizzi 4 canestri su 5 tiri, e la probabilità che realizzi almeno 3 canestri su 5 tiri

a) $P = 0,6 \quad n = 5 \quad K = 4$

$P_5(4) = \frac{5!}{4!\,1!} p^4 q = 5 \cdot 0,6^4 \, 0,4 = 0,25$

b) $P\{\text{almeno 3 canestri su 5 tiri}\} = P_5(3) + P_5(4) + P_5(5) =$

$= \frac{5!}{3!\,2!} 0,6^3 \cdot 0,4^2 + \frac{5!}{4!\,1!} 0,6^4 \cdot 0,4 + \frac{5!}{5!\,0!} 0,6^5 \, 0,4^0 =$

$= 0,6826$

Esercizio A8

3) a) $1/D_{7,7} = 0,000198\% \; \left(= \frac{1}{P_7}\right)$

b) $\frac{C_{7,2} \cdot P_5}{P_7} = \frac{1}{7} \cdot \frac{1}{6} = 2,38\%$

c) $n = 4 \quad p = 1/7$

$- P_4(3) = \frac{4!}{3!\,1!} p^3 (1-p) = 1\%$

d) $- P_4(0) + P_4(1) + P_4(3) = 98,96\%$

Esercizio A9

probabilità difetto $p = \frac{1}{10}$ $\Rightarrow$ prob. protesi non difettosa $q = \frac{9}{10}$

a) $P_8(4) = \binom{8}{4} p^4 q^{8-4} = \frac{8!}{4!4!} \left(\frac{1}{10}\right)^4 \left(\frac{9}{10}\right)^4 = \frac{\cancel{8}^{2}\cdot 7\cdot \cancel{6}^{2}\cdot 5}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}\, 10^{-4} \cdot 0{,}6561 = 0{,}46\%$

b) $P = \sum_{K=3}^{8} P_8(K) = 1 - \sum_{K=0}^{2} P_8(K) = P_8(3) + P_8(4) + P_8(5) + P_8(6) + P_8(7) + P_8(8)$

$= \binom{8}{3}\left(\frac{1}{10}\right)^3\left(\frac{9}{10}\right)^5 + \binom{8}{4}\left(\frac{1}{10}\right)^4\left(\frac{9}{10}\right)^4 + \binom{8}{5}\left(\frac{1}{10}\right)^5\left(\frac{9}{10}\right)^3 + \binom{8}{6}\left(\frac{1}{10}\right)^6\left(\frac{9}{10}\right)^2 + \binom{8}{7}\left(\frac{1}{10}\right)^7\left(\frac{9}{10}\right)^1 + \binom{8}{8}\left(\frac{1}{10}\right)^8\left(\frac{9}{10}\right)^0 =$

$= 1 - \binom{8}{2}\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^6 - \binom{8}{1}\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^7 - \binom{8}{0}\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^8 = 3{,}8\%$

Altro modo

prob. non difetto $p = \frac{9}{10}$ prob. difetto $q = \frac{1}{10}$

a) $P\{4 \text{ difettose}\} = P\{4 \text{ non difettose}\} = P_8(4) = \binom{8}{4} p^4 q^{8-4} = \frac{8!}{4!4!}\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)^4$

b) probabilità $P\{\text{almeno 3 difettose}\} = P\{\text{non difettose} \leq 5\} =$

$= \sum_{K=0}^{5} P_8(K) = P_8(0) + P_8(1) + P_8(2) + P_8(3) + P_8(4) + P_8(5) =$

$= \binom{8}{0}\left(\frac{9}{10}\right)^0\left(\frac{1}{10}\right)^8 + \binom{8}{1}\left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^7 + \binom{8}{2}\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)^6 + \binom{8}{3}\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^5 + \binom{8}{4}\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)^4 +$

$+ \binom{8}{5}\left(\frac{9}{10}\right)^5\left(\frac{1}{10}\right)^4 = 1 - \sum_{K=6}^{8} P_8(K) = 1 - \binom{8}{6}\left(\frac{9}{10}\right)^6\left(\frac{1}{10}\right)^2 - \binom{8}{7}\left(\frac{9}{10}\right)^7\left(\frac{1}{10}\right)^1 - \binom{8}{8}\left(\frac{9}{10}\right)^8\left(\frac{1}{10}\right)^0$

$\boxed{\text{N.B.}}$

$\binom{n}{k} = \binom{n}{n-k}$

$\binom{8}{0} = \binom{8}{8}$

$\binom{8}{2} = \binom{8}{6}$