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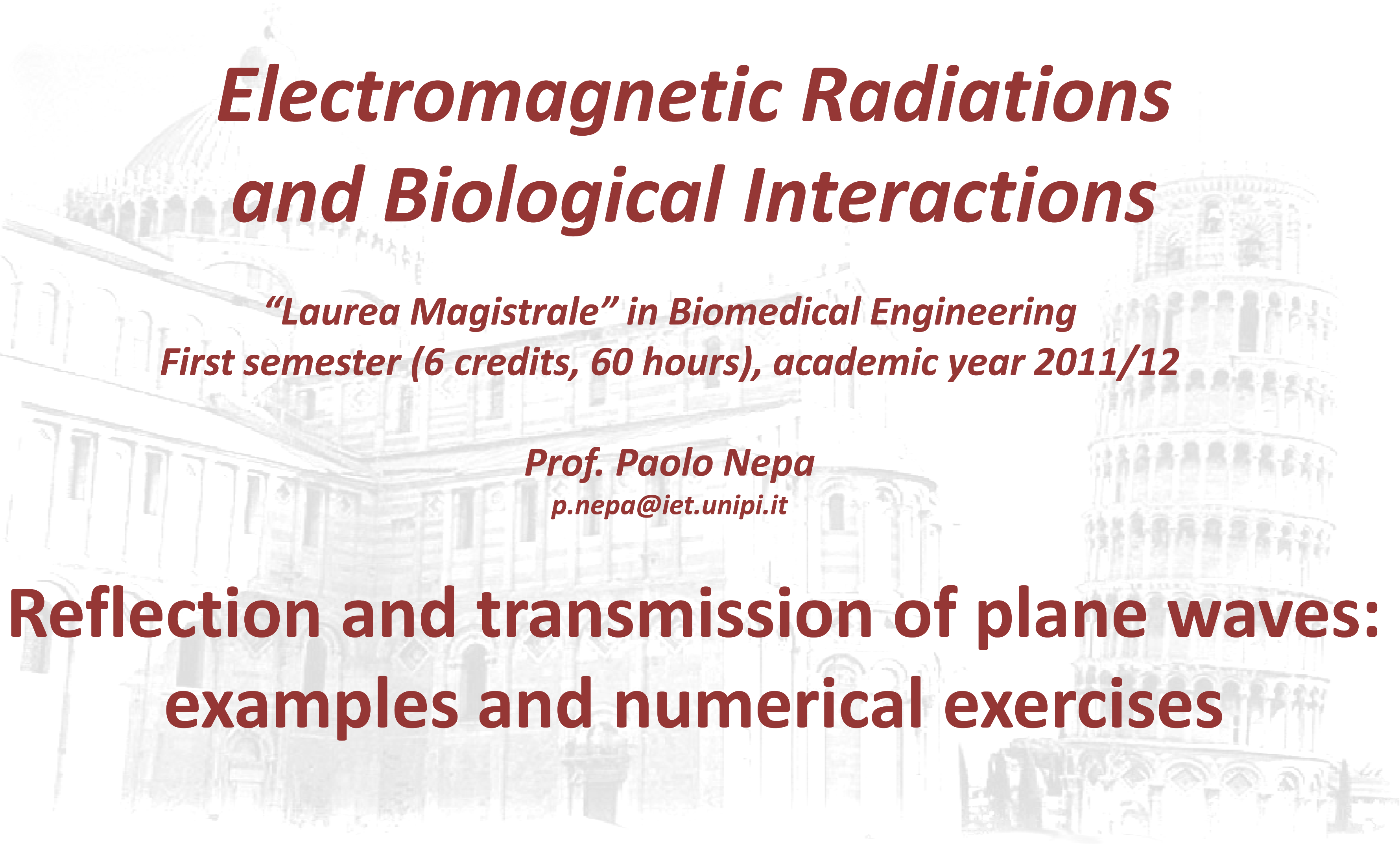

Electromagnetic Radiations and Biological Interactions

"Laurea Magistrale" in Biomedical Engineering
First semester (6 credits, 60 hours), academic year 2011/12

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Reflection and transmission of plane waves: examples and numerical exercises

Edited by Dr. Anda Guraliuc

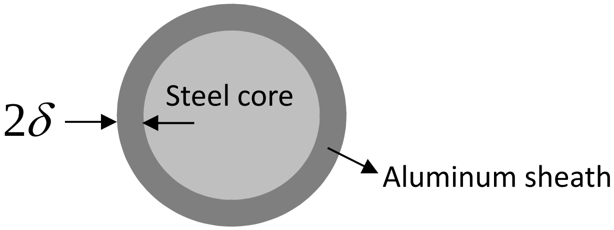
Microwave&Radiation Laboratory

Construction of a 60Hz high-voltage power line

Design considerations:

- ✓ the metal must have strength to support the long length of line.
- ✓ the metal must have good conductivity to reduce the ohmic losses

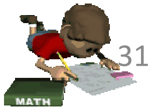
For the strengthen: steel is an option, but it has low conductivity ($\sigma_{steel} = 0.1\sigma_{cooper}$). The solution consists in using a core of steel (it provides strength) surrounded by a sheath of aluminum, which has a better conductivity ($\sigma_{aluminum} = 0.61\sigma_{cooper}$).



The 60Hz currents will flow primarily in the aluminum sheath as long as its thickness is a few skin depths.

$$@60\text{Hz}: \boxed{\delta_{aluminum} = \frac{1}{\sqrt{\pi f \mu \sigma}} \approx 1.1 cm} \qquad \sigma_{aluminum} = 3.63 \cdot 10^{7} S/m$$

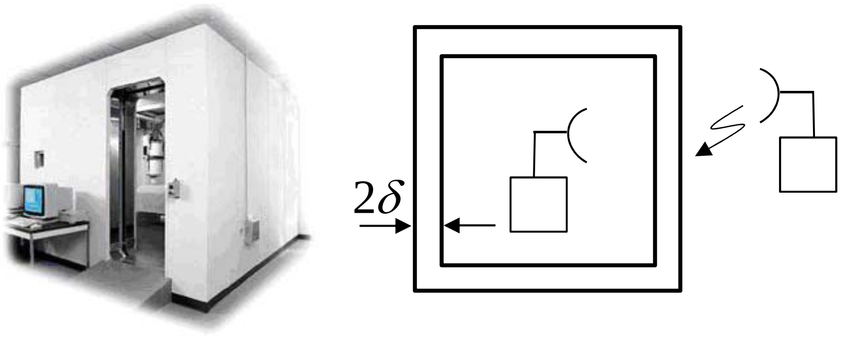
For a sheath thickness of a few centimeters, the majority of the current is confined in the aluminum sheath and very little goes into the steel core.



Electromagnetic shielding

Shields are used to prevent unwanted EH-fields and other electromagnetic disturbances from entering a protected electromagnetic zone.

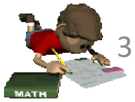
Shielded room



<u>Ex</u>: sensitive electronic equipment inside the shielded room is protected from the electromagnetic fields generated by a radar transmitter outside the room.

- ✓ The transmitter fields will impinge on the shield wall and induce currents in the wall.
- ✓ If the conducting wall thickness is several skin depths, the induced current on the outside will decay significantly before reaching the other side of the wall, reducing its level and preventing its reception by the electronics inside the room.
- ✓ For a transmitter at 3GHz, and room walls of steel ($\sigma_r = 0.1\sigma_{cooper}, \mu_r = 2000$) the steel skin depth is:

$$\boxed{\delta = 0.09\mu m}$$



Magnetically shielded room (with ferrites)

Ex: magnetically shielded room can be used in medical studies like: storage of iron in the liver, particles in lungs, brain studies, heart functions.

✓ To measure the iron concentration, the basic idea is to apply a field to the region of the liver and measure the field response produced by molecules in the body tissues. The response of the tissue in comparison with the strength of applied field is called magnetic susceptibility of the tissue. Hence to determine iron concentration only the susceptibility of the liver needs to be measured.

➢ The body produces magnetic fields in two main ways: by electric currents and by ferromagnetic particles. The electric currents are the ion currents generated by the muscles, nerves, and other organs.

➢ The ion current generated by heart muscle, which provides the basis for the electrocardiogram, also produces a magnetic field over the chest.

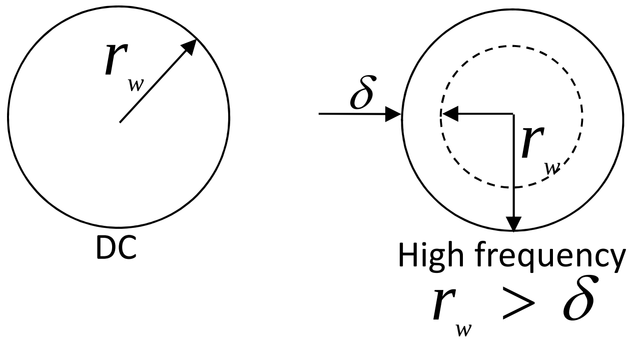
➢ The ion current generated by the brain, which provides the basis for the electroencephalogram, also produces a magnetic field over the head.

➢ Ferromagnetic particles are insoluble contaminants of the body; the most important are the ferromagnetic dust particles in the lungs (magnetite). Magnetic fields can give information about the internal organs not otherwise available.

These magnetic fields are very weak and can be measured using a magnetic shielded room.

High frequency resistance of a wire

Consider a circular cross section conductor of radius r_w.



✓DC: the current will be uniformly distributed over the wire cross section.

✓ the DC resistance per unit length is:

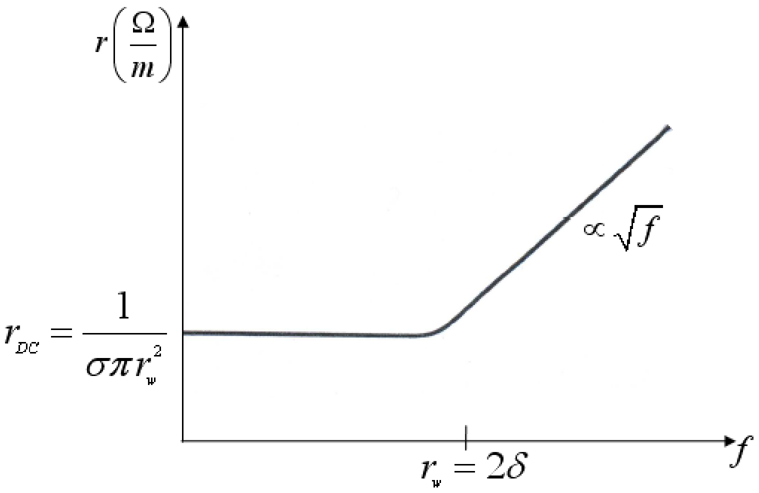
$$r_{DC} = \frac{1}{\sigma\pi r_w^2}[\Omega/\mathrm{m}] \quad r_w << \delta$$

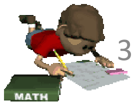
- ➢ if the frequency of the current increases, the current will be concentrated close to the wire surface
- ➢ at higher frequency, where the wire radius is greater than skin depth, the cross-sectional area occupied by the current is smaller, and the resistance per unit length is:

$$r_{HF} = \frac{1}{\sigma 2\pi r_w\delta}[\Omega/\mathrm{m}] \quad r_w >> \delta$$

Skin depth: $\boxed{\delta = \frac{1}{\sqrt{\pi f \mu\sigma}} \propto \frac{1}{\sqrt{f}}}$

$\Longrightarrow$ r_{HF} increases at a rate of $\sqrt{f}$





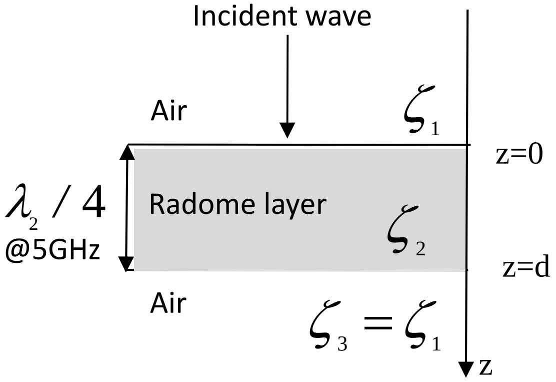
Radome design

Radome (Radar Dome) is a protective dielectric enclosure for a microwave antenna. A ground-based C-band (NATO: 500-1000MHz, IEEE:4-8GHz) microwave landing system used for airplanes landing must be protected from the weather by enclosing it in a radome.

Consider: the center frequency band 5GHz; thermoplastic PEI material (lossless, nonmagnetic, $\varepsilon_{2r}=3$);
Assume a flat planar radome

Calculate: (1) the minimum thickness of the radome that will give no reflections at 5GHz.
(2) for the frequency 4GHz, what percentage of the incident power is reflected?
(3) for the frequency 6GHz, what percentage of the incident power is reflected?

Thickness of the radome layer: design at 5 GHz

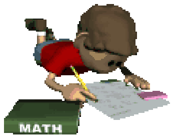


$$\lambda_{2@5GHz} = \frac{\lambda_1}{\sqrt{\varepsilon_{2r}}} = \frac{c}{f\sqrt{\varepsilon_{2r}}} = \frac{3 \cdot 10^8 \, m/s}{\left(5 \cdot 10^9 \, Hz\right)\sqrt{3}} = 3.46cm$$

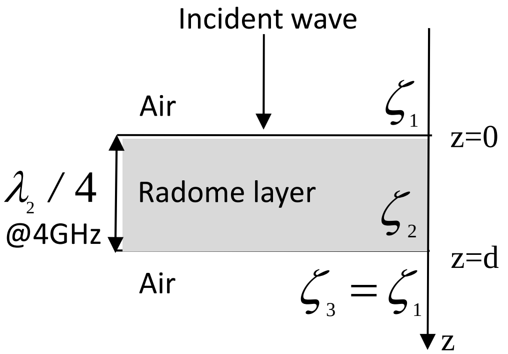
In order not to affect the operation of the microwave landing system:

$$d = N\frac{\lambda_2}{2}, N = 1,2,3....$$

$$\boxed{d_{min} = \frac{\lambda_2}{2} = 1.73cm}$$



Reflected power@4GHz



$$f=4GHz$$

$$\lambda_1=\frac{3\cdot 10^8\,m/s}{4\cdot 10^9\,Hz}=7.5cm$$

$$\lambda_2=\frac{\lambda_1}{\sqrt{\varepsilon_{2r}}}=\frac{7.5}{\sqrt{3}}=4.33cm \qquad d_{min}=1.73cm\neq\frac{\lambda_2}{2}$$

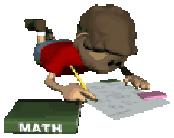
$$\zeta_2=\frac{\zeta_1}{\sqrt{\varepsilon_{2r}}}=\frac{377}{\sqrt{3}}=218\Omega$$

$$k_2=\beta_2=\frac{2\pi}{\lambda_2}=1.45cm^{-1}$$

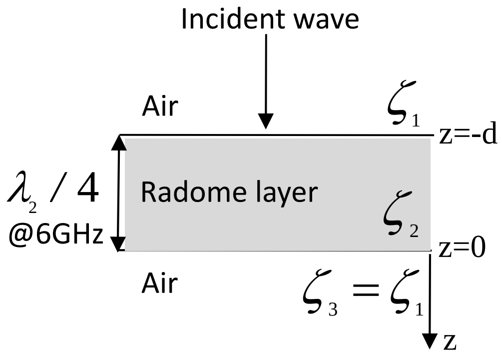
$$\boxed{\Gamma=\frac{(\zeta_2-\zeta_1)(\zeta_3+\zeta_2)+(\zeta_2+\zeta_1)(\zeta_3-\zeta_2)e^{-j2k_2d}}{(\zeta_2+\zeta_1)(\zeta_3+\zeta_2)+(\zeta_2-\zeta_1)(\zeta_3-\zeta_2)e^{-j2k_2d}}=0.321\angle 130^\circ}$$

<u>Reflected power back to air:</u>

$$\boxed{\frac{S^r}{S^i}\times 100=|\Gamma|^2\times 100=10.3\%}$$



Reflected power@6GHz



$$f=6GHz$$

$$\lambda_1=\frac{3\cdot 10^8\, m/s}{6\cdot 10^9\, Hz}=5cm$$

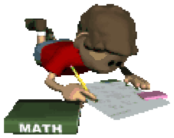
$$\lambda_2=\frac{\lambda_1}{\sqrt{\varepsilon_{2r}}}=\frac{5}{\sqrt{3}}=2.89cm \qquad d_{min}=1.73cm\neq\frac{\lambda_2}{2}$$

$$\zeta_2=\frac{\zeta_1}{\sqrt{\varepsilon_{2r}}}=\frac{377}{\sqrt{3}}=218\Omega$$

$$k_2=\beta_2=\frac{2\pi}{\lambda_2}=2.17cm^{-1}$$

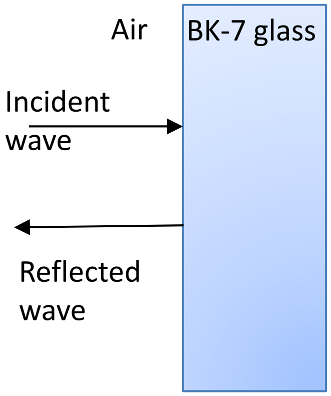
$$\boxed{\Gamma=\frac{(\zeta_2-\zeta_1)(\zeta_3+\zeta_2)+(\zeta_2+\zeta_1)(\zeta_3-\zeta_2)e^{-j2k_2d}}{(\zeta_2+\zeta_1)(\zeta_3+\zeta_2)+(\zeta_2-\zeta_1)(\zeta_3-\zeta_2)e^{-j2k_2d}}=0.321\angle 130°}$$

Reflected power back to air: $\boxed{\frac{S^r}{S^i}\times 100=|\Gamma|^2\times 100=10.3\%}$



⇨ The effective reflection coefficient varies symmetrically around the center frequency , being down by the same amount in magnitude at 4GHz and 6GHz.

Reflectance of glass



Consider a beam of light is incident normally from air on a BK-7 glass interface; BK-7 glass refraction index *n*=1.52

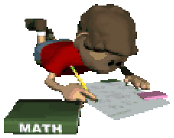
Calculate the reflection coefficient and the percent of incident energy reflected

$$\mu_1 = \mu_2 = \mu_0 ; \varepsilon_{2r} = 2.25 \qquad n_2 = \sqrt{\varepsilon_{2r}\mu_{2r}} = 1.52 = \frac{c}{v} = \frac{1/\sqrt{\varepsilon_0\mu_0}}{1/\sqrt{\varepsilon_2\mu_2}}$$

$$\boxed{\Gamma = \frac{\zeta_2 - \zeta_1}{\zeta_2 + \zeta_1} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1-1.52}{1+1.52} = -0.206}$$

$$\boxed{\frac{S^r}{S^i} \times 100 = |\Gamma|^2 \times 100 = 4.26\%}$$

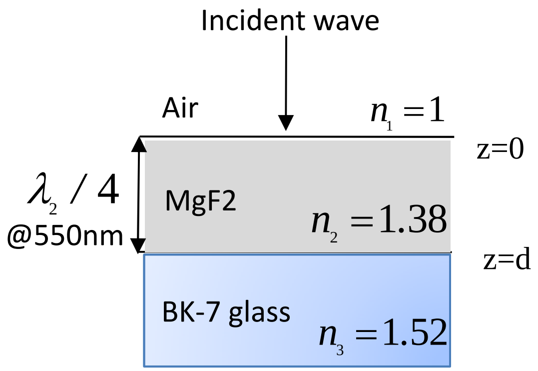
- ➢ 4% of the incident power is reflected by the glass interface. In some applications, this loss may be significant. A camera lens consists of three or more separate lenses, which represents six or more air-glass interfaces.
- ➢ If 4% of the incoming energy reflects back every time light passes through one of these interfaces, up to 22% of original energy is lost during each traverse of light through the lens.
- ➢ These losses can be reduced by introducing antireflection coating on the glass surface.



Coated glass surface

Consider a beam of light is incident normally from air on a BK-7 glass interface; BK-7 glass refraction index *n*=1.52

Calculate the refractive index and the minimum thickness of a film (MgF2 *n*=1.38) to be deposited on the glass surface such that no normally incident visible light of free-space wavelength 550nm (~545THz) is reflected.



$$\lambda_1 = 550nm \quad \text{(green light)}$$

$$\left(n = \sqrt{\varepsilon_r}, \zeta = \zeta_0 / \sqrt{\varepsilon_r}\right)$$

$$\boxed{n_2 = \sqrt{n_1 n_3} = \sqrt{1 \times 1.52} = 1.23}$$

$$\left(\zeta_2 = \sqrt{\zeta_1 \zeta_3} \rightarrow n_2 = \sqrt{n_1 n_3}\right)$$

$$\boxed{d = d_{min} = \frac{\lambda_2}{4} = \frac{\lambda_1}{4n_2} = \frac{550 \cdot 10^{-9}}{4 \cdot 1.23} = 0.112 \mu m}$$

In practice it is difficult to manufacture antireflection coating materials with desired refractive index, so a common material is MgF2 with *n*=1.38.

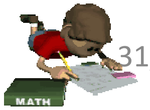
$$\lambda_2 = \frac{\lambda_1}{n_2} = \frac{550}{1.38} = 399nm \Longrightarrow d = d_{min} = \frac{\lambda_2}{4} = 99.6nm$$

$$\zeta_1 = 377\Omega$$

$$\zeta_2 = \frac{\zeta_1}{n_2} = 273.19\Omega$$

$$\zeta_3 = \frac{\zeta_1}{n_3} = 248.03\Omega$$

$$\Longrightarrow \Gamma = -0.112 \Longrightarrow \boxed{\frac{S^r}{S^i} \times 100 = \left|\Gamma\right|^2 \times 100 = 1.26\% < 4\%}$$



Effectiveness over the visible spectrum

$\lambda_1 = 400nm$ (violet light)

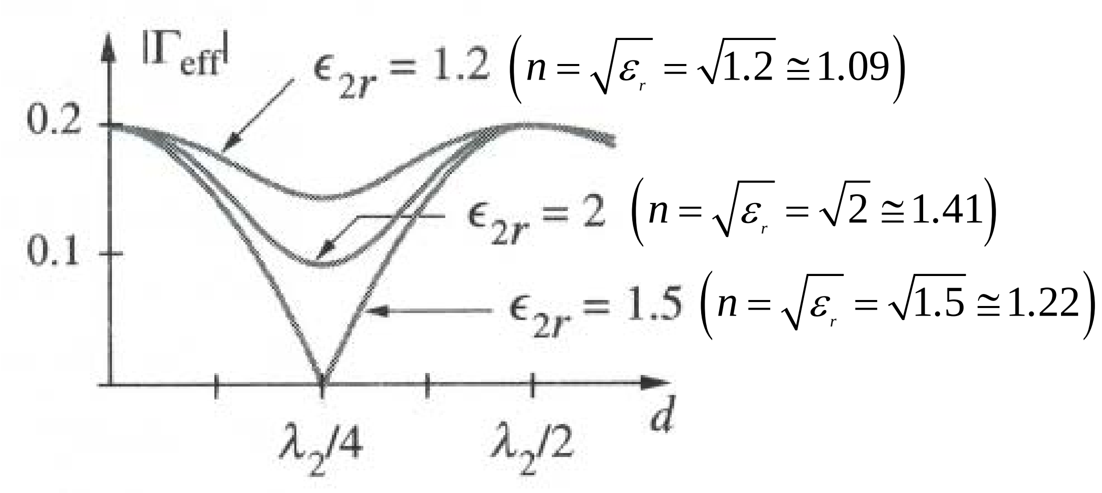
$$\Gamma = 0.1485\angle 163°$$

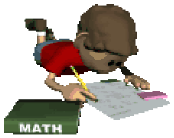
$$\frac{S^r}{S^i} \times 100 = |\Gamma|^2 \times 100 = 2.21\% < 4\%$$

$\lambda_1 = 750nm$ (red light)

$$\Gamma = 0.13\angle 164.6°$$

$$\frac{S^r}{S^i} \times 100 = |\Gamma|^2 \times 100 = 1.7\% < 4\%$$



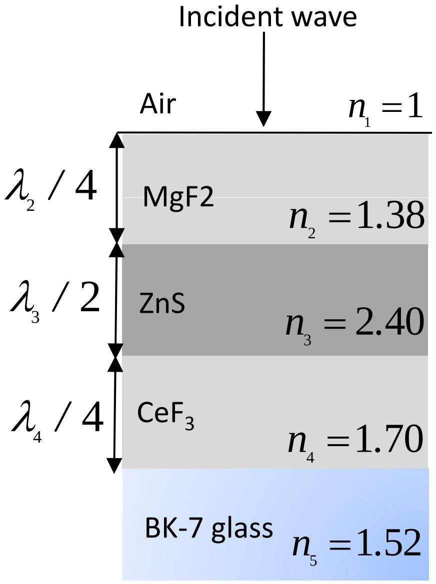


Multiple-layered coatings

- Antireflection coatings are required in most optical applications in order to reduce unwanted reflections at the surface of optical elements.
- It is desirable to reduce the surface reflectivity (or reflection coefficient) to an extremely low value over an extended spectral region to maintain proper color balance and provide optimum efficiency.
- A single layer is not enough since it reduce reflectivity only from 4% to 2.2%.
- To achieve the requirements, a triple-layer coating can be implemented.

<u>Consider</u>: a wideband nonmagnetic antireflection coating system for an air-glass interface, including three layers of coating materials.

<u>Calculate and plot</u>: the reflection coefficient over the wavelength range 400-750nm.

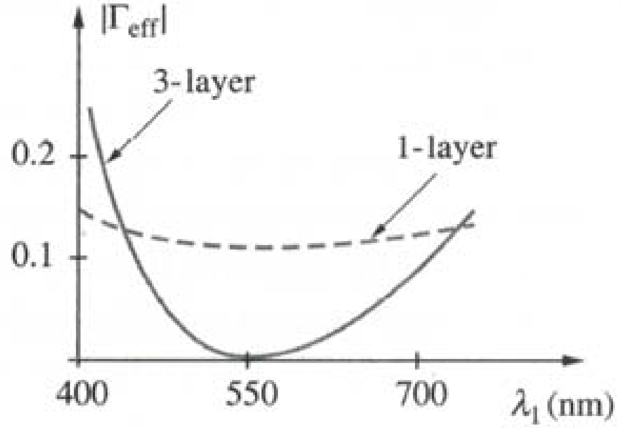


$\lambda_1 = 550nm$ (green light) $\zeta_1 = 377\Omega$

$$d_2 = \frac{\lambda_2}{4} = \frac{\lambda_1}{4n_2} = 99.4nm \quad d_3 = \frac{\lambda_3}{2} = \frac{\lambda_1}{2n_3} = 115nm \quad d_4 = \frac{\lambda_4}{4} = \frac{\lambda_1}{4n_4} = 80.9nm$$

$$\zeta_2 = \frac{\zeta_1}{n_2} = 273.19\Omega \quad \zeta_3 = \frac{\zeta_1}{n_3} = 157\Omega \quad \zeta_4 = \frac{\zeta_1}{n_4} = 222\Omega \quad \zeta_5 = \frac{\zeta_1}{n_5} = 248\Omega$$

http://www.kruschwitz.com/materials.htm



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