UNIVERSITÀ DI PISA

Electromagnetic Radiations and Biological Interactions

"Laurea Magistrale" in Biomedical Engineering
First semester (6 credits), academic year 2011/12

Prof. Paolo Nepa
p.nepa@iet.unipi.it

Electrostatics and Magnetostatics

Edited by Dr. Anda Guraliuc

Microwave&Radiation Laboratory

Lecture Content

- Electric field
- Electric displacement field
- Magnetic field
- Magnetic induction
- Current density and charge density

Electric Field (E)

E is the effect produced by the existence of an electric charge, e.g. an electron, ion, or proton, in the volume of space or medium that surrounds it. It derives from Coulomb law which describe the interaction between electrically charged particles.

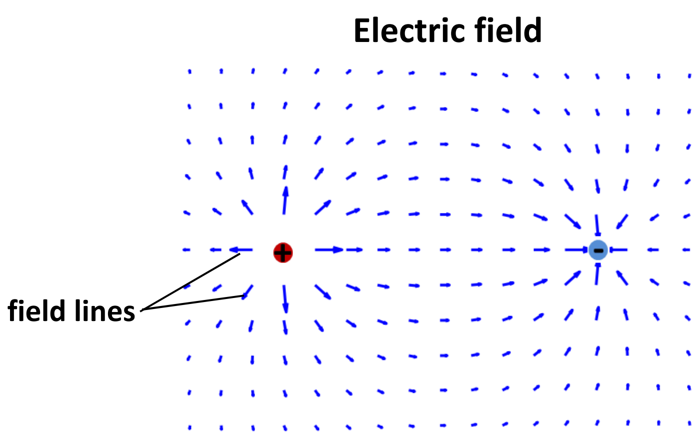
The electric field intensity:

$$\underline{E} = \frac{\underline{F}}{q} [V/m]$$

- F = the electric force experienced by the particle
- q = particle charge
- E = the electric field where the particle is located

$$\underline{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \underline{r}$$

- Q = the charge of the particle creating the electric force
- r = the distance from the particle with charge Q to the E-field evaluation point
- $\underline{r}$ = the unit vector pointing from the particle with charge Q to the E-field evaluation point
- ε_0 = the electric constant $\varepsilon_0 = 8.85 \cdot 10^{-12}\, F/m$



The electric field generated by a positive and negative charge

Electric Displacement Field (D)

The displacement accounts for the effects of free charges within materials. In free space, the electric displacement field is equivalent to flux density, a concept that lends understanding to Gauss's law.

In a dielectric material the presence of an electric field E causes the bound charges in the material (atomic nuclei and their electrons) to slightly separate, inducing a local electric dipole moment.

$$\underline{D} \equiv \varepsilon_0 \underline{E} + \underline{P} \ [C/m^2]$$

- *P - polarization field*
- ε_0 *- the vacuum permittivity*

In a linear, homogeneous, isotropic dielectric with instantaneous response to changes in the electric field, P depends linearly on the electric field:

$$\underline{P} = \varepsilon_0 \chi \underline{E}$$ with χ – electric susceptibility

$$\underline{D} = \varepsilon_0 \underbrace{(1+\chi)}_{\varepsilon_r} \underline{E} = \varepsilon \underline{E}$$

ε_r – relative permittivity

In linear anisotropic media: $\underline{D} = \varepsilon_0 \underline{\underline{\varepsilon_r}} \underline{E} \Leftrightarrow \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} (\varepsilon_r)_{xx} & (\varepsilon_r)_{xy} & (\varepsilon_r)_{xz} \\ (\varepsilon_r)_{yx} & (\varepsilon_r)_{yy} & (\varepsilon_r)_{yz} \\ (\varepsilon_r)_{zx} & (\varepsilon_r)_{zy} & (\varepsilon_r)_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$

tensor

Relative permittivity

Material	ε_r
Vacuum	1
Concrete	4.5
Diamond	5.5-10
Graphite	10-15
Paper	3.85
Polystyrene	2.4-2.7
PTFE/Teflon	2.1
Pyrex (Glass)	4.7 (3.7-10)
Rubber	7
Salt	3-15
Silicon	11.0 - 12.0
Silicone Oil	2.2-2.9
Silicone Rubber	3.2-9.8
Water (Distilled)	76.5-80

Magnetic Field (H)

➢ The magnetic field is the effect produced by moving electric charges, by electric fields that vary in time, and by the 'intrinsic' magnetic field of elementary particles associated with the spin of the particle. It derives from Ampere's law which expresses the interaction between two loops of wires carrying currents.

➢The magnetic field can be defined in many equivalent ways based on the effects it has on its environment:

- a particle having an electric charge, q, and moving in a B-field with a velocity, v, experiences a force, F, called the Lorentz force.

- the magnetic field can be defined in terms of the torque it produces on a magnetic dipole.

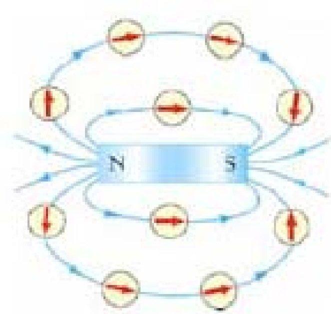
➢ The H-field is defined as a modification of B due to magnetic fields produced by material media. In vacuum the B and H fields are indistinguishable. Inside a material, they may differ in relative magnitude and even direction. Often, though, they differ only by a material dependent multiplicative constant.

➢ B-field is measured in [Wb/m^2] while the H-field is measured in [A/m].

➢ To further distinguish B from H, B is sometimes called the magnetic flux density or the magnetic induction.

$$\underline{H} \equiv \frac{\underline{B}}{\mu_0} - \underline{M}$$

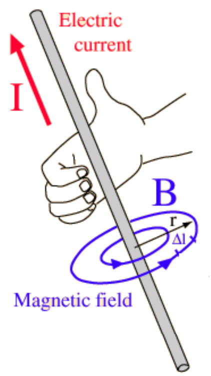
- $\underline{B}$ = the magnetic induction
- $\underline{M}$= magnetization ($\underline{M} = \frac{\mu}{V}$)
- μ_0= the magnetic constant $\mu_0 = 4\pi \cdot 10^{-7}\, H/m$



lines direction – magnetic field direction
lines density – field strength

Magnetic Inductance (B)

The magnetic field **B** generated by a steady current *I* is described by the Biot-Savart law:

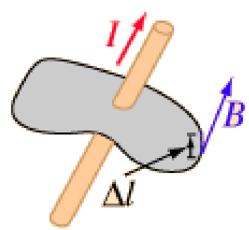


$$\underline{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\underline{l} \times \underline{r}}{r^2} [Wb/m^2]$$

$d\underline{l}$ - infinitesimal length of conductor carrying electric current *I*

$\underline{r}$ - unit vector to specify the direction of the vector distance r from the current to the field point.

A general way to relate current I to the B-field is through Ampère's law:



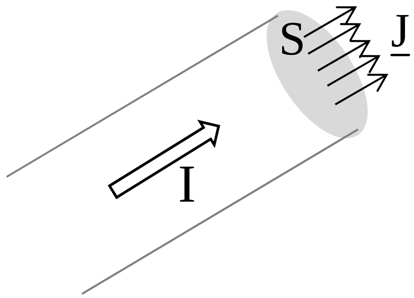
$$\oint \underline{B} \cdot d\underline{l} = \mu_0 I$$

The line integral is over any arbitrary loop and *I* is the current enclosed by that loop. Ampère's law is always valid for steady currents and can be used to calculate the B-field for certain highly symmetric situations such as an infinite wire or an infinite solenoid.

Current density (*J*) & Charge density (*ρ*)

Current density is a measure of the density of flow of a conserved charge. Usually the charge is the electric charge, in which case the associated current density is the electric current per unit area of cross section, but the term current density can also be applied to other conserved quantities. It is defined as a vector whose magnitude is the current per cross-sectional area. Current density *J* is measured in [A/m^2].

$$I = \iint_S \underline{J} \cdot \hat{n} \cdot dS$$



Charge density is the amount of electric charge in a line, surface, or volume. Charge density ρ is measured in [C/m^3].

$$Q = \iiint_V \rho \cdot dV$$

References

1. G. Manara, A. Monorchio, P.Nepa, “*Appunti di Campi Elettromagnetici*”
2. J. Slater, N. Frank, “*Electromagnetism*”
3. M. Schwartz, “*Principle of Electrodynamics*”
4. http://www.wikipedia.org/