UNIVERSITÀ DI PISA

Electromagnetic Radiations and Biological Interactions

"Laurea Magistrale" in Biomedical Engineering
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Wave Reflection and Transmission

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Microwave&Radiation Laboratory

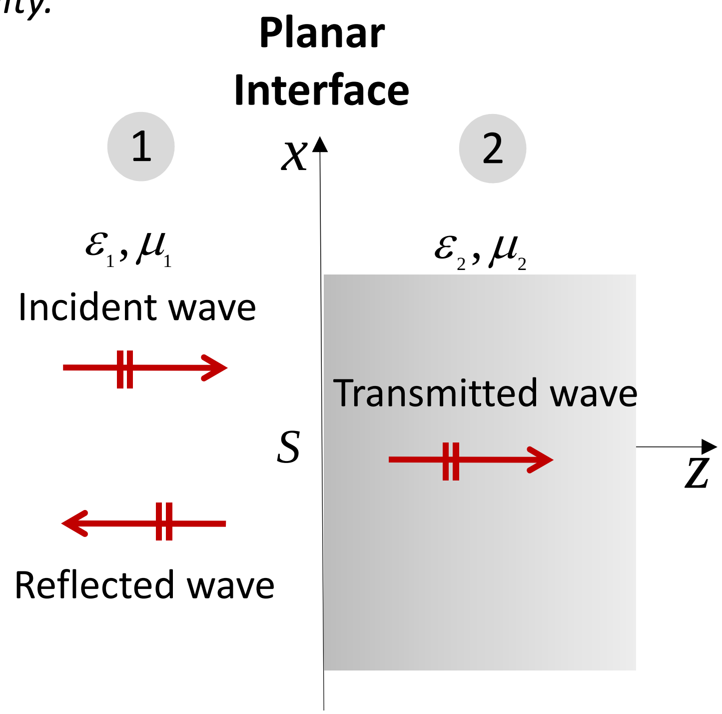
Lecture Content

➢ Plane Wave Reflection and Transmission

- Reflection and transmission coefficients (at normal Incidence, single interface)
- Stationary wave
- Transmitted power density
- Special cases: Dielectric-Conductor, Dielectric-PEC,

Problem

- *Two different homogeneous media separated by a planar boundary located at S.*
- *A monochromatic plane wave is impinging upon the interface* S *at normal incidence (assume z as direction of propagation).*
- *The incident wave is linearly polarized (assume electric field direction along* x *axis).*
- *Medium "2" extends to infinity.*

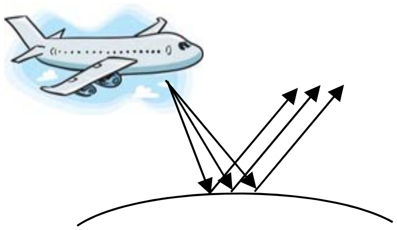
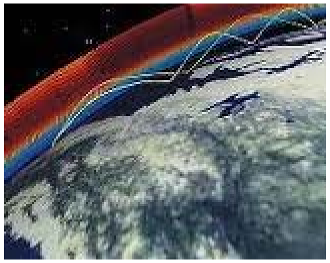


Due to symmetry of the problem, reflected and transmitted waves will propagate along a direction perpendicular to the interface and will exhibit the same polarization as the incident wave.

Applications

- *EM wave radiated by a UMTS/GSM base station and transmitted inside a building*
- *EM wave radiated by a satellite/airplane: reflection and transmission at the ground surface*
- *EM penetration in a human body illuminated by an external antenna*
- *Ionospheric propagation at HF frequencies*
- *etc.*



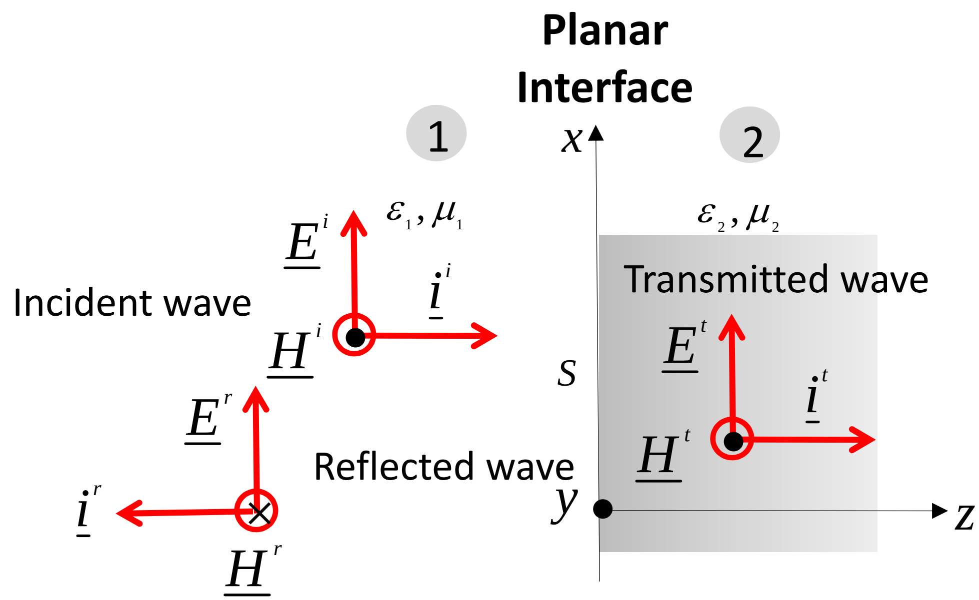
The radius of curvature of the boundary at the reflection point must be large w.r.t. the radiation wavelength: it implies that the boundary can be approximated by a planar interface.

The distance of the source (antenna) from the reflection point must be large w.r.t. the radiation wavelength and antenna size (reflection point in the antenna far-field region): it implies that the incident spherical wave can be locally approximated by a plane wave.

E and H field expressions

***Total** field in medium 1*
(incident wave + reflected wave):

$$\begin{cases} \underline{E}_1(z) = E_0^i e^{-jk_1 z}\, \underline{i}_x + E_0^r e^{+jk_1 z}\, \underline{i}_x \\ \underline{H}_1(z) = \dfrac{E_0^i}{\zeta_1} e^{-jk_1 z}\, \underline{i}_y - \dfrac{E_0^r}{\zeta_1} e^{+jk_1 z}\, \underline{i}_y \end{cases}$$



Field in medium 2 (transmitted wave):

$$\begin{cases} \underline{E}_2(z) = E_0^t e^{-jk_2 z}\, \underline{i}_x \\ \underline{H}_2(z) = \dfrac{E_0^t}{\zeta_2} e^{-jk_2 z}\, \underline{i}_y \end{cases}$$

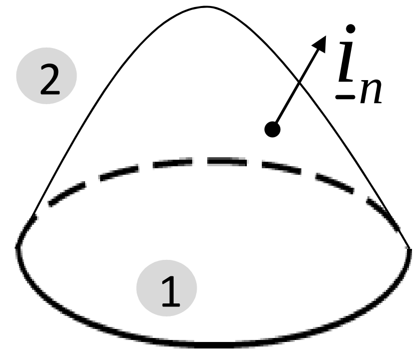
- Characteristic impedance $\zeta_{1,2} = \sqrt{\dfrac{\mu_{1,2}}{\varepsilon_{1,2}}}$
- Propagation constant $k_{1,2} = \omega\sqrt{\varepsilon_{1,2}\mu_{1,2}}$

The complex amplitude of the incident plane wave is assumed known (it is due to a transmitting antenna far away from the planar interface).

Boundary conditions

The tangential components of the electric and magnetic fields must be continuous at any point on the interface (no free charges or currents exist at the boundary).

$$\underline{i}_n \times \underline{E}_1 = \underline{i}_n \times \underline{E}_2$$
$$\underline{i}_n \times \underline{H}_1 = \underline{i}_n \times \underline{H}_2$$

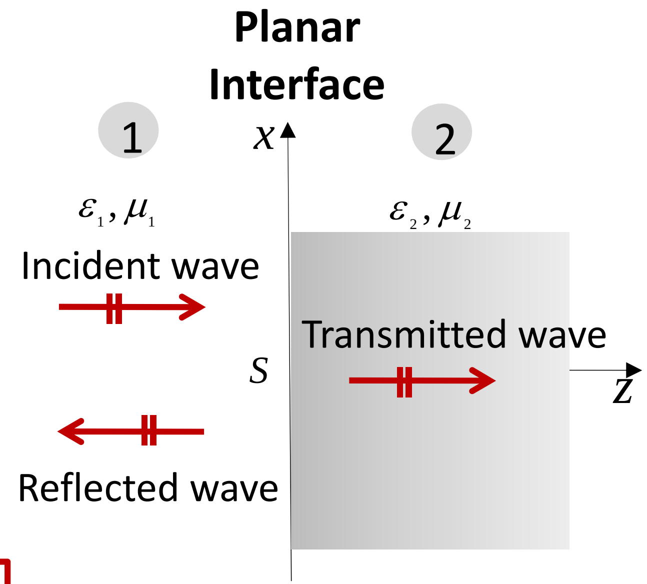


$$\begin{cases} \underline{E}_1(z) = E_0^{\,i} e^{-jk_1 z}\underline{i}_x + E_0^{\,r} e^{+jk_1 z}\underline{i}_x \\ \underline{H}_1(z) = \dfrac{E_0^{\,i}}{\zeta_1} e^{-jk_1 z}\underline{i}_y - \dfrac{E_0^{\,r}}{\zeta_1} e^{+jk_1 z}\underline{i}_y \end{cases} \qquad \begin{cases} \underline{E}_2(z) = E_0^{\,t} e^{-jk_2 z}\underline{i}_x \\ \underline{H}_2(z) = \dfrac{E_0^{\,t}}{\zeta_2} e^{-jk_2 z}\underline{i}_y \end{cases}$$

$$\begin{cases} \underline{i}_z \times \underline{E}_1\big|_{z=0^-} = \underline{i}_z \times \underline{E}_2\big|_{z=0^+} \\ \underline{i}_z \times \underline{H}_1\big|_{z=0^-} = \underline{i}_z \times \underline{H}_2\big|_{z=0^+} \end{cases} \Rightarrow \begin{cases} E_x\big|_{z=0^-} = E_x\big|_{z=0^+} \\ H_y\big|_{z=0^-} = H_y\big|_{z=0^+} \end{cases} \Rightarrow \begin{cases} E_0^{\,i} + E_0^{\,r} = E_0^{\,t} \\ \dfrac{E_0^{\,i}}{\zeta_1} - \dfrac{E_0^{\,r}}{\zeta_1} = \dfrac{E_0^{\,t}}{\zeta_2} \end{cases}$$

Reflection and transmission coefficients

$$\begin{cases} E_0^i + E_0^r = E_0^t \\ E_0^i - E_0^r = \dfrac{\zeta_1}{\zeta_2} E_0^t \end{cases} \Rightarrow \begin{cases} 2E_0^i = \dfrac{\zeta_2 + \zeta_1}{\zeta_2} E_0^t \\ 2E_0^r = \dfrac{\zeta_2 - \zeta_1}{\zeta_2} E_0^t \end{cases}$$



Reflection coefficient

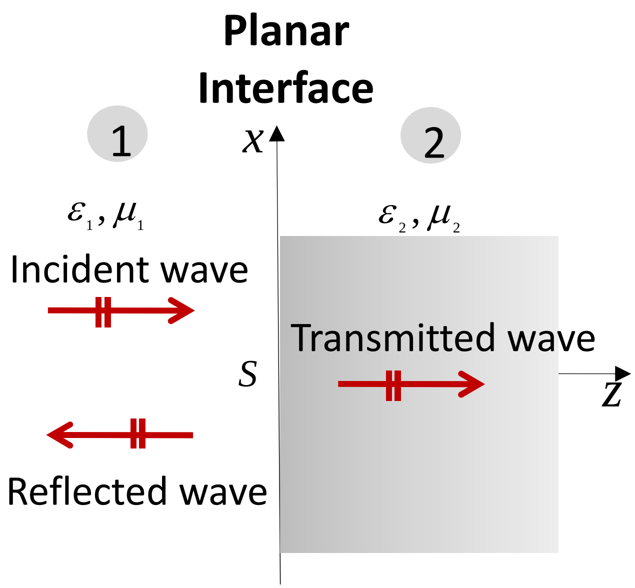
$$\Gamma = \frac{E_x^r(z=0)}{E_x^i(z=0)} = \frac{E_0^r}{E_0^i} = \frac{\zeta_2 - \zeta_1}{\zeta_2 + \zeta_1}, \quad E_0^r = \Gamma E_0^i \qquad |\Gamma| \le 1$$

Transmission coefficient

$$\tau = \frac{E_x^t(z=0)}{E_x^i(z=0)} = \frac{E_0^t}{E_0^i} = \frac{2\zeta_2}{\zeta_1 + \zeta_2}, \quad E_0^t = \tau E_0^i$$

$$\Rightarrow \quad 1 + \Gamma = \tau$$

E and H field expressions (linear polarization)



$$\Gamma = \frac{\zeta_2 / \zeta_1 - 1}{\zeta_2 / \zeta_1 + 1}$$

$$\tau = \frac{2\zeta_2 / \zeta_1}{\zeta_2 / \zeta_1 + 1}$$

$$E_0^{\,r} = \Gamma E_0^{\,i}$$

$$E_0^{\,t} = \tau E_0^{\,i}$$

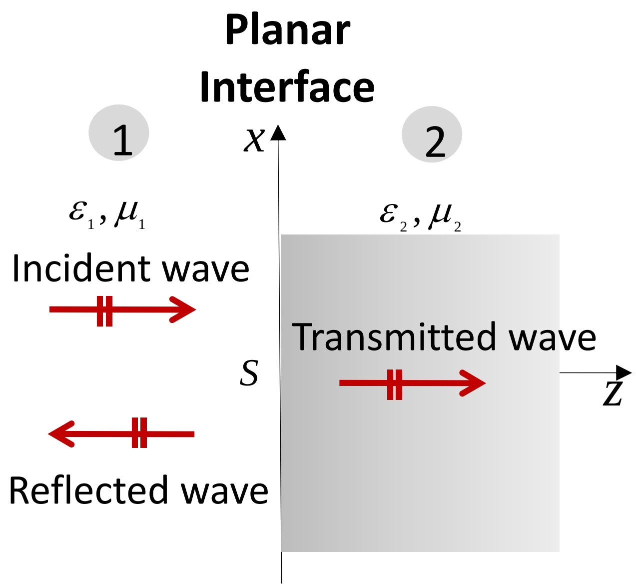
Total field in medium 1:

$$\begin{cases} \underline{E}_1(z) = E_0^{\,i} e^{-jk_1 z}[1 + \Gamma e^{+j2k_1 z}]\underline{i}_x \\ \underline{H}_1(z) = \dfrac{E_0^{\,i}}{\zeta_1} e^{-jk_1 z}[1 - \Gamma e^{+j2k_1 z}]\underline{i}_y \end{cases}$$

Field in medium 2:

$$\begin{cases} \underline{E}_2(z) = \tau E_0^{\,i} e^{-jk_2 z} \underline{i}_x \\ \underline{H}_2(z) = \dfrac{\tau}{\zeta_2} E_0^{\,i} e^{-jk_2 z} \underline{i}_y \end{cases}$$

E and H field expressions (arbitrary polarization)



A plane wave can be always split into two linearly polarized plane waves (orthogonally polarized): reflection coefficient exhibits the same expression for both components (the same happens for the transmission coefficient).

$$\Gamma = \frac{\zeta_2/\zeta_1 - 1}{\zeta_2/\zeta_1 + 1} \qquad \tau = \frac{2\zeta_2/\zeta_1}{\zeta_2/\zeta_1 + 1}$$

Total field in medium 1:

$$\begin{cases} \underline{E}_1(z) = \left(E_{0x}^i \underline{i}_x + E_{0y}^i \underline{i}_y\right) e^{-jk_1 z}[1 + \Gamma e^{+j2k_1 z}] \\ \underline{H}_1(z) = \left(-\frac{E_{0y}^i}{\zeta_1} \underline{i}_x + \frac{E_{0x}^i}{\zeta_1} \underline{i}_y\right) e^{-jk_1 z}[1 - \Gamma e^{+j2k_1 z}] \end{cases}$$

Field in medium 2:

$$\begin{cases} \underline{E}_2(z) = \tau e^{-jk_2 z}\left(E_{0x}^i \underline{i}_x + E_{0y}^i \underline{i}_y\right) \\ \underline{H}_2(z) = \frac{\tau}{\zeta_2} e^{-jk_2 z}\left(-E_{0y}^i \underline{i}_x + E_{0x}^i \underline{i}_y\right) \end{cases}$$

The sense of polarization (or handedness) of the reflected wave is reversed with respect to that one of the incident wave (handedness does not change for the transmitted wave).

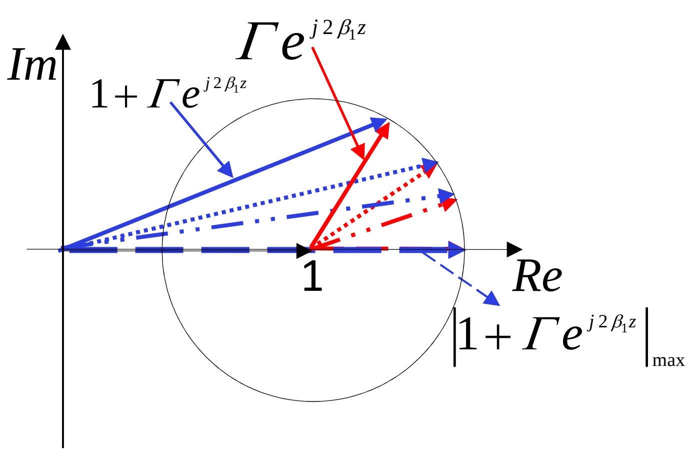
Standing wave pattern (medium 1: lossless)

$$k_1 = \beta_1 = \omega\sqrt{\varepsilon_1\mu_1} = real \qquad \lambda_1 = 2\pi / \beta_1 \qquad \zeta_1 = \sqrt{\mu_1 / \varepsilon_1} = real$$

$$\left|\underline{E}_1(z)\right| = \left|E_0^i\right|\left|1+\Gamma e^{j2\beta_1 z}\right| \qquad \left|\underline{H}_1(z)\right| = \left|E_0^i\right| / \zeta_1 \left|1-\Gamma e^{j2\beta_1 z}\right|$$

$$\Gamma = \left|\Gamma\right| e^{j\phi} \qquad \left|1+\Gamma e^{j2\beta_1 z}\right| = \left|1+\left|\Gamma\right| e^{j(\phi+2\beta_1 z)}\right| = \left|1+\left|\Gamma\right| e^{j(\phi+4\pi z/\lambda_1)}\right|$$

Simultaneous presence of incident and reflected waves gives rise to a standing wave pattern. Note that a standing wave exist only in medium 1. The magnitude of the electric (magnetic) field in medium 1 can be analyzed to determine the locations of the maximum and minimum values of the electric (magnetic) field standing wave pattern. The standing wave pattern exhibits a repetition period of half a wavelength.



$$\left|1+\left|\Gamma\right| e^{j(\phi+2\beta_1 z)}\right| \le 1+\left|\Gamma\right|$$

$$\Uparrow$$

$$\phi + 2\beta_1 z = 2n\pi,$$

$$n = 0, -1, -2, \ldots$$

$$\Gamma e^{j2\beta_1 z} = +\left|\Gamma\right|$$

$$\left|E_1\right|_{\max} = \left|E_0^i\right|(1+\left|\Gamma\right|),$$

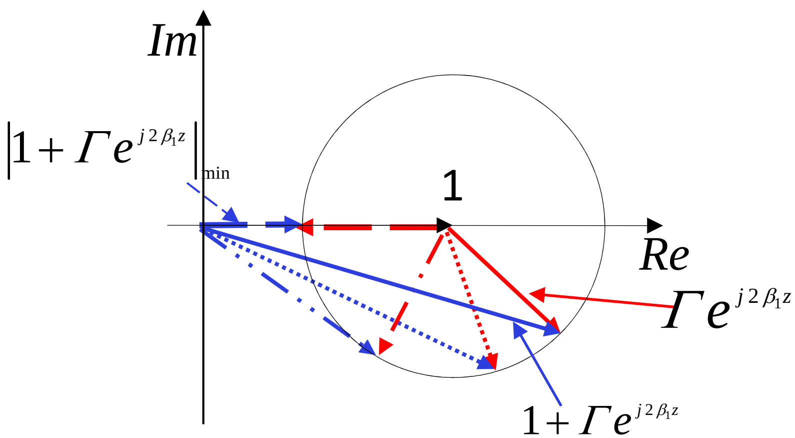
$$\left|H_1\right|_{\min} = \left|E_0^i\right| / \zeta_1 (1-\left|\Gamma\right|)$$

Standing wave pattern (medium 1: lossless)

$$k_1 = \beta_1 = \omega\sqrt{\varepsilon_1\mu_1} = real \qquad \lambda_1 = 2\pi / \beta_1 \qquad \zeta_1 = \sqrt{\mu_1 / \varepsilon_1} = real$$

$$\left|\underline{E}_1(z)\right| = \left|E_0^i\right|\left|1+\Gamma e^{j2\beta_1 z}\right| \qquad \left|\underline{H}_1(z)\right| = \left|E_0^i\right| / \zeta_1 \left|1-\Gamma e^{j2\beta_1 z}\right|$$

$$\Gamma = \left|\Gamma\right| e^{j\phi} \qquad \left|1+\Gamma e^{j2\beta_1 z}\right| = \left|1+\left|\Gamma\right| e^{j(\phi+2\beta_1 z)}\right| = \left|1+\left|\Gamma\right| e^{j(\phi+4\pi z/\lambda_1)}\right|$$



$$1-\left|\Gamma\right| \le \left|1+\left|\Gamma\right| e^{j(\phi+2\beta_1 z)}\right|$$

$$\Uparrow$$

$$\phi + 2\beta_1 z = 2n\pi - \pi,$$

$$n = 0, -1, -2, \ldots$$

$$\Gamma e^{j2\beta_1 z} = -\left|\Gamma\right|$$

$$\left|E_1\right|_{min} = \left|E_0^i\right|(1-\left|\Gamma\right|),$$

$$\left|H_1\right|_{max} = \left|E_0^i\right| / \zeta_1 (1+\left|\Gamma\right|)$$

The spatial distance between the location of a field maximum and the location of the closest field minimum is equal to a quarter of wavelength.

Standing wave pattern (lossless media)

$$\zeta_2 > \zeta_1 \quad \Rightarrow \quad \Gamma > 0\left(real\ and\ \ positive\right)$$

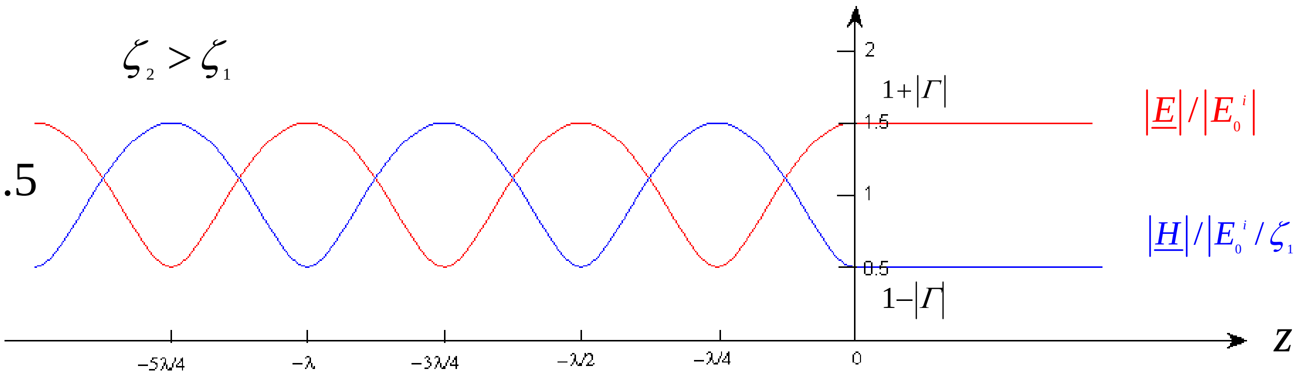
$$\Gamma = \frac{\zeta_2/\zeta_1 - 1}{\zeta_2/\zeta_1 + 1}$$

$$\left|\underline{E}_1(z)\right| = \left|E_0^i\right|\left|1+\Gamma e^{j2\beta_1 z}\right| \qquad \left|\underline{E}_2(z)\right| = \left|E_0^i\right|\left|\tau\right|$$

$$\left|\underline{H}_1(z)\right| = \left|E_0^i\right|/\zeta_1\left|1-\Gamma e^{j2\beta_1 z}\right| \qquad \left|\underline{H}_1(z)\right| = \left|E_0^i\right|\left|\tau\right|/\zeta_2$$

$\zeta_2 > \zeta_1$

$\Gamma = 0.5$



$$\left|1+\Gamma e^{j2\beta_1 z}\right|_{\max} = 1+\left|\Gamma\right| \text{ when } 2\beta_1(-z) = n(2\pi), \quad n = 0,1,2,\ldots$$

$$z = -\frac{n\pi}{\beta_1} = -\frac{n\pi}{(2\pi)/\lambda_1} = -n\frac{\lambda_1}{2}$$

$$\left|E_1\right|_{\max} = \left|E_0^i\right|(1+\left|\Gamma\right|), \left|H_1\right|_{\min} = \left|E_0^i\right|/\zeta_1(1-\left|\Gamma\right|)$$

$$\left|1+\Gamma e^{j2\beta_1 z}\right|_{\min} = 1-\left|\Gamma\right| \text{ when } 2\beta_1(-z) = (2n+1)\pi, \quad n = 0,1,2,\ldots$$

$$z = -\frac{(2n+1)\pi}{2(2\pi/\lambda_1)} = -(2n+1)\frac{\lambda_1}{4}$$

$$\left|E_1\right|_{\min} = \left|E_0^i\right|(1-\left|\Gamma\right|), \left|H_1\right|_{\max} = \left|E_0^i\right|/\zeta_1(1+\left|\Gamma\right|)$$

Standing wave pattern (lossless media)

$$\zeta_1 > \zeta_2 \;\Rightarrow\; \Gamma < 0 \; (real\ and\ negative)$$

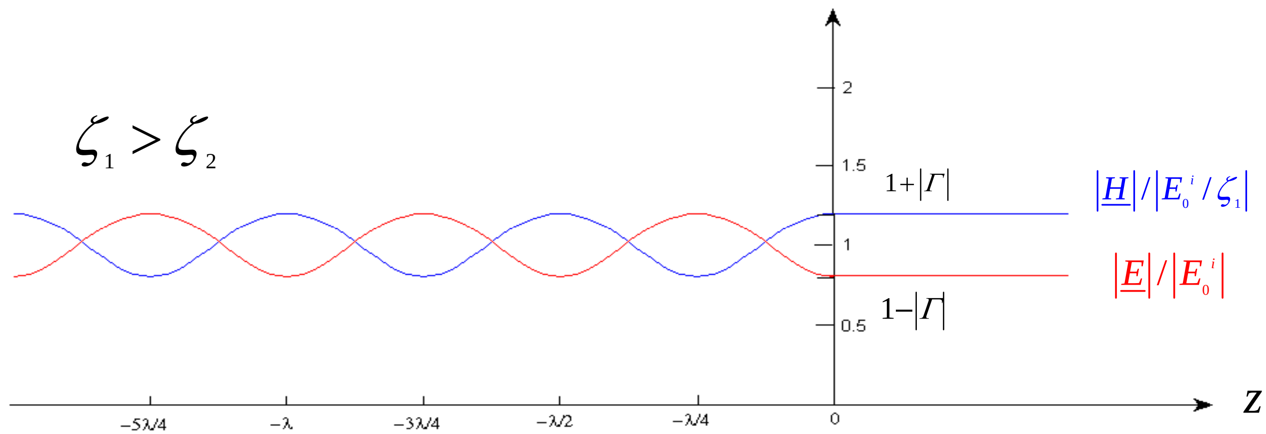
$$\Gamma = \frac{\zeta_2/\zeta_1 - 1}{\zeta_2/\zeta_1 + 1}$$

$$\left|\underline{E}_1(z)\right| = \left|E_0^i\right|\left|1 + \Gamma e^{j2\beta_1 z}\right|$$
$$\left|\underline{H}_1(z)\right| = \left|E_0^i\right| / \zeta_1 \left|1 - \Gamma e^{j2\beta_1 z}\right|$$

$$\left|\underline{E}_2(z)\right| = \left|E_0^i\right|\left|\tau\right|$$
$$\left|\underline{H}_1(z)\right| = \left|E_0^i\right|\left|\tau\right| / \zeta_2$$

$$\zeta_1 > \zeta_2$$

$$\Gamma = -0.2$$



The locations of minimums and peaks are reversed, but the equations for the maximum and minimum electric field magnitudes in terms of $|\Gamma|$ are the same as previous case.

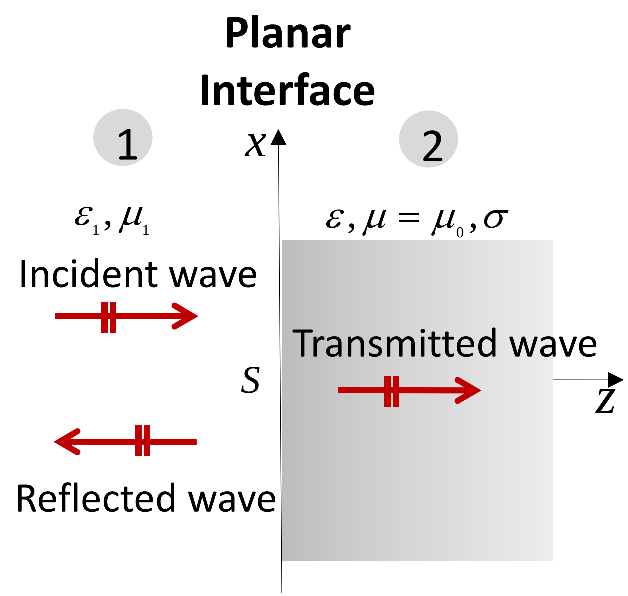
Dielectric-conductor: medium 2 with losses

- characteristic impedance

$$\zeta_2 = \sqrt{\frac{\mu_0}{\varepsilon_{eff}}} = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r\left(1 - j\frac{\sigma}{\omega\varepsilon_0\varepsilon_r}\right)}}$$

- propagation constant

$$k_2 = \omega\sqrt{\mu_0\varepsilon_{eff}} = \omega\sqrt{\varepsilon_0\varepsilon_r\mu_0\left(1 - j\frac{\sigma}{\omega\varepsilon_0\varepsilon_r}\right)} = \beta_2 - j\alpha_2$$



Filed in medium 2 (transmitted wave):

$$\begin{cases} \underline{E}_2 = E_0^t e^{-jk_2 z}\underline{i}_x = E_0^t e^{-\alpha_2 z}e^{-j\beta_2 z}\underline{i}_x \\ \underline{H}_2 = \dfrac{E_0^t}{\zeta_2}e^{-jk_2 z}\underline{i}_y = \dfrac{E_0^t}{\zeta_2}e^{-\alpha_2 z}e^{-j\beta_2 z}\underline{i}_y \end{cases} \Rightarrow \begin{cases} \underline{E}_2 = \tau E_0^i e^{-\alpha_2 z}e^{-j\beta_2 z}\underline{i}_x \\ \underline{H}_2 = \dfrac{\tau E_0^i}{\zeta_2}e^{-\alpha_2 z}e^{-j\beta_2 z}\underline{i}_y \end{cases} \qquad \tau = \frac{2\zeta_2}{\zeta_1 + \zeta_2}$$

$$\underline{J} = \sigma\underline{E}_2 = J(z=0)e^{-\alpha_2 z}e^{-j\beta_2 z}\underline{i}_x$$

The majority of current exists within a few skin depths of the surface.

Standing wave pattern (medium 2: lossy)

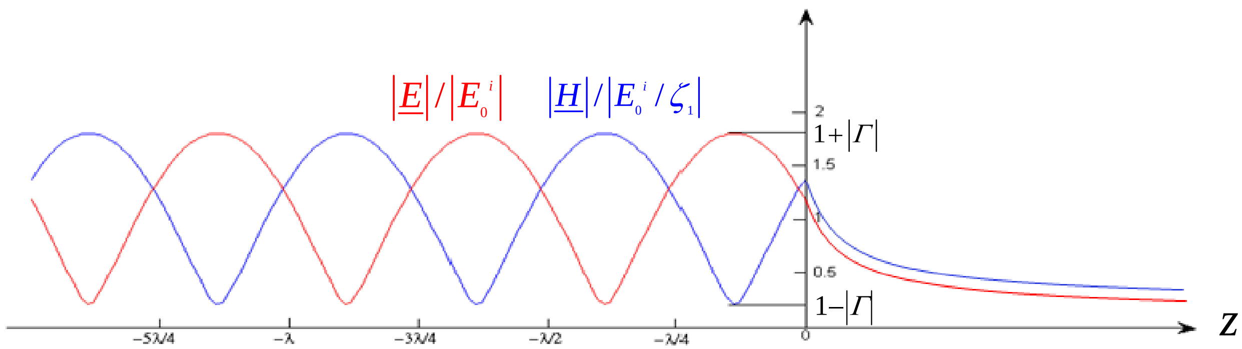
$$k_2 = \beta_2 - j\alpha_2 \qquad \zeta_2 = R - jX \qquad \Gamma = \frac{\zeta_2/\zeta_1 - 1}{\zeta_2/\zeta_1 + 1}$$

ζ_2 *is complex* $\Rightarrow$ Γ *is complex*

$$\left|\underline{E}_1(z)\right| = \left|E_0^i\right|\left|1 + \Gamma e^{j2\beta_1 z}\right| \qquad \left|\underline{E}_2(z)\right| = \left|E_0^i\right|\left|\tau\right| e^{-\alpha_2 z}$$

$$\left|\underline{H}_1(z)\right| = \left|E_0^i\right| / \zeta_1 \left|1 - \Gamma e^{j2\beta_1 z}\right| \qquad \left|\underline{H}_1(z)\right| = \left|E_0^i / \zeta_2\right|\left|\tau\right| e^{-\alpha_2 z}$$

$|\Gamma| = 0.8$



Standing Wave Ratio

The standing wave ratio, SWR, is defined as the ratio of the maximum electric field magnitude to the minimum electric field magnitude:

$$s = SWR = \frac{\left|\underline{E}_1\right|_{\max}}{\left|\underline{E}_1\right|_{\min}} = \frac{1+\left|\Gamma\right|}{1-\left|\Gamma\right|} \quad \Rightarrow \quad \left|\Gamma\right| = \frac{s-1}{s+1}$$

$$1 \le SWR \le \infty$$

$\Uparrow$ $\Gamma = 0$ *no reflection* (for $SWR = 1$)

$\Uparrow$ $\left|\Gamma\right| = 1$ *total reflection* (for $SWR = \infty$)

Standing Wave Pattern

Standing Wave Pattern Animation (SWR)

http://www.youtube.com/watch?v=s5MBno0PZjE

http://www.youtube.com/watch?v=z8ya4LqG1CQ

Note: Poynting Vector

$$\underline{E} = \underline{E}_0 e^{-jkz}$$
$$\underline{H} = \underline{H}_0 e^{-jkz} = \frac{1}{\zeta}\underline{i}_z \times \underline{E}_0 e^{-jkz}$$
$$k = \beta - j\alpha \ \& \ \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\Rightarrow \ \underline{S} = \frac{1}{2}\underline{E}\times\underline{H}^* = \frac{1}{2\zeta^*}\underline{E}_0 \times \left(\underline{i}_z \times \underline{E}_0^*\right) e^{-jkz} e^{jk^* z}$$
$$\underline{A}\times\left(\underline{B}\times\underline{C}\right) = \underline{B}\cdot\left(\underline{A}\cdot\underline{C}\right) - \underline{C}\cdot\left(\underline{A}\cdot\underline{B}\right)$$
$$\underline{i}_z \cdot \underline{E}_0 = 0$$
$$\underline{E}_0 \cdot \underline{E}_0^* = \left|\underline{E}_0\right|^2$$

$$\Rightarrow \underline{S} = \frac{1}{2\zeta^*}\left|\underline{E}_0\right|^2 e^{-2\alpha z}\underline{i}_z$$

In a lossless medium (non dissipative):

- $\underline{S}$ has the same direction as the wave propagation
- $\underline{S} = S\underline{i}_z$ with S real and constant with respect to z

$$\underline{S} = \frac{1}{2\zeta}\left|\underline{E}_0\right|^2 \underline{i}_z = S\underline{i}_z, \quad S = \frac{1}{2\zeta}\left|\underline{E}_0\right|^2 = \frac{1}{2}\zeta\left|\underline{H}_0\right|^2 \qquad \underline{S} = \frac{1}{2\zeta}\left|\underline{E}_0\right|^2 \underline{i}_z = \frac{1}{2\zeta}\left(\left|E_{0x}\right|^2 + \left|E_{0y}\right|^2\right)\underline{i}_z$$

In a medium with no losses and an arbitrary propagation direction: $\underline{S} = \frac{1}{2\zeta}\left|\underline{E}_0\right|^2 \underline{i} = \frac{1}{2}\zeta\left|\underline{H}_0\right|^2 \underline{i} = S\underline{i}$

In a lossy medium:

- $\underline{S}$ has the same direction as the wave propagation
- $\underline{S} = S\underline{i}_z$ with S having both a real and an imaginary part
- S decreases as $e^{-2\alpha z}$

$$\underline{S} = \frac{1}{2\zeta^*}\left|\underline{E}_0\right|^2 e^{-2\alpha z}\underline{i}_z$$

$$\mathrm{Re}\left\{\underline{S}\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{\zeta^*}\right\}\left|\underline{E}_0\right|^2 e^{-2\alpha z}\underline{i}_z \qquad S(z) = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{\zeta^*}\right\}\left|\underline{E}_0\right|^2 e^{-2\alpha z} = \frac{1}{2}\mathrm{Re}\left\{\zeta^*\right\}\left|\underline{H}_0\right|^2 e^{-2\alpha z}$$

Note: Poynting Vector

$$\begin{cases} \underline{E} = \left[E_0^{\,i} e^{-j\beta z} + E_0^{\,r} e^{+j\beta z}\right]\underline{i}_x \\ \underline{H} = \left[\dfrac{E_0^{\,i}}{\zeta} e^{-j\beta z} - \dfrac{E_0^{\,r}}{\zeta} e^{+j\beta z}\right]\underline{i}_y \end{cases}$$

$$\underline{S} = \frac{1}{2}\underline{E}\times\underline{H}^* = \frac{1}{2}\left[\frac{\left|E_0^{\,i}\right|^2}{\zeta} - \frac{\left|E_0^{\,r}\right|^2}{\zeta} - \frac{\left(E_0^{\,i}\right)\left(E_0^{\,r}\right)^*}{\zeta} e^{-j2\beta z} + \frac{\left(E_0^{\,r}\right)\left(E_0^{\,i}\right)^*}{\zeta} e^{j2\beta z}\right]\underline{i}_z$$

$$\left(E_0^{\,i}\right)\left(E_0^{\,r}\right)^* e^{-j2\beta z} = \left[\left(E_0^{\,i}\right)^*\left(E_0^{\,r}\right)e^{j2\beta z}\right]^*$$

$$(a+jb)-(a+jb)^* = 2jb$$

$\Rightarrow$

$$\operatorname{Re}\{\underline{S}\} = \frac{1}{2\zeta}\left[\left(E_0^{\,i}\right)^2 - \left(E_0^{\,r}\right)^2\right]\underline{i}_z = \left(\frac{1}{2\zeta}\left|E_0^{\,i}\right|^2 - \frac{1}{2\zeta}\left|E_0^{\,r}\right|^2\right)\underline{i}_z = \frac{1}{2\zeta}\left|E_0^{\,i}\right|^2\left(1-\left|\Gamma\right|^2\right)\underline{i}_z$$

$$\Gamma = \frac{E_0^{\,r}}{E_0^{\,i}} \qquad S = \left(\frac{1}{2\zeta}\left|E_0^{\,i}\right|^2 - \frac{1}{2\zeta}\left|E_0^{\,r}\right|^2\right) = \frac{1}{2\zeta}\left|E_0^{\,i}\right|^2\left(1-\left|\Gamma\right|^2\right)$$

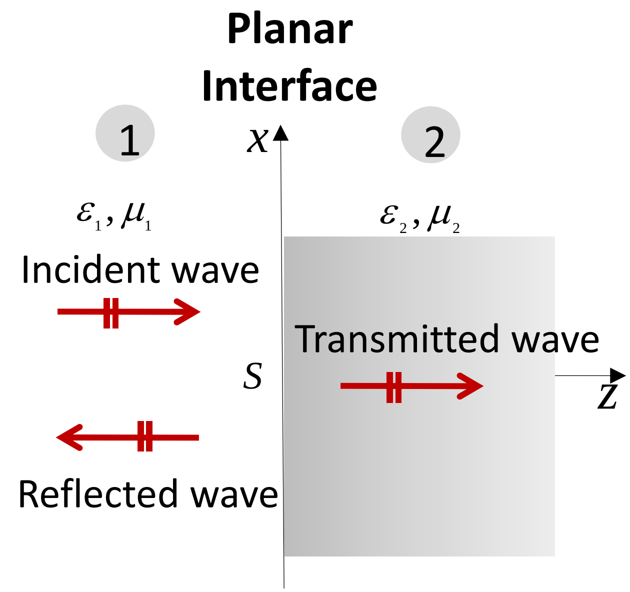
Transmitted Power Density (lossless media)

Medium 1:

Average power density for incident and reflected waves [W/m²]

$$S^i = \frac{1}{2\zeta_1}\left|E_0^{\,i}\right|^2$$

$$S^r = \frac{1}{2\zeta_1}\left|E_0^{\,r}\right|^2 = \frac{1}{2\zeta_1}\left|E_0^{\,i}\right|^2\left|\Gamma\right|^2 = S^i\left|\Gamma\right|^2$$



Medium 2:

Average power density for transmitted wave [W/m²]

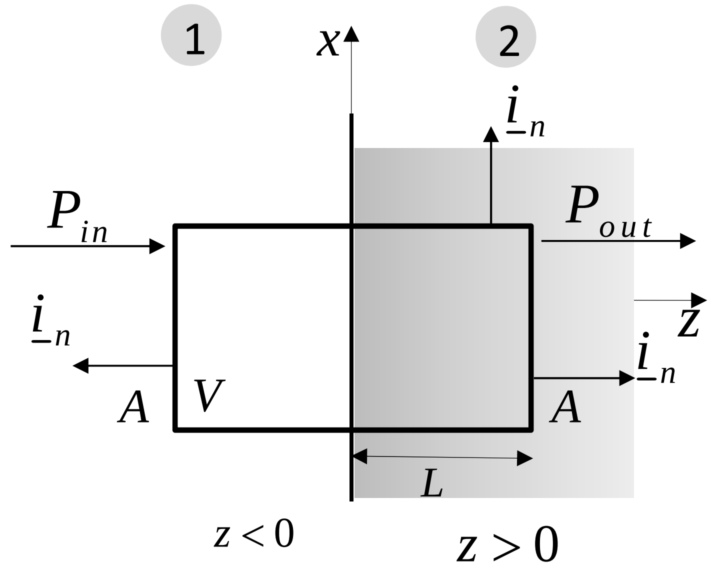
$$S^t = \frac{1}{2\zeta_2}\left|E_0^{\,t}\right|^2 = \frac{1}{2\zeta_2}\left|E_0^{\,t}\right|^2\left|\tau\right|^2 = \frac{\zeta_1}{\zeta_2}\cdot\frac{1}{2\zeta_1}\left|E_0^{\,i}\right|^2\left|\tau\right|^2 = S^i\left|\tau\right|^2\frac{\zeta_1}{\zeta_2}$$

$$S^t = \frac{1}{2\zeta_2}\left|E_0^{\,t}\right|^2 = S^i - S^r = \frac{1}{2\zeta_1}\left|E_0^{\,i}\right|^2 - \frac{1}{2\zeta_1}\left|E_0^{\,r}\right|^2 = \frac{1}{2\zeta_1}\left|E_0^{\,i}\right|^2(1-\left|\Gamma\right|^2) = S^i(1-\left|\Gamma\right|^2)$$

$$\Rightarrow \boxed{S^t = S^i\left[1-\left|\Gamma\right|^2\right]}$$ *Transmitted Average Power Density [power per unit area, W/m²]*

Transmitted Power Density (lossless media)

$$S^t = S^i\left[1-|\Gamma|^2\right]$$



$$P_{source} = 0.\ P_{diss} = 0: \qquad \oiint_S \mathrm{Re}\{\underline{S}\}\cdot \underline{i}_n dS + P_{diss} = P_{source} \qquad \oiint_S \mathrm{Re}\{\underline{S}\}\cdot \underline{i}_n dS = 0$$

$$\iint_{A(z<0)} \mathrm{Re}\{\underline{S}\}\cdot(-\underline{i}_z)dA + \iint_{A(z>0)} \mathrm{Re}\{\underline{S}\}\cdot \underline{i}_z dA = 0 \iff P_{in} = P_{out}$$

$$\frac{A}{2\zeta_1}\left[|E_0^i|^2 - |E_0^r|^2\right]\underline{i}_z\cdot(-\underline{i}_z) + \frac{A}{2\zeta_2}|E_0^t|^2\,\underline{i}_z\cdot\underline{i}_z = 0 \iff S^i\left[1-|\Gamma|^2\right] = S^t$$

If the transmitted average power density is evaluated just after the interface (z=0$^+$), last equation is still valid even if medium “2” is lossy (it can be shown by considering *L*→0 in previous case and noting that the dissipated power in the differential volume must vanish as well: average power flow across the boundary must be conserved).

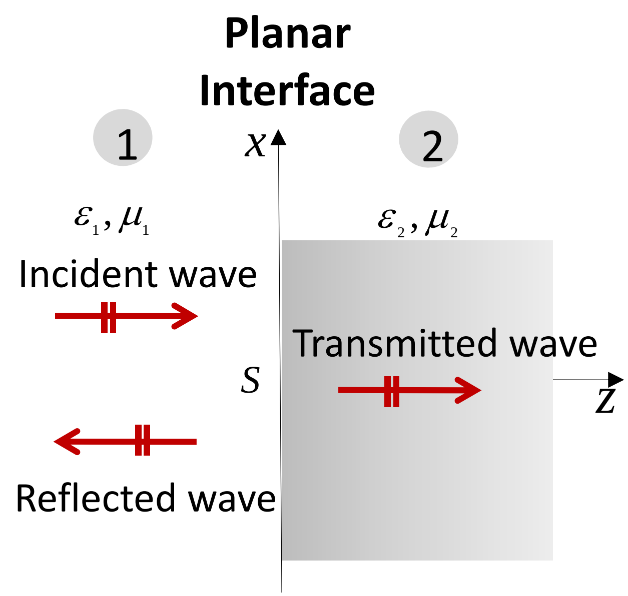
Medium 2: conductor

- characteristic impedance

$$\zeta_2 = \sqrt{\frac{\mu_0}{\varepsilon_{eff}}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r \left(1 - j\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r}\right)}}$$

- propagation constant

$$k_2 = \omega\sqrt{\mu_0 \varepsilon_{eff}} = \omega\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \left(1 - j\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r}\right)} = \beta_2 - j\alpha_2$$



$$\tau = \frac{2\zeta_2}{\zeta_1 + \zeta_2}$$

$$\begin{cases} \underline{E}_2 = E_0^{\,t} e^{-jk_2 z} \underline{i}_x = E_0^{\,t} e^{-\alpha_2 z} e^{-j\beta_2 z} \underline{i}_x \\ \underline{H}_2 = \dfrac{E_0^{\,t}}{\zeta_2} e^{-jk_2 z} \underline{i}_y = \dfrac{E_0^{\,t}}{\zeta_2} e^{-\alpha_2 z} e^{-j\beta_2 z} \underline{i}_y \end{cases} \Rightarrow \begin{cases} \underline{E}_2 = \tau E_0^{\,i} e^{-\alpha_2 z} e^{-j\beta_2 z} \underline{i}_x \\ \underline{H}_2 = \dfrac{\tau E_0^{\,i}}{\zeta_2} e^{-\alpha_2 z} e^{-j\beta_2 z} \underline{i}_y \end{cases}$$

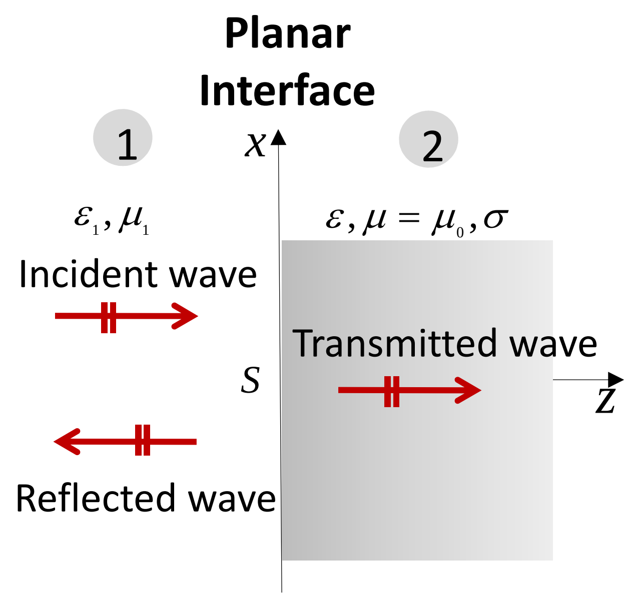
Dielectric-conductor: medium 2 with losses

- characteristic impedance

$$\zeta_2 = \sqrt{\frac{\mu_0}{\varepsilon_{eff}}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r \left(1 - j\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r}\right)}}$$

- propagation constant

$$k_2 = \omega \sqrt{\mu_0 \varepsilon_{eff}} = \omega \sqrt{\varepsilon_0 \varepsilon_r \mu_0 \left(1 - j\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r}\right)} = \beta_2 - j\alpha_2$$



Filed in medium 2 (transmitted wave):

$$\begin{cases} \underline{E}_2 = E_0^t e^{-jk_2 z} \underline{i}_x = E_0^t e^{-\alpha_2 z} e^{-j\beta_2 z} \underline{i}_x \\ \underline{H}_2 = \dfrac{E_0^t}{\zeta_2} e^{-jk_2 z} \underline{i}_y = \dfrac{E_0^t}{\zeta_2} e^{-\alpha_2 z} e^{-j\beta_2 z} \underline{i}_y \end{cases} \Rightarrow \begin{cases} \underline{E}_2 = \tau E_0^i e^{-\alpha_2 z} e^{-j\beta_2 z} \underline{i}_x \\ \underline{H}_2 = \dfrac{\tau E_0^i}{\zeta_2} e^{-\alpha_2 z} e^{-j\beta_2 z} \underline{i}_y \end{cases}$$

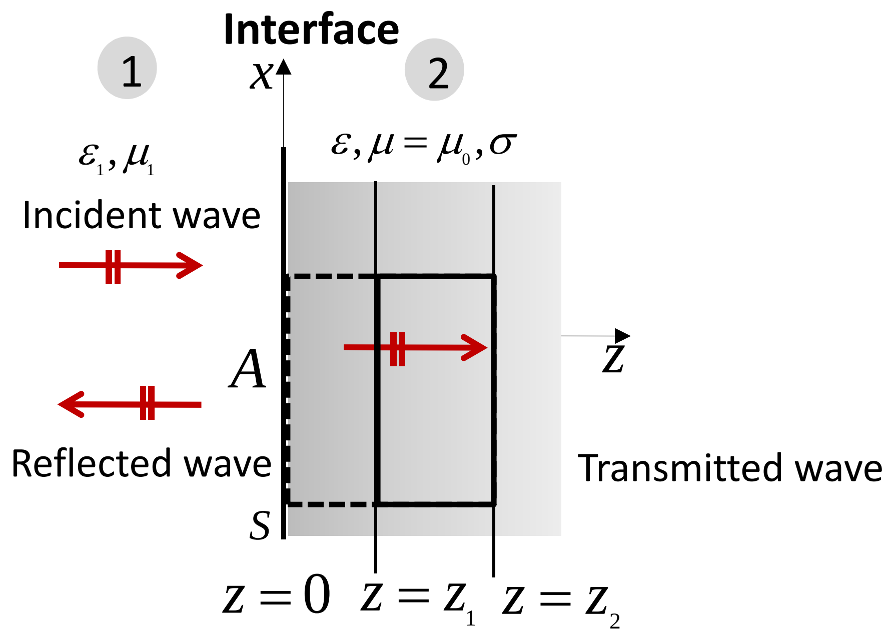
$$\tau = \frac{2\zeta_2}{\zeta_1 + \zeta_2}$$

Transmitted Power Density (medium 2: conductor)

$$S^t(z=0)=\frac{1}{2}\mathrm{Re}\left\{\frac{1}{\zeta_2}\right\}\left|E_0^{\,i}\right|^2\left|\tau\right|^2=S^i\left(1-\left|\Gamma\right|^2\right)$$

$$S^t(z>0)=\frac{1}{2}\mathrm{Re}\left\{\frac{1}{\zeta_2}\right\}\left|E_0^{\,t}\right|^2 e^{-2\alpha_2 z}=$$

$$=S^t(z=0)e^{-2\alpha_2 z}=S^i\left(1-\left|\Gamma\right|^2\right)e^{-2\alpha_2 z}$$



The power dissipated by Joule effect:

$$P_{Joule}=\iiint_V \frac{1}{2}\sigma\left|\underline{E}_2\right|^2 dV=P_{in}-P_{out}=S^t(z_1)A-S^t(z_2)A=$$

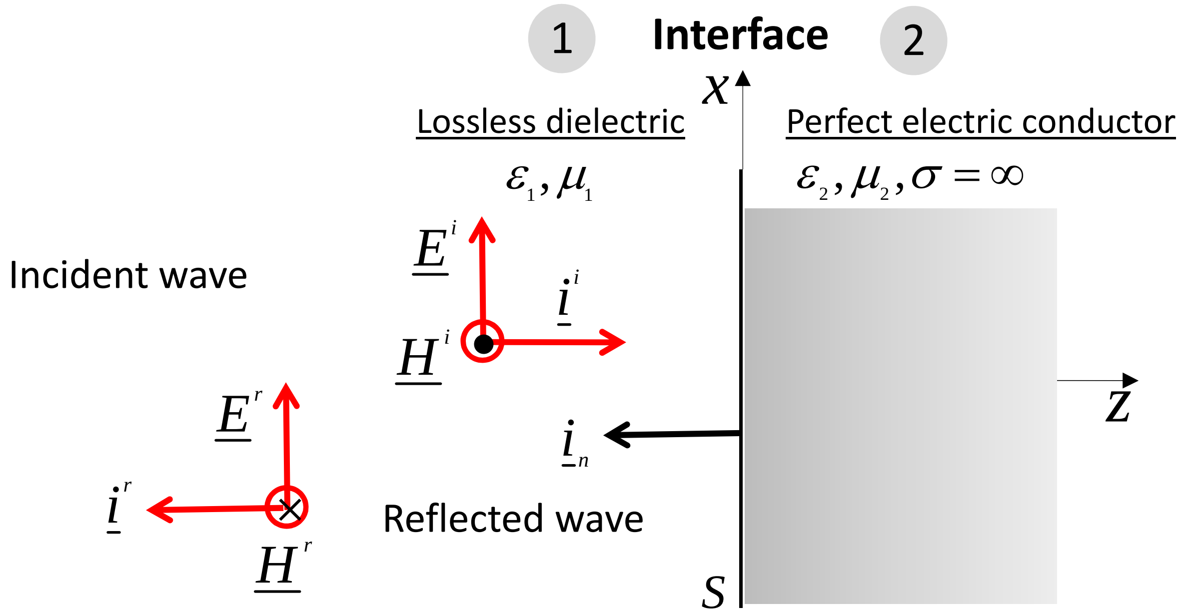
$$=A\frac{1}{2}\mathrm{Re}\left\{\frac{1}{\zeta_2}\right\}\left|E_0^{\,t}\right|^2\left(e^{-2\alpha z_1}-e^{-2\alpha z_2}\right)$$

If z_1=0, and z_2=z :

$$P_{Joule}=\iiint_V \frac{1}{2}\sigma\left|\underline{E}_2\right|^2 dV=P_{in}-P_{out}=S^t(0)A-S^t(z)A=A\frac{1}{2}\mathrm{Re}\left\{\frac{1}{\zeta_2}\right\}\left|E_0^{\,t}\right|^2\left(1-e^{-2\alpha z}\right)=S^i\left(1-\left|\Gamma\right|^2\right)\left(1-e^{-2\alpha z}\right)$$

$$P_{Joule}=A\frac{1}{2}\mathrm{Re}\left\{\frac{1}{\zeta_2}\right\}\left|E_0^{\,t}\right|^2\left(1-e^{-2\alpha z}\right)=AS^i\left(1-\left|\Gamma\right|^2\right)\left(1-e^{-2\alpha z}\right)\cong AS^i\left(1-\left|\Gamma\right|^2\right)\quad if\quad z>>1/\alpha=\delta$$

Dielectric – PEC (Perfect Electric Conductor)



$$\underline{E}_2 = 0 \;\&\; \underline{H}_2 = 0$$

$$\begin{cases} \underline{E}_1(z) = E_0^i e^{-jk_1 z}\,\underline{i}_x + E_0^r e^{+jk_1 z}\,\underline{i}_x \\ \underline{H}_1(z) = \dfrac{E_0^i}{\zeta_1} e^{-jk_1 z}\,\underline{i}_y - \dfrac{E_0^r}{\zeta_1} e^{+jk_1 z}\,\underline{i}_y \end{cases}$$

Boundary conditions

$\underline{i}_n \times \underline{E}_1 = 0$

$\underline{i}_n \times \underline{H}_1 = \underline{J}_s$ Surface current

$D_1 \cdot \underline{i}_n = \rho_s$ Surface charge density

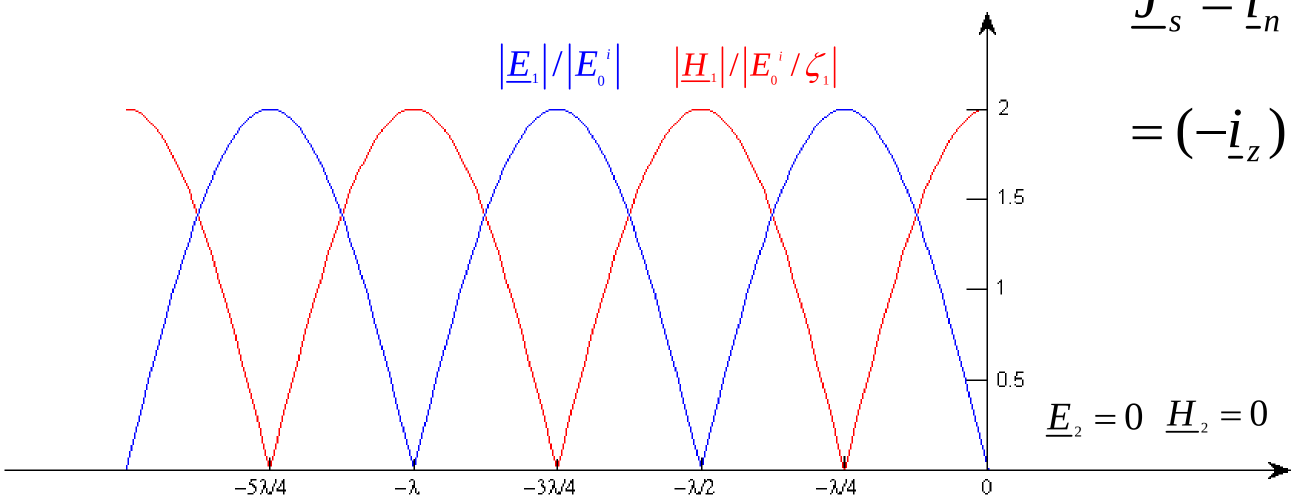
Dielectric – PEC (Perfect Electric Conductor)

$$\underline{i}_n \times \underline{E}_1 = 0 \quad E_x\Big|_{z=0^-} = E_x\Big|_{z=0^+} \quad E_0^i + E_0^r = 0 \quad E_0^r = -E_0^i \quad \Longrightarrow \quad \Gamma = -1$$

$$\left(\Gamma = \frac{\zeta_2/\zeta_1 - 1}{\zeta_2/\zeta_1 + 1} \to -1 \quad if \quad |\zeta_2/\zeta_1| \to 0 \right)$$

Total reflection

$$\begin{cases} \underline{E}_1(z)\underline{i}_x = E_0^i[e^{-j\beta_1 z} - e^{+j\beta_1 z}]\underline{i}_x = -2jE_0^i \sin(\beta_1 z)\underline{i}_x \\ \underline{H}_1(z)\underline{i}_y = \frac{E_0^i}{\zeta_1}[e^{-j\beta_1 z} + e^{+j\beta_1 z}]\underline{i}_y = \frac{2E_0^i}{\zeta_1}\cos(\beta_1 z)\underline{i}_y \end{cases}$$



$$\underline{E}_2 = 0 \quad \underline{H}_2 = 0$$

$$\underline{J}_s = \underline{i}_n \times \underline{H}_1(z=0) = $$

$$= (-\underline{i}_z) \times \frac{2E_0^i}{\zeta_1}\underline{i}_y = \frac{2E_0^i}{\zeta_1}\underline{i}_x$$

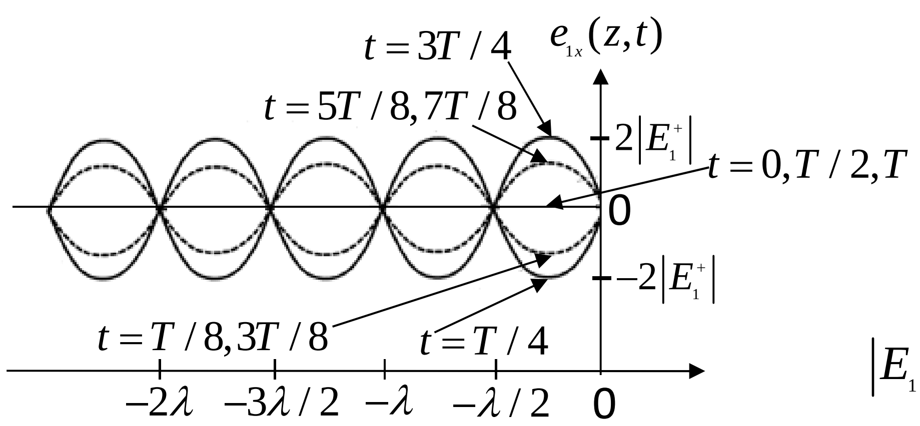
Dielectric – PEC (Perfect Electric Conductor)

$$\underline{e}_1(z,t) = \mathrm{Re}\left\{\underline{E}_1 e^{j\omega t}\right\} =$$
$$= \mathrm{Re}\left\{2e^{-j\pi/2}\left|E_0^{\,i}\right| e^{j\phi}\sin\left(\beta_1 z\right)e^{j\omega t}\underline{i}_x\right\}$$
$$= 2\left|E_0^{\,i}\right|\sin\left(\beta_1 z\right)\cos(\omega t+\phi-\pi/2)\underline{i}_x$$
$$= 2\left|E_0^{\,i}\right|\sin\left(\beta_1 z\right)\sin(\omega t+\phi)\underline{i}_x$$

$$\underline{h}_1(z,t) = \mathrm{Re}\left\{\underline{H}_1 e^{j\omega t}\right\} =$$
$$= \mathrm{Re}\left\{\frac{2\left|E_0^{\,i}\right|}{\zeta_1} e^{j\phi}\cos\left(\beta_1 z\right)e^{j\omega t}\underline{i}_y\right\}$$
$$= \frac{2\left|E_0^{\,i}\right|}{\zeta_1}\cos\left(\beta_1 z\right)\cos(\omega t+\phi)\underline{i}_y$$

$$E_0^{\,i} = \left|E_0^{\,i}\right| e^{j\phi}$$

Assuming that $\phi = 0$ (the phase of the incident electric field is 0° at the interface), the instantaneous electric field in medium 1 is: $\underline{E}_1 = 2\left|E_0^{\,i}\right|\sin\left(\beta_1 z\right)\sin\left(\omega t\right)\underline{i}_x$



Location of nulls and peaks in the standing wave electric field pattern :

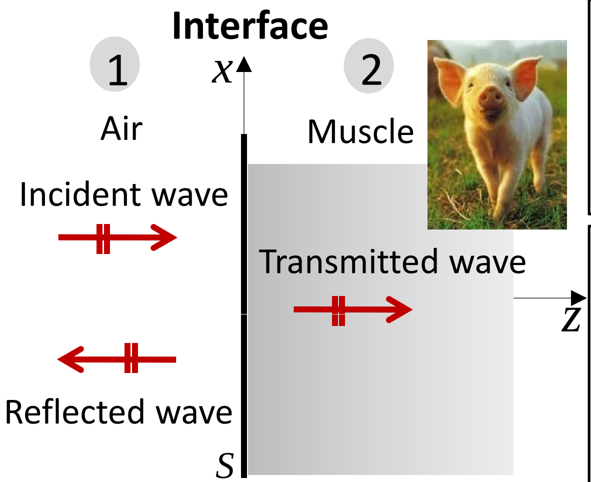
$$\left|E_1\right|_{\min} = 0 \qquad \sin\beta_1 z = 0 \Rightarrow \beta_1(-z) = n\pi$$
$$z = -n\pi/\beta_1 = -n\lambda_1/2, n = 0,1,2,\ldots$$

$$\left|E_1\right|_{\max} = 2\left|E_0^{\,i}\right| \qquad \sin\beta_1 z = 1 \Rightarrow \beta_1(-z) = (2n+1)\frac{\pi}{2}$$
$$z = -(2n+1)\lambda_1/4, n = 0,1,2,\ldots$$

Microwave treatment of hypothermia in newborn piglets

Newly born piglets are very vulnerable to cold temperatures, and many of them die because of hypothermia. Hypothermia can be treated by placing the piglets under infrared lamps, which are not very effective and are very costly. An alternative to treat hypothermia is by microwaves, which is a more expensive technique, but more effective and consumes less power.

Consider a plane wave normally incident at the air-muscle tissue interface. Calculate the reflection coefficient, the propagation constant and depth penetration at 915MHz and 2.45GHz. What is the percentage of incident power absorbed by the muscle tissue at 915MHz and 2.45GHz?



f=915MHz

$$\varepsilon_{rm} = 51, \sigma_m = 1.6 S/m$$

$$\sigma_m / (2\pi f \varepsilon_0 \varepsilon_{rm}) \cong 0.61$$

$$\zeta_0 = 377\Omega$$

$$\zeta_m = \frac{\zeta_0}{\sqrt{1 - j\dfrac{\sigma}{\omega\varepsilon_0\varepsilon_{rm}}}} = 48.7\angle 15.8^\circ\ \Omega$$

$$\Gamma = \frac{\zeta_m - \zeta_0}{\zeta_m + \zeta_0} \cong 0.779\angle 176^\circ$$

$$\delta = \frac{c}{\omega\sqrt{\dfrac{\varepsilon_r}{2}\left(\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon_0\varepsilon_r}\right)^2} - 1\right)}} = 2.47cm$$

$$\frac{S^t}{S^i}\times 100 = \left(1 - |\Gamma|^2\right)\times 100 = 39.3\%$$

f=2.45GHz

$$\varepsilon_{rm} = 47, \sigma_m = 2.21 S/m$$

$$\sigma_m / (2\pi f \varepsilon_0 \varepsilon_{rm}) \cong 0.345$$

$$\zeta_0 = 377\Omega$$

$$\zeta_m = 53.5\angle 9.52^\circ\ \Omega$$

$$\Gamma \cong 0.755\angle 177^\circ$$

$$\delta = 1.67cm$$

$$\frac{S^t}{S^i}\times 100 = \left(1 - |\Gamma|^2\right)\times 100 = 43\%$$