UNIVERSITÀ DI PISA

Electromagnetic Radiations and Biological Interactions

"Laurea Magistrale" in Biomedical Engineering

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Reflection and transmission of plane waves through multiple interfaces

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Lecture Content

➢Reflection and transmission of plane waves through multiple interfaces

- Boundary conditions
- Reflection and transmission coefficients
- Absorbed average power

Motivations

- Human body is made of different tissues like skin, fat, muscle, blood etc.

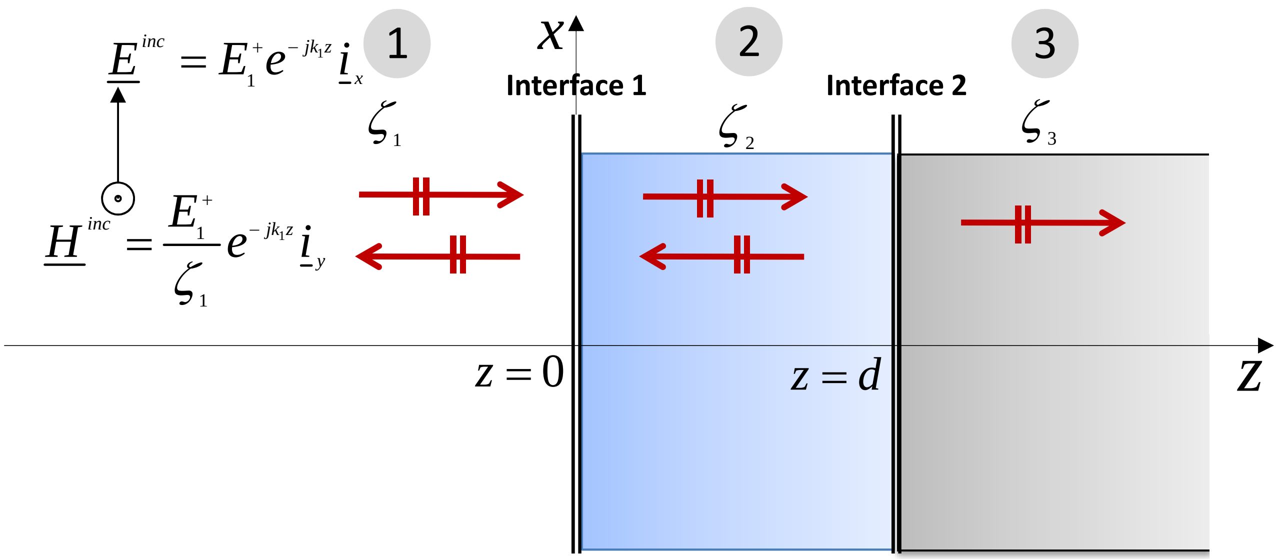
- The effects of RF/microwave interaction with biological tissues are the result of the EM waves penetration into the living system and their propagation into it.

- The biological effects of RF/microwaves do not depend on the external power density, but on the field inside the tissue or the body. Hence, the internal fields have to be determined for any meaningful and general quantification of biological interaction.

- Shielding of electromagnetic fields is of interest to protect biomedical environments as well as the measurements involved (electromagnetic absorbers, shielding effectiveness).

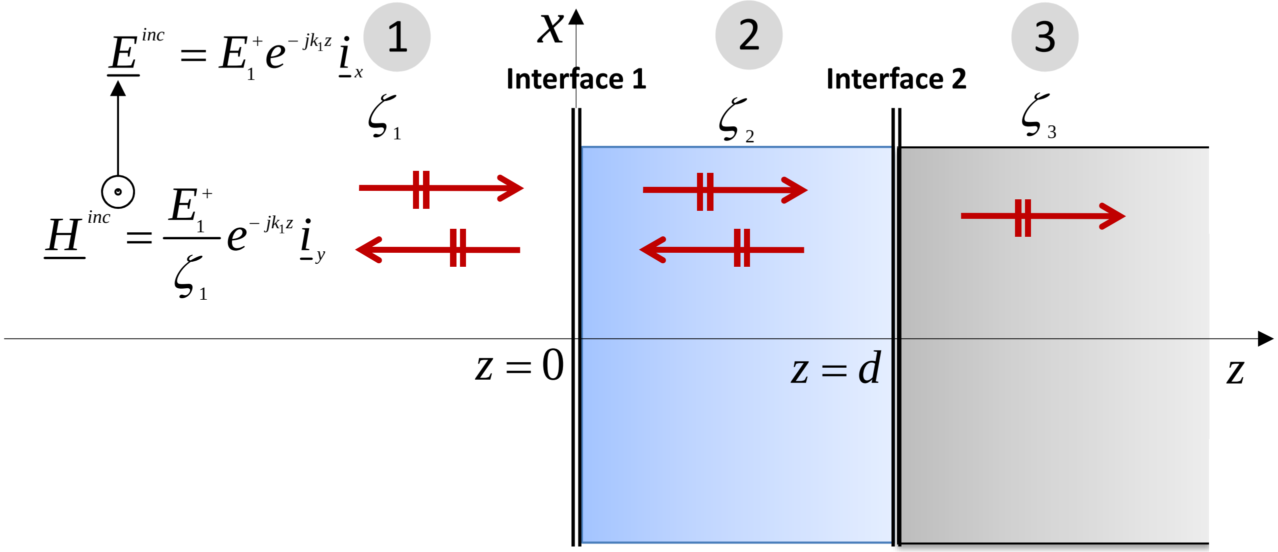
The problem

Hypothesis: multiple planar interfaces; monochromatic plane wave linearly polarized; orthogonal incidence; medium 3 is infinite in extent.



Medium 1 (z<0):	Medium 2 (0<z<d):	Medium 3 (z>d):
$\underline{E}_1 = \left(E_1^+ e^{-jk_1 z} + E_1^- e^{jk_1 z}\right)\underline{i}_x$	$\underline{E}_2 = \left(E_2^+ e^{-jk_2 z} + E_2^- e^{jk_2 z}\right)\underline{i}_x$	$\underline{E}_3 = E_3^+ e^{-jk_3 z} \underline{i}_x$
$\underline{H}_1 = \left(\frac{E_1^+}{\zeta_1} e^{-jk_1 z} - \frac{E_1^-}{\zeta_1} e^{jk_1 z}\right)\underline{i}_y$	$\underline{H}_2 = \left(\frac{E_2^+}{\zeta_2} e^{-jk_2 z} - \frac{E_2^-}{\zeta_2} e^{jk_2 z}\right)\underline{i}_y$	$\underline{H}_3 = \frac{E_3^+}{\zeta_3} e^{-jk_3 z} \underline{i}_y$

Boundary conditions



Interface 1

$$\underline{i}_z \times \underline{E}_1\Big|_{z=0^-} = \underline{i}_z \times \underline{E}_2\Big|_{z=0^+}$$

$$\underline{i}_z \times \underline{H}_1\Big|_{z=0^-} = \underline{i}_z \times \underline{H}_2\Big|_{z=0^+}$$

Interface 2

$$\underline{i}_z \times \underline{E}_2\Big|_{z=d^-} = \underline{i}_z \times \underline{E}_3\Big|_{z=d^+}$$

$$\underline{i}_z \times \underline{H}_2\Big|_{z=d^-} = \underline{i}_z \times \underline{H}_3\Big|_{z=d^+}$$

Reflection and Transmission Coefficients

Medium 1 (z<0):

$$\underline{E}_1 = \left(E_1^+ e^{-jk_1 z} + E_1^- e^{jk_1 z}\right)\underline{i}_x$$

$$\underline{H}_1 = \left(\frac{E_1^+}{\zeta_1} e^{-jk_1 z} - \frac{E_1^-}{\zeta_1} e^{jk_1 z}\right)\underline{i}_y$$

Medium 2 (0<z<d):

$$\underline{E}_2 = \left(E_2^+ e^{-jk_2 z} + E_2^- e^{jk_2 z}\right)\underline{i}_x$$

$$\underline{H}_2 = \left(\frac{E_2^+}{\zeta_2} e^{-jk_2 z} - \frac{E_2^-}{\zeta_2} e^{jk_2 z}\right)\underline{i}_y$$

Medium 3 (z>d):

$$\underline{E}_3 = E_3^+ e^{-jk_3 z}\underline{i}_x$$

$$\underline{H}_3 = \frac{E_3^+}{\zeta_3} e^{-jk_3 z}\underline{i}_y$$

$$\begin{cases} E_1^+ + E_1^- = E_2^+ + E_2^- \\ \dfrac{E_1^+}{\zeta_1} - \dfrac{E_1^-}{\zeta_1} = \dfrac{E_2^+}{\zeta_2} - \dfrac{E_2^-}{\zeta_2} \\ E_2^+ e^{-jk_2 d} + E_2^- e^{jk_2 d} = E_3^+ e^{-jk_3 d} \\ \dfrac{E_2^+}{\zeta_2} e^{-jk_2 d} - \dfrac{E_2^-}{\zeta_2} e^{jk_2 d} = \dfrac{E_3^+}{\zeta_3} e^{-jk_3 d} \end{cases}$$

The latter equations are similar to those for the single interface problem.

*Reflection coefficient **at z=d*** & *Transmission coefficient **at z=d***

$$\Gamma' = \frac{\zeta_3 - \zeta_2}{\zeta_3 + \zeta_2} \qquad \tau' = \frac{2\zeta_3}{\zeta_3 + \zeta_2}$$

$$\begin{cases} E_2^- e^{jk_2 d} = \Gamma' E_2^+ e^{-jk_2 d} \\ E_3^+ e^{-jk_3 d} = \tau' E_2^+ e^{-jk_2 d} \end{cases} \Longrightarrow$$

Reflection and Transmission Coefficients

$$\begin{cases} E_1^+ + E_1^- = E_2^+ + E_2^- \\ \dfrac{E_1^+}{\zeta_1} - \dfrac{E_1^-}{\zeta_1} = \dfrac{E_2^+}{\zeta_2} - \dfrac{E_2^-}{\zeta_2} \\ E_2^- e^{jk_2 d} = \Gamma' E_2^+ e^{-jk_2 d} \end{cases} \quad \begin{cases} E_1^+ + E_1^- = E_2^+\left(1+\Gamma' e^{-j2k_2 d}\right) = E_2^+(1+a) \\ E_1^+ - E_1^- = \dfrac{\zeta_1}{\zeta_2} E_2^+\left(1-\Gamma' e^{-j2k_2 d}\right) = \dfrac{\zeta_1}{\zeta_2} E_2^+(1-a) \\ E_2^- e^{jk_2 d} = \Gamma' E_2^+ e^{-jk_2 d} \end{cases}$$

$$\boxed{a = \Gamma' e^{-j2k_2 d}}$$

$$E_1^+ = \frac{E_2^+}{2}\left[(1+a) + \frac{\zeta_1}{\zeta_2}(1-a)\right], \quad E_1^- = \frac{E_2^+}{2}\left((1+a) - \frac{\zeta_1}{\zeta_2}(1-a)\right) \quad E_3^+ e^{-jk_3 d} = \tau' E_2^+ e^{-jk_2 d}$$

$$\Gamma = \left.\frac{E_1^- e^{jk_1 z}}{E_1^+ e^{-jk_1 z}}\right|_{z=0} = \frac{E_1^-}{E_1^+} = \frac{\zeta_2(1+a) - \zeta_1(1-a)}{\zeta_2(1+a) + \zeta_1(1-a)} = \frac{(\zeta_2 - \zeta_1) + a(\zeta_2 + \zeta_1)}{(\zeta_2 + \zeta_1) + a(\zeta_2 - \zeta_1)}$$

$$\tau = \frac{\left.E_3^+ e^{-jk_3 z}\right|_{z=d}}{\left.E_1^+ e^{-jk_1 z}\right|_{z=0}} = \frac{E_3^+ e^{-jk_3 d}}{E_1^+} = \frac{2\tau' \zeta_2 e^{-jk_2 d}}{\zeta_2(1+a) + \zeta_1(1-a)} \quad \Longrightarrow$$

$$a = \Gamma' e^{-j2k_2 d} = \frac{\zeta_3 - \zeta_2}{\zeta_3 + \zeta_2} e^{-j2k_2 d}$$

Reflection and Transmission Coefficients

Reflection coefficient

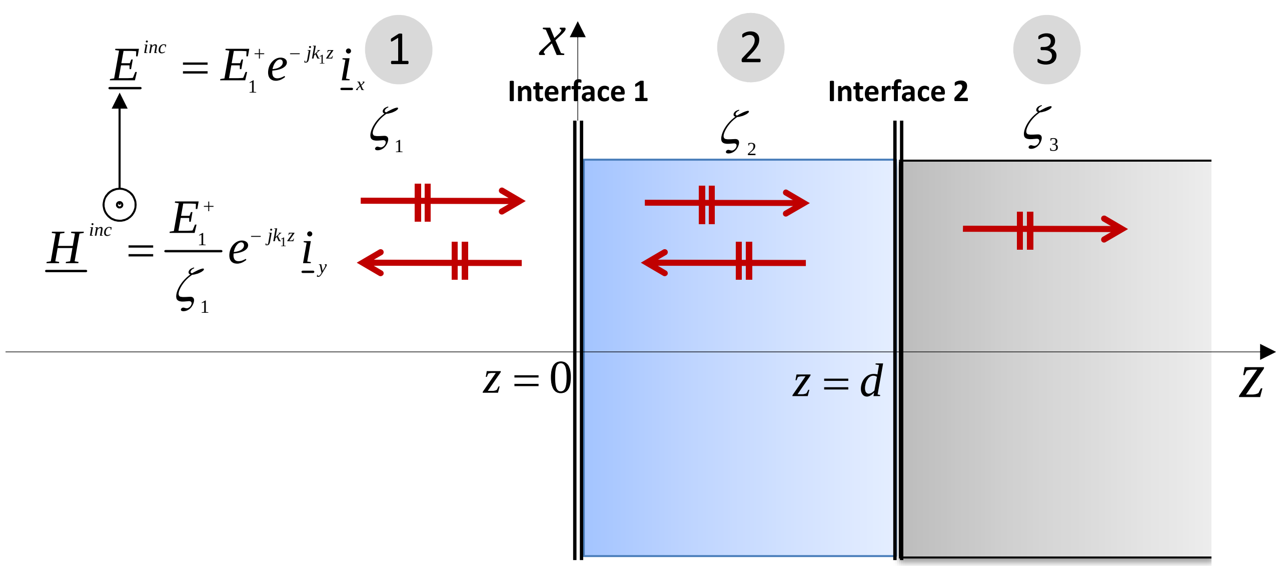
$$\Gamma = \frac{(\zeta_2-\zeta_1)(\zeta_3+\zeta_2)+(\zeta_2+\zeta_1)(\zeta_3-\zeta_2)e^{-j2k_2d}}{(\zeta_2+\zeta_1)(\zeta_3+\zeta_2)+(\zeta_2-\zeta_1)(\zeta_3-\zeta_2)e^{-j2k_2d}}$$

$$E_1^- e^{jk_1z}\Big|_{z=0} = \Gamma E_1^+ e^{-jk_1z}\Big|_{z=0},$$
$$E_1^- = \Gamma E_1^+$$

Transmission coefficient

$$\tau = \frac{4\zeta_3\zeta_2 e^{-jk_2d}}{(\zeta_2+\zeta_1)(\zeta_3+\zeta_2)+(\zeta_2-\zeta_1)(\zeta_3-\zeta_2)e^{-j2k_2d}}$$

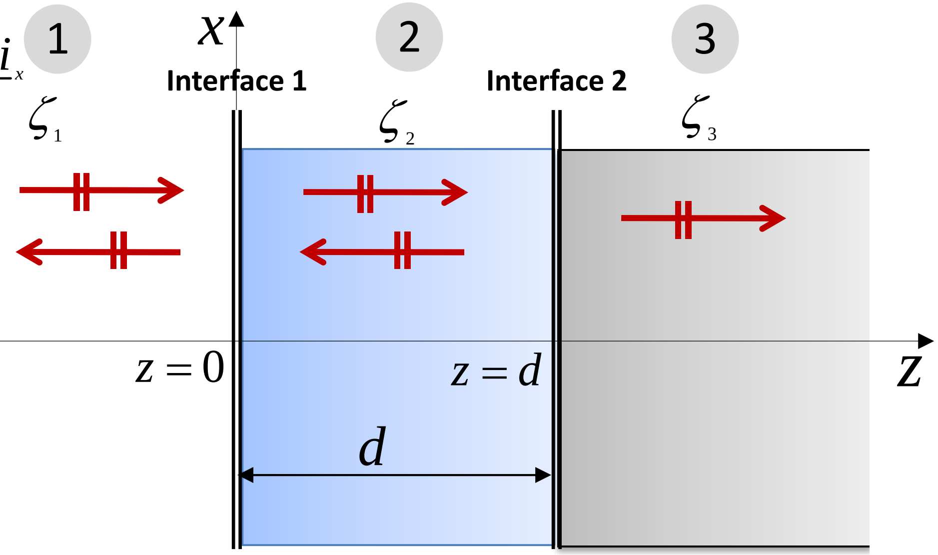
$$E_3^+ e^{-jk_3z}\Big|_{z=d} = \tau E_1^+ e^{-jk_1z}\Big|_{z=0},$$
$$E_3^+ e^{-jk_3d} = \tau E_1^+$$



Medium ‘2”: (electrically) thin layer

$$\underline{E}^{inc} = E_1^+ e^{-jk_1 z}\, \underline{i}_x$$

$$\underline{H}^{inc} = \frac{E_1^+}{\zeta_1} e^{-jk_1 z}\, \underline{i}_y$$



<u>Hypothesis:</u> in medium “2’,

$$d \ll \lambda_2$$

$$\left(and\ d_2 \ll 1/\alpha_2 \ \ for\ lossy\ media\right)$$

$$\Downarrow$$

$$a = \Gamma' e^{-j2k_2 d} = \Gamma' e^{-2\alpha_2 d} e^{-j4\pi\frac{d}{\lambda_2}} \cong \Gamma' = \frac{\zeta_3 - \zeta_2}{\zeta_3 + \zeta_2}$$

$$\Gamma = \frac{(\zeta_2 - \zeta_1)(\zeta_3 + \zeta_2) + (\zeta_2 + \zeta_1)(\zeta_3 - \zeta_2) e^{-j2k_2 d}}{(\zeta_2 + \zeta_1)(\zeta_3 + \zeta_2) + (\zeta_2 - \zeta_1)(\zeta_3 - \zeta_2) e^{-j2k_2 d}} \Rightarrow \boxed{\Gamma = \frac{\zeta_3 - \zeta_1}{\zeta_3 + \zeta_1}}$$

Represents the reflection coefficient between medium 1 and medium 3, when medium 2 doesn’t exist.

If the thickness of an intermediate layer is thinner than the wavelength and the penetration depth (<u>both evaluated in such a medium</u>), the above layer can be neglected.

Approximated expressions for electrically thin layers

$$|kd| << 1 \rightarrow d << \lambda_2, d << 1/\alpha_2$$

$$\left(e^x = 1 + x + x^2/2 + \dots, |x| < 1\right)$$

$$1 \pm a = 1 \pm \Gamma' e^{-j2k_2 d} \cong 1 \pm \Gamma'\left(1 - j2k_2 d\right)$$

$$\Rightarrow \quad \Gamma = \frac{(\zeta_2 - \zeta_1)(\zeta_3 + \zeta_2) + (\zeta_2 + \zeta_1)(\zeta_3 - \zeta_2)(1 - j2k_2 d)}{(\zeta_2 + \zeta_1)(\zeta_3 + \zeta_2) + (\zeta_2 - \zeta_1)(\zeta_3 - \zeta_2)(1 - j2k_2 d)}$$

$$\tau = \frac{4\zeta_3\zeta_2(1 - j2k_2 d)}{(\zeta_2 + \zeta_1)(\zeta_3 + \zeta_2) + (\zeta_2 - \zeta_1)(\zeta_3 - \zeta_2)(1 - j2k_2 d)}$$

Medium '2": thick and lossy

<u>Hypothesis:</u> medium 2 is dissipative $k_2 = \beta_2 - j\alpha_2$ *and* $d >> \delta_2 = 1/\alpha_2$

$$a = \Gamma' e^{-j2\beta_2 d} e^{-2\alpha_2 d} \to 0$$

$$\left(\Gamma = \frac{(\zeta_2 - \zeta_1) + a(\zeta_2 + \zeta_1)}{(\zeta_2 + \zeta_1) + a(\zeta_2 - \zeta_1)} \right)$$

$$\Gamma = \frac{(\zeta_2 - \zeta_1)(\zeta_3 + \zeta_2) + (\zeta_2 + \zeta_1)(\zeta_3 - \zeta_2) e^{-j4\pi \frac{d}{\lambda_2}}}{(\zeta_2 + \zeta_1)(\zeta_3 + \zeta_2) + (\zeta_2 - \zeta_1)(\zeta_3 - \zeta_2) e^{-j4\pi \frac{d}{\lambda_2}}} \cong \frac{(\zeta_2 - \zeta_1)}{(\zeta_2 + \zeta_1)}$$

Represents the reflection coefficient between medium 1 and medium 2, when medium 3 doesn't exist.

If an intermediate layer is dissipative and thick with respect to its penetration depth, next interface doesn't influence the reflection phenomena at previous interfaces.

Transmission through a dielectric slab (air-dielectric-air)

Hypothesis: medium 1 and medium 3 are equal: $\zeta_3 = \zeta_1$

$$\left(\Gamma = \frac{(\zeta_2 - \zeta_1) + a(\zeta_2 + \zeta_1)}{(\zeta_2 + \zeta_1) + a(\zeta_2 - \zeta_1)}, a = \Gamma' e^{-j2k_2d_2} = \frac{\zeta_3 - \zeta_2}{\zeta_3 + \zeta_2} e^{-j2k_2d_2} \right) \qquad \Gamma = 0 : is\ it\ possible??$$

$$\Gamma = 0 \Leftrightarrow a = \frac{\zeta_1 - \zeta_2}{\zeta_1 + \zeta_2} \quad \Rightarrow \quad a = \frac{\zeta_3 - \zeta_2}{\zeta_3 + \zeta_2} e^{-j2k_2d} = \left. \frac{\zeta_1 - \zeta_2}{\zeta_1 + \zeta_2} \right|_{\zeta_1 = \zeta_3} \quad \Longrightarrow$$

$$\Rightarrow e^{-j2k_2d} = 1 \Leftrightarrow \alpha_2 = 0\ (k_2 = \beta_2) \ \ \text{and}\ \ 4\pi \frac{d}{\lambda_2} = N 2\pi \rightarrow d = N\frac{\lambda_2}{2}, N = 1, 2, 3\ldots$$

If the first and third layers are equal, it is possible to eliminate the reflected wave $(\Gamma = 0)$
if the second layer thickness meets the condition:

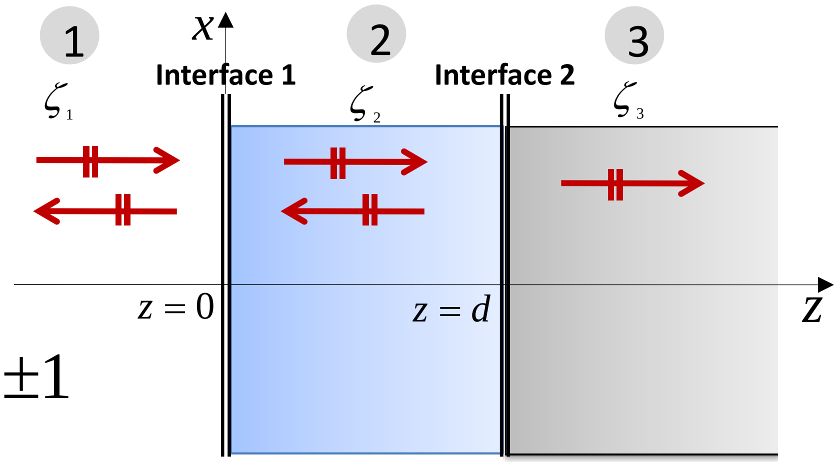
$$d = N\frac{\lambda_2}{2} = N\frac{\lambda_0}{2\sqrt{\varepsilon_{r2}}}$$

A quarter-wavelength coating

Hypothesis: three different and **lossless** media (all characteristic impedances are real valued).

$$\Gamma = 0 : is\ it\ possible??$$

$$\Gamma = 0 \Leftrightarrow a = \frac{\zeta_1 - \zeta_2}{\zeta_1 + \zeta_2}$$



$$\Gamma = 0 \Leftrightarrow \frac{\zeta_3 - \zeta_2}{\zeta_3 + \zeta_2} e^{-j2k_2d} = \frac{\zeta_1 - \zeta_2}{\zeta_1 + \zeta_2} \Rightarrow e^{-j2k_2d} = \pm 1$$

$$\boxed{e^{-j2k_2d} = +1} \Rightarrow 4\pi \frac{d}{\lambda_2} = N2\pi \rightarrow d = N\frac{\lambda_2}{2}, N = 1,2,3\ldots \text{and since } \frac{\zeta_3 - \zeta_2}{\zeta_3 + \zeta_2} = \frac{\zeta_1 - \zeta_2}{\zeta_1 + \zeta_2} \Rightarrow \zeta_1 = \zeta_3$$

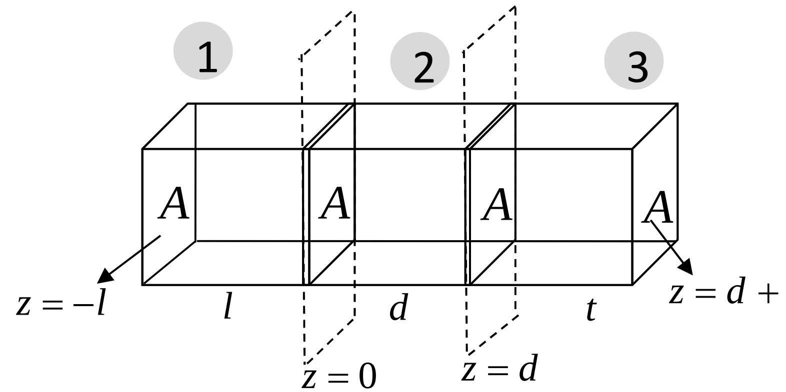
$$\boxed{e^{-j2k_2d} = -1} \Rightarrow 4\pi \frac{d}{\lambda_2} = (2N+1)\pi \rightarrow d = \frac{\lambda_2}{4} + N\frac{\lambda_2}{2}, N = 1,2,3\ldots \text{and since } \frac{\zeta_3 - \zeta_2}{\zeta_3 + \zeta_2} = -\frac{\zeta_1 - \zeta_2}{\zeta_1 + \zeta_2}$$

$$\Rightarrow \zeta_2 = \sqrt{\zeta_1 \zeta_3}$$

To eliminate the reflected wave at the interface between two different medi, it is needed to cover the interface with a quarter-wavelength coating made of a proper dielectric material (ex: glass anti-reflection coating): $\zeta_2 = \sqrt{\zeta_1 \zeta_3}, d = \lambda_2 / 4 + N\lambda_2 / 2, N = 1,2,3\ldots$

Transmitted/absorbed average power

Medium “1” is assumed to be lossless (ex: air)



The average power flow through the surface $z=-l$ or $z=0$:

$$P(z=-l)=A\frac{1}{2}\frac{|E_1^+|^2}{\zeta_1}\left(1-|\Gamma|^2\right)=AS^{inc}\left(1-|\Gamma|^2\right)$$

The active power flow through the surface $z=d$:

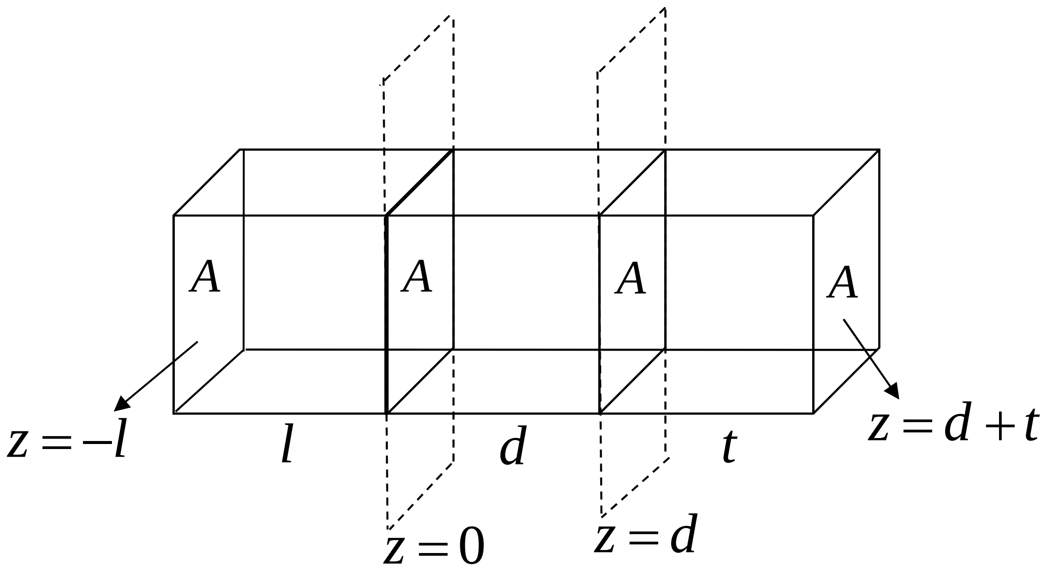
$$P(z=d)=A\frac{1}{2}\mathrm{Re}\left\{\frac{1}{\zeta_3}\right\}\left|E_3^+e^{-jk_3d}\right|^2=\frac{A}{2}\mathrm{Re}\left\{\frac{1}{\zeta_3}\right\}|\tau|^2|E_1^+|^2e^{-2\alpha_3d}=A\mathrm{Re}\left\{\frac{\zeta_1}{\zeta_3}\right\}|\tau|^2\left(\frac{1}{2\zeta_1}|E_1^+|^2\right)=$$

$$=A\mathrm{Re}\left\{\frac{\zeta_1}{\zeta_3}\right\}|\tau|^2S^{inc}$$

The average power dissipated in medium 2:

$$P_{diss}=P(z=0)-P(z=d)=AS^{inc}\left[\left(1-|\Gamma|^2\right)-\mathrm{Re}\left\{\frac{\zeta_1}{\zeta_3}\right\}|\tau|^2\right]\cong AS^{inc}\left(1-|\Gamma|^2\right)\ if\ d>>1/\alpha_2=\delta_2$$

Transmitted/Absorbed average power



The average power flow through the surface at $z=d+t$

$$P_3\left(z=d+t\right)=P_3\left(z=d\right)e^{-2\alpha_3 t}=\frac{A}{2}\mathrm{Re}\left\{\frac{1}{\zeta_3}\right\}\left|E_3^+\right|^2 e^{-2\alpha_3 t}=\frac{A}{2}\mathrm{Re}\left\{\frac{1}{\zeta_3}\right\}\left|E_1^+\right|^2\left|\tau\right|^2 e^{-2\alpha_3 t}=$$

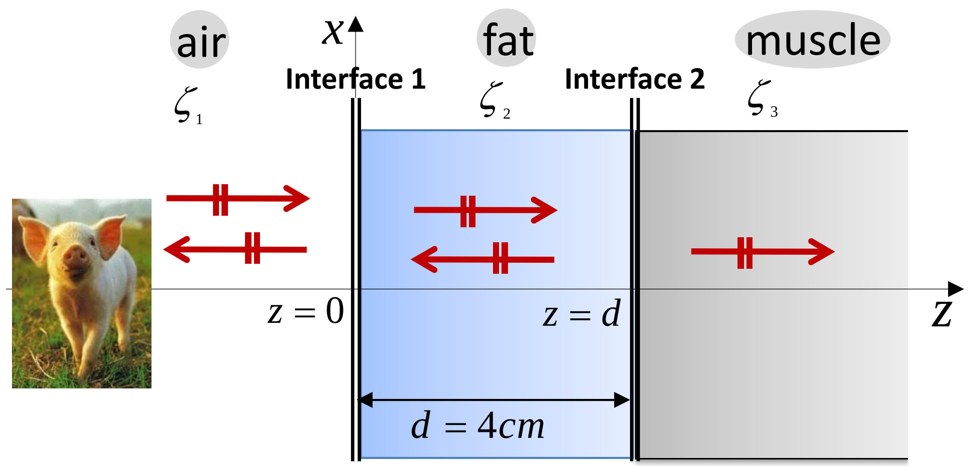
$$=A\,\mathrm{Re}\left\{\frac{\zeta_1}{\zeta_3}\right\}\left|\tau\right|^2 S^{inc}e^{-2\alpha_3 t}$$

The average power dissipated in medium 3:

$$P_{diss}=P_3(z=d)-P_3(z=d+t)=P_3(z=d)\left(1-e^{-2\alpha_3 t}\right)\cong P_3(z=d)\ if\ t>>\delta_3=1/\alpha_3$$

Exercise

Problem: a plane wave normally incident at the surface of the body of a pig. The body of the pig can be approximately modeled as a layer of fat tissue of a certain thickness followed by muscle tissue (assumed infinite in extent). For a mature pig the thickness of the fat layer should be considered 4cm. Calculate the percentage of the incident microwave power reflected back into air, the percentage of the power dissipated in the fat layer, the percentage of power transmitted into the muscle tissue at 915MHz and 2.45GHz.



@915MHz: $\varepsilon_{rf} = 5.6, \sigma_f = 0.1 S/m$

$\varepsilon_{rm} = 51, \sigma_m = 1.6 S/m$

@2.45GHz: $\varepsilon_{rf} = 5.5, \sigma_f = 155 mS/m$

$\varepsilon_{rm} = 47, \sigma_m = 2.21 S/m$

$\varepsilon_0 = 8.854 \cdot 10^{-12} F/m$

$\zeta_0 = 377\Omega$

f=915 MHz

fat: $\frac{\sigma_f}{\omega\varepsilon_0\varepsilon_{rf}} = 0.351 \quad k_f = \beta_f - j\alpha_f = 46 - j7.84m^{-1} \quad \delta_f = \frac{1}{\alpha_f} = 12.7cm \quad \zeta_f = \frac{\zeta_0}{\sqrt{1 - \frac{j\sigma_f}{\omega\varepsilon_0\varepsilon_{rf}}}} \cong 155\angle 9.7^\circ \Omega$

muscle: $\frac{\sigma_m}{\omega\varepsilon_0\varepsilon_{rm}} = 0.617 \qquad \delta_m = \frac{1}{\alpha_m} = 2.47cm \qquad \zeta_m = \frac{\zeta_0}{\sqrt{1 - j\frac{\sigma_m}{\omega\varepsilon_0\varepsilon_{rm}}}} \cong 48.7\angle 15.8^\circ \Omega$

Reflection and transmission coefficients:

$$\Gamma = \frac{(\zeta_f - \zeta_0)(\zeta_m + \zeta_f) + (\zeta_f + \zeta_0)(\zeta_m - \zeta_f)e^{-j2k_f d}}{(\zeta_f + \zeta_0)(\zeta_m + \zeta_f) + (\zeta_f - \zeta_0)(\zeta_m - \zeta_f)e^{-j2k_f d}} \cong 0.414\angle 124^\circ$$

$$\tau = \frac{4\zeta_m \zeta_f e^{-jk_f d}}{(\zeta_f + \zeta_0)(\zeta_m + \zeta_f) + (\zeta_f - \zeta_0)(\zeta_m - \zeta_f)e^{-j2k_f d}} \cong 0.261\angle 47.8^\circ$$

Reflected average power back to air: $|\Gamma|^2 \times 100 \cong 17.1\%$

Transmitted average power into muscle: $\frac{\mathrm{Re}\left\{\frac{1}{2\zeta_m}\right\}|E^i|^2|\tau|^2}{\frac{1}{2\zeta_0}|E^i|^2} \times 100 = \zeta_0 \,\mathrm{Re}\left\{\frac{1}{\zeta_m}\right\}|\tau|^2 \times 100 = 52.7\%$

Absorbed power by the fat layer: $\%P_{abs} = 100 - 17.1 - 52.7 \cong 30.2\%$

f=2.45 GHz

fat: $\frac{\sigma_f}{\omega\varepsilon_0\varepsilon_{rf}} = 0.207$; $\quad k_f = \beta_f - j\alpha_f = (120.91 - j12.38)m^{-1}$ $\quad \delta_f = 8.1cm$ $\quad \zeta_f = \frac{\zeta_0}{\sqrt{1 - j\frac{\sigma_f}{\omega\varepsilon_0\varepsilon_{rf}}}} \cong 159.07\angle 5.85°$

muscle: $\frac{\sigma_m}{\omega\varepsilon_0\varepsilon_{rm}} = 0.345$; $\quad k_f = \beta_f - j\alpha_f = (59.79 - j59.8)m^{-1}$ $\quad \delta_m = 1.67cm$ $\quad \zeta_m = \frac{\zeta_0}{\sqrt{1 - j\frac{\sigma_m}{\omega\varepsilon_0\varepsilon_{rm}}}} \cong 53.5\angle 9.52°$

Reflection and transmission coefficients:

$$|\Gamma| = \left|\frac{(\zeta_f - \zeta_0)(\zeta_m + \zeta_f) + (\zeta_f + \zeta_0)(\zeta_m - \zeta_f)e^{-j2k_f d}}{(\zeta_f + \zeta_0)(\zeta_m + \zeta_f) + (\zeta_f - \zeta_0)(\zeta_m - \zeta_f)e^{-j2k_f d}}\right| \cong 0.249$$

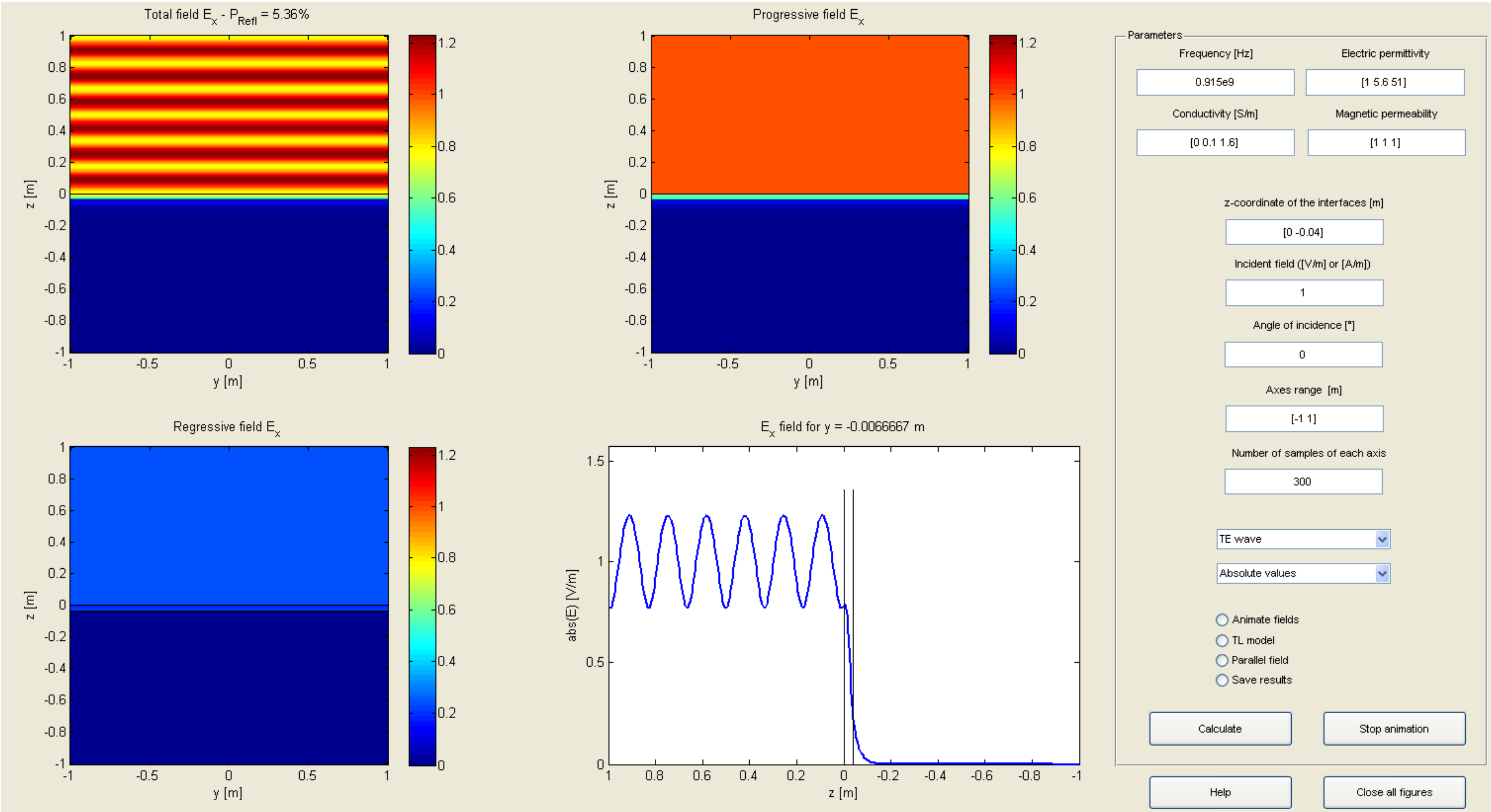
$$|\tau| = \left|\frac{4\zeta_m\zeta_f e^{-jk_f d}}{(\zeta_f + \zeta_0)(\zeta_m + \zeta_f) + (\zeta_f - \zeta_0)(\zeta_m - \zeta_f)e^{-j2k_f d}}\right| \cong 0.194$$

Reflected power back to air: $|\Gamma|^2 \times 100 \cong 6.2\%$

Transmitted power into muscle: $\frac{\mathrm{Re}\left\{\frac{1}{2\zeta_m}\right\}|E^i|^2|\tau|^2}{\frac{1}{2\zeta_0}|E^i|^2} \times 100 = \zeta_0 \,\mathrm{Re}\left\{\frac{1}{\zeta_m}\right\}|\tau|^2 \times 100 = 26.7\%$

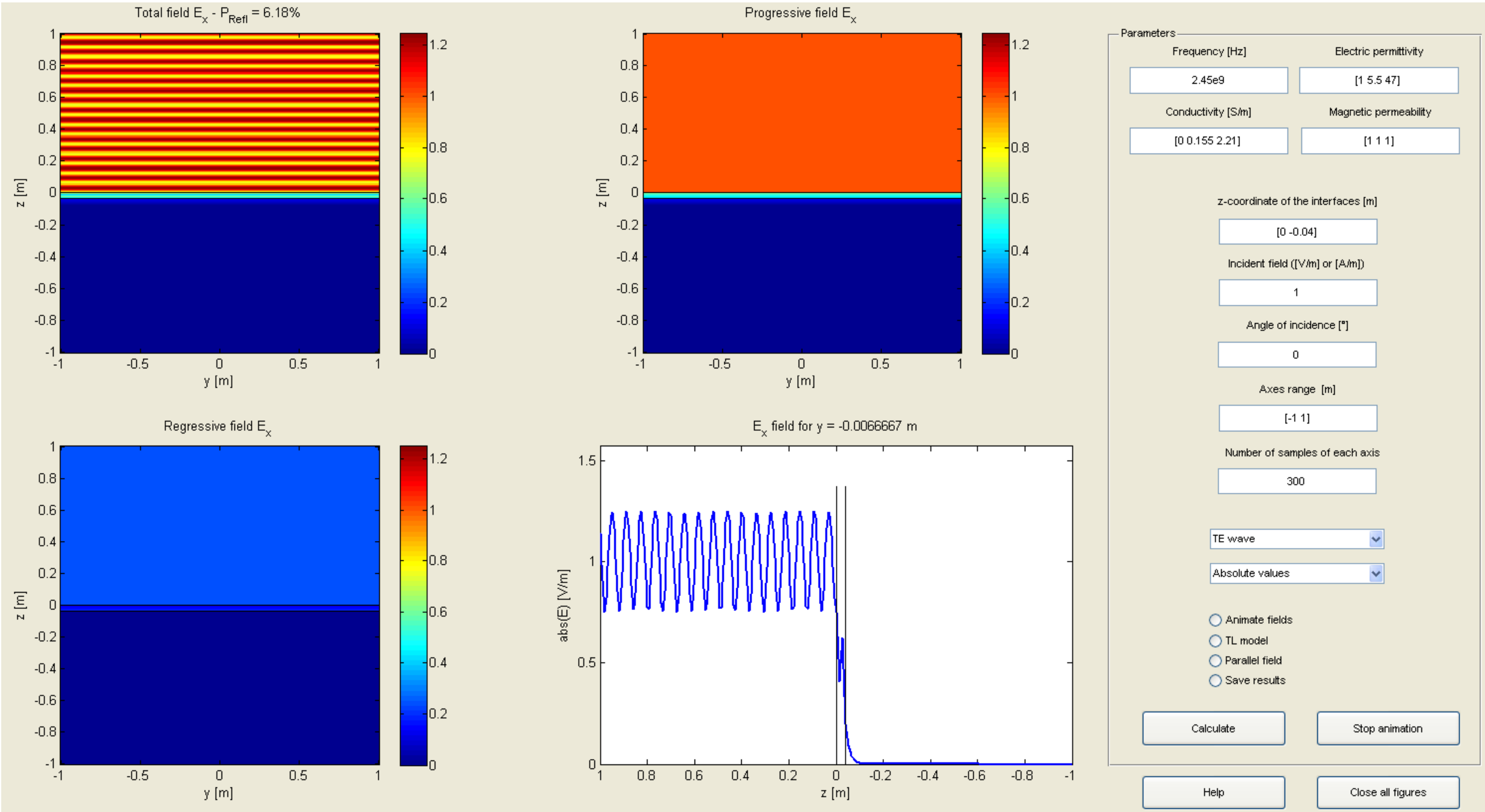
Absorbed power by the fat layer: $\%P_{abs} = 100 - 6.2 - 26.7 \cong 67.1\%$

@915MHz – E-field



http://www.mathworks.com/matlabcentral/fileexchange/16724-gui-for-tetm-electromagnetic-plane-waves-propagation-through-multilayered-structures

@2.45GHz – E-field



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